DOCUMENT RESUME

ED 071 896 SE 015 533

TITLE	Project Physics Teacher Guide 3, The Triumph of Mechanics.
INSTITUTION	Harvard Univ., Cambridge, Mass. Harvard Project Physics.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C. Bureau of Research.
BUREAU NO	BR-5-1038
PUB DATE	68
CONTRACT	OEC-5-10-058
NOTE	358p.; Authorized Interim Version
EDRS PRICE	MF-$0.65 HC-$13.16
DESCRIPTORS	Energy; *Instructional Materials; Kinetic Molecular Theory; Multimedia Instruction; *Physics; Secondary Grades; *Secondary School Science; *Teaching Guides; Teaching Procedures; *Thermodynamics
IDENTIFIERS	Harvard Project Physics

ABSTRACT

Teaching procedures of Project Physics Unit 3 are presented to help teachers make effective use of learning materials. Unit contents are discussed in connection with teaching aid perspective, multi-media schedules, schedule blocks, and resource charts. Brief analyses are made for transparencies, 16mm films, and reader articles. Included is information about the background and development of each unit chapter, demonstration procedures, apparatus operations, notes on the student handbook, and explanation of film loops. Stroboscopic photographs are analyzed along with related film loops. Additional background articles are included to deal with physics aspects at the beginning of the 19th Century, conservation laws, elastic and inelastic collisions, energy reference levels, temperatures of outer space, feedback, atmospheric pressure determination, and photographing of standing waves. Solutions to the study guide in the text are provided, and answers to test items are suggested. The third unit of the text, along with marginal notes on each section, is also compiled in the teacher guide. The work of Harvard Project Physics has been financially supported by: the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education, and Harvard University. (CC)

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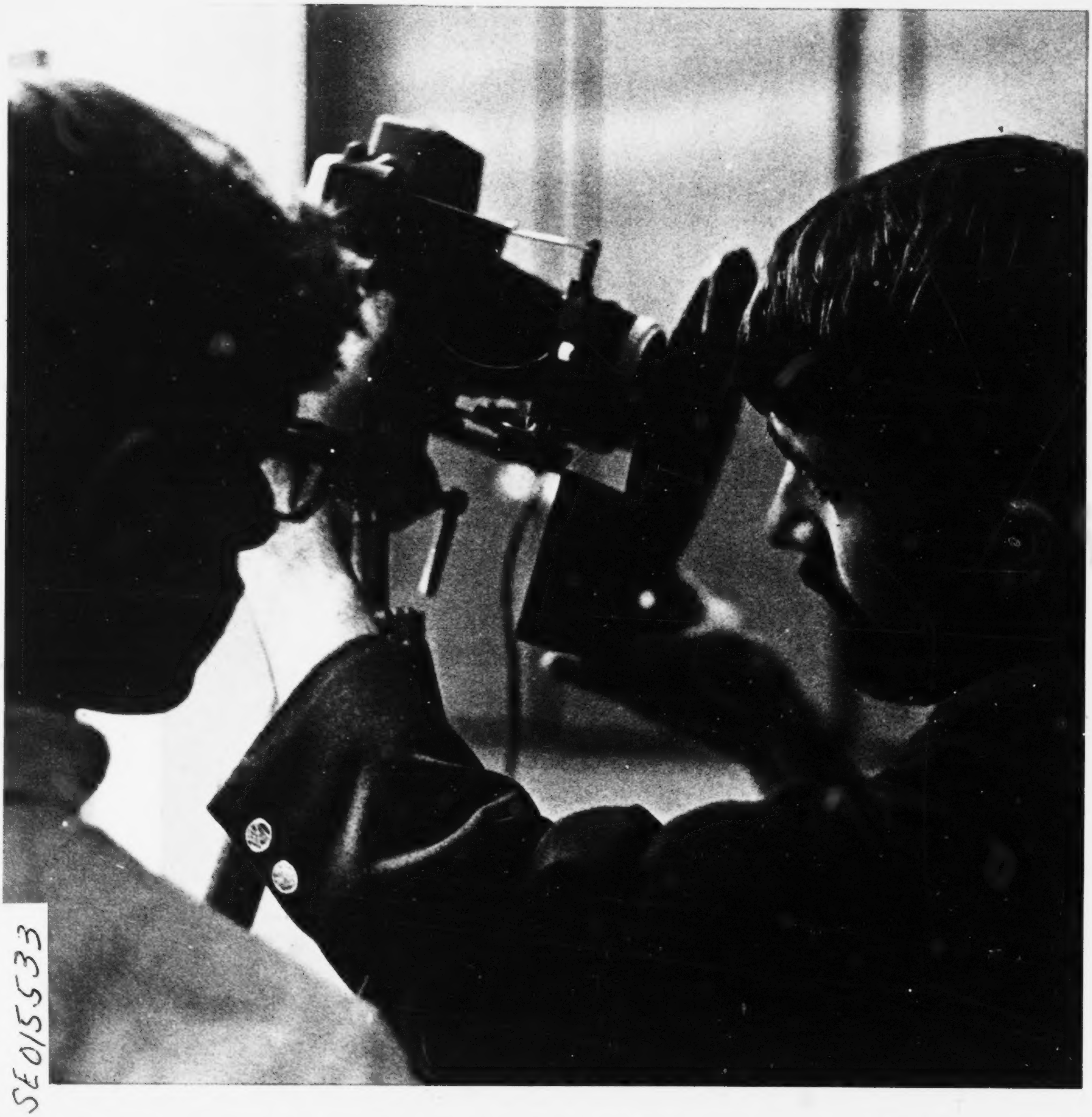
Project Physics **Teacher Guide**

An Introduction to Physics

The Triumph of Mechanics



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Project Physics **Text**

An Introduction to Physics 3 The Triumph of Mechanics

Authorized Interim Version 1968-69

Distributed by **Holt, Rinehart and Winston, Inc.** New York Toronto

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Welcome to the study of physics. This volume, more of a student's guide than a text of the usual kind, is part of a whole group of materials that includes a student handbook, laboratory equipment, films, programmed instruction, readers, transparencies, and so forth. Harvard Project Physics has designed the materials to work together. They have all been tested in classes that supplied results to the Project for use in revisions of earlier versions.

The Project Physics course is the work of about 200 scientists, scholars, and teachers from all parts of the country, responding to a call by the National Science Foundation in 1963 to prepare a new introductory physics course for nationwide use. Harvard Project Physics was established in 1964, on the basis of a two-year feasibility study supported by the Carnegie Corporation. On the previous pages are the names of our colleagues who helped during the last six years in what became an extensive national curriculum development program. Some of them worked on a full-time basis for several years; others were part-time or occasional consultants, contributing to some aspect of the whole course; but all were valued and dedicated collaborators who richly earned the gratitude of everyone who cares about science and the improvement of science teaching.

Harvard Project Physics has received financial support from the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education and Harvard University. In addition, the Project has had the essential support of several hundred participating schools throughout the United States and Canada, who used and tested the course as it went through several successive annual revisions.

The last and largest cycle of testing of all materials is now completed; the final version of the Project Physics course will be published in 1970 by Holt, Rinehart and Winston, Inc., and will incorporate the final revisions and improvements as necessary. To this end we invite our students and instructors to write to us if in practice they too discern ways of improving the course materials.

The Directors
Harvard Project Physics

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Prologue The triumph of Isaac Newton in uniting motion and astronomy is one of the glories of the human mind, a turning point in the development of science and man. Never before had a scientific theory been so successful in predicting the future, and never before had the possibilities for future development in science seemed so unlimited.

So it is not surprising that after his death in 1727 Newton was practically deified, especially in England, by poems such as this one:

> Newton the unparallel'd, whose Name
> No Time will wear out of the Book of Fame,
> Celestial Science has promoted more,
> Than all the Sages that have shone before.
> Nature compell'd his piercing Mind obeys,
> And gladly shows him all her secret Ways;
> Gainst Mathematics she has no defence,
> And yields t'experimental Consequence;
> His tow'ring Genius, from its certain Cause
> Ev'ry Appearance a priori draws
> And shews th'Almight Architect's unalter'd Laws.

(From J. T. Desagulier, The Newtonian System of the World, the Best Model of Government, an Allegorical Poem).

Newton's work not only led others to new heights in science, but altered profoundly man's view of the universe. Physicists after Newton explained the motion of planets around the sun by treating the solar system as a huge machine. Although the parts of the solar system are held together by gravitational forces rather than by nuts and bolts, the motion of these parts relative to each other, according to Newton's theory, is still fixed forever once the system has been put together.

We call this model of the solar system the Newtonian world-machine. It is a theoretical system, not a real one, because the mathematical equations which govern its motions take account of only a few of the properties of the real solar system and leave out others. In particular, the equations take no account of the structure and chemical composition of the planets, or the heat, light, electricity and magnetism which are involved. The Newtonian system takes account only of the masses, positions and velocities of the parts of the system, and the gravitational forces among them.

The idea of a world machine does not derive entirely from Newton. In his Principles of Philosophy, René Descartes, the most influential French philosopher of the seventeenth century, clearly stated the idea that the world is like a machine. He wrote:

> I do not recognize any difference between the machines that artisans make and the different bodies that nature alone composes, unless it be that the effects of the machines depend only upon the adjustment of certain tubes or springs, or other instruments, that, having necessarily some proportion with the hands of those who make them, are always so large that their shapes and motions can be seen, while the tubes and springs that cause the

> effects of natural bodies are ordinarily too small to be perceived by our senses. And it is certain that all the laws of Mechanics belong to Physics, so that all the things that are artificial, are at the same time natural.

"The Ancient of Days" by William Blake, an English poet who had little sympathy with the Newtonian style of "natural philosophy."

Robert Boyle (1627-1691), a British scientist who studied the properties of air (see Chapter 11), expressed the mechanistic viewpoint even in his religious writings. He argued that a God who could design a universe that would run by itself like a machine was more wonderful, and more deserving of human worship, than a God who simply created several different kinds of matter and gave each a natural tendency to behave in the way it does. Boyle also thought it was insulting to God to believe that the world-machine would be so badly designed as to require any further divine intervention after it had once been created. He suggested that an engineer's skill in designing "an elaborate engine" is more deserving of praise if the engine never needs supervision to regulate it or keep it from getting "out of order." "Just so," he continued,

> ...it more sets off the wisdom of God in the fabric of the universe, that he can make so vast a machine perform all those many things, which he designed it should, by the meer contrivance of brute matter managed by certain laws of local motion and upheld by his ordinary and general concourse, than if he employed from time to time an intelligent overseer, such as nature is fancied to be, to regulate, assist, and controul the motions of the parts....

According to Boyle and many other scientists in the seventeenth and eighteenth centuries, God was the first and greatest theoretical physicist. God, they said, set down the laws of matter and motion; human scientists can best glorify the works of God by discovering and proclaiming these laws.

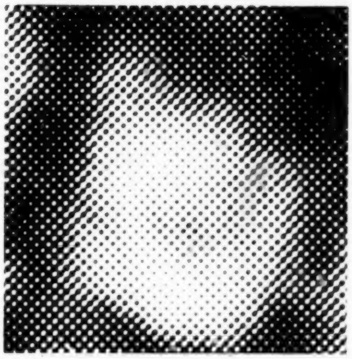
Our main concern in this unit is with physics after Newton. In mechanics the Newtonian theory was developed to accommodate a wider range of concepts. Conservation laws became increasingly important. These powerful principles offered a new way of thinking about Newtonian mechanics, and so were important in the application of Newton's theory to other areas of physics.

Newton's mechanics treats directly only a small range of experiences, those concerning the motion of simple bodies. Will the same theory work when applied to phenomena on earth as well as to those in the heavens? Are solids, liquids and gases really just machines, or mechanical systems, which can be explained by using the same ideas about matter and motion which Newton used in explaining the solar system?

At first sight, it might seem unlikely that everything can be reduced to matter and motion, because we feel, hear, smell and see many things that seem different from matter and motion. What about colors, sounds, odors, hardness and softness, temperature and so forth? Newton himself believed that the mechanical view would be useful in investigating these other properties. In the Preface to the Principia he wrote:

> I wish we could derive the rest of the phenomena of Nature by the same kind of reasoning from mechanical principles, for I am induced by many reasons to suspect that they may all depend upon certain forces by which the particles of bodies, by some causes hitherto unknown, are mutually impelled towards one another, and cohere in regular figures, or are repelled and recede from one another. These forces being unknown, philosophers have hitherto attempted the search of Nature in vain; but I hope the principles here laid down will afford some light either to this or some truer method of Philosophy.

Knowing the laws of motion, scientists after Newton strove to apply them in many different areas. We shall see in this unit how successful the Newtonians were in explaining the physical world.



A small area from the center of the picture has been enlarged to show what the picture is "really" like. Is the picture only a collection of dots? Knowing the underlying structure doesn't spoil our other reactions to the picture, but rather gives us another dimension of understanding it.

The Cultural Impact of the Newtonian Viewpoint

The century following the death of Newton in 1727 was a period of consolidation and further application of Newton's discoveries and methods. The effects of these discoveries and methods were felt also in fields outside science.

During the 1700's, the so-called Age of Reason or Century of Enlightenment, the viewpoint which we have previously called the Newtonian cosmology became firmly entrenched in European science and philosophy. The impact of Newton's achievements may be summarized thus: he had shown that man, by observing and reasoning, could uncover the workings of the physical universe. Therefore, his followers argued, man should be able to understand not only nature but also society and the human mind by the same method. As the French science writer Fontenelle (1657-1757) expressed it:

> The geometric spirit is not so bound up with geometry that it cannot be disentangled and carried into other fields. A work of morals, or politics, of criticism, perhaps even of eloquence, will be the finer, other things being equal, if it is written by the hand of a geometer.

The English philosopher John Locke (1632-1704) reinforced Newton's influence; he argued that the goal of philosophy should be to solve problems that affect our daily life, and to deal with things that we can actually know about by observation and reasoning. "Reason must be our best judge and guide in all things," he said. Locke thought that the concept of "natural law" could be used in religion as well as in physics, and the notion of a non-emotional, non-fanatical religion appealed to many Europeans who hoped to avoid a revival of the bitter religious wars of the 1600's.

Locke advanced the theory that the mind of the new-born baby contains no "innate ideas;" it is like a blank piece of paper on which anything may be written. If this is true, then it is futile to search within ourselves for a God-given sense of what is true or morally right. Instead, we must look at nature and society to discover there any "natural laws" that may exist. Conversely, if we want to improve the quality of man's mind, we must improve the society in which he lives.

Locke's view also seems to imply an "atomistic" structure of society: each person is separate from other individuals in the sense that he has no "organic" relation to them. Previously, political theories had been based on the idea of society as an organism in which each person has a prescribed place and function. Later theories based on Locke's ideas asserted that government should have no function except to protect the freedom and property of the individual person.

Although "reason" was the catchword of the eighteenth-century philosophers, we do not have to accept their own judgment that their theories about improving religion and society were necessarily the most reasonable. It might be more accurate to say that men like Voltaire approached a subject with strong opinions, and then tried to use rational arguments to justify those opinions; but these men would not give up a doctrine such as the equal rights of all men merely because they could not find a strictly mathematical or scientific proof for it. Newtonian physics, religious toleration and republican government were all advanced by the same movement, but this does not mean there is really a logical connection among them.

The dominant theme of the 1700's was moderation—the happy medium, based on toleration of different opinions, restraint of excess in any direction, and balance of opposing forces. Even reason was not allowed to ride roughshod over religious faith; atheism, which some philosophers thought to be the logical consequence of unlimited rationality, was still regarded with horror by most Europeans. The Constitution of the United States of America, with its ingenious system of "checks and balances" to prevent any faction from getting too much power, is perhaps the most enduring achievement of this period. It attempts to establish in politics a stable equilibrium of opposing forces similar to the balance between the sun's gravitational attraction and the tendency of a planet to fly off in a straight line. If the gravitational attraction increased without a corresponding increase in planetary velocity, the planet would fall into the sun; if its velocity increased without a corresponding increase in gravitational attraction, the planet would escape from the solar system, and its inhabitants would soon freeze to death. Just as Newtonian mechanics avoided the extremes of hot and cold by keeping the earth at the right distance from the sun, so the political philosophers hoped to avoid the extremes of dictatorship and anarchy by devising a system of government that was neither too strong nor too weak.

According to James Wilson (1742-1798), who played a major role in drafting the American Constitution,

> In government, the perfection of the whole depends on the balance of the parts, and the balance of the parts consists in the independent exercise of their separate powers, and, when their powers are separately exercised, then in their mutual influence and operation on one another. Each part acts and is acted upon, supports and is supported, regulates and is regulated by the rest.
>
> It might be supposed, that these powers, thus mutually checked and controlled, would remain in a state of inaction. But there is a necessity for movement in human affairs; and these powers are forced to move, though still to move in concert. They move, indeed, in a line of direction somewhat different from that, which each acting by itself would have taken; but, at the same time, in a line partaking of the natural directions of the whole—the true line of public liberty and happiness.

In literature many men welcomed the new viewpoint as a source of novel metaphors, allusions and concepts which they could use in their poems and essays. Newton's discovery that white light is composed of colors was reflected in many poems of the 1700's (see Unit 4). Samuel Johnson advocated the literary use of words drawn from the natural sciences, defining many such words in his Dictionary and illustrating their application in his "Rambler" essays.

Other writers, such as Pope and Swift, distrusted the new cosmology and so used it for purposes of satire. In his epic poem The Rape of the Lock, Pope exaggerates the new scientific vocabulary for comic effect. Swift, sending Gulliver on his travels to Laputa, describes an academy of scientists and mathematicians whose experiments and theories were as absurd as those of the Fellows of the Royal Society must have seemed to the layman of the 1700's.

The first really powerful reaction against Newtonian cosmology was the Romantic movement. This movement was started in Germany around 1780 by young writers inspired by Johann Wolfgang von Goethe. The most familiar examples of

Romanticism in English literature are the poems and novels of Blake, Coleridge, Wordsworth, Shelley, Byron and Scott.

The Romantics disliked the notion that everything should be measured by the use of numbers; they emphasized the importance of quality rather than quantity. They preferred to study the individual, unique person or experience, rather than make abstractions and generalizations. They exalted emotion and feeling over reason and calculation. They abhorred the theory that the universe is like a piece of clockwork, made of inert matter set into motion by a God who can never afterwards show His presence. As the historian and philosopher of science, E. A. Burtt said,

> ...it was of the greatest consequence for succeeding thought that now the great Newton's authority was squarely behind that view of the cosmos which saw in man a puny, irrelevant spectator (so far as a being wholly imprisoned in a dark room can be called such) of the vast mathematical system whose regular motions according to mechanical principles constituted the world of nature. The gloriously romantic universe of Dante and Milton, that set no bounds to the imagination of man as it played over space and time, had now been swept away. Space was identified with the realm of geometry, time with the continuity of number. The world that people had thought themselves living in—a world rich with colour and sound, redolent with fragrance, filled with gladness, love and beauty, speaking everywhere of purposive harmony and creative ideals—was crowded now into minute corners in the brains of scattered organic beings. The really important world outside was a world hard, cold, colourless, silent, and dead; a world of quantity, a world of mathematically computable motions in mechanical regularity. The world of qualities as immediately perceived by man became just a curious and quite minor effect of that infinite machine beyond.

The Romantics insisted that phenomena should not be analyzed and reduced to their separate parts by mechanistic explanations; instead, they argued that the whole is greater than the sum of its parts, because the whole (whether it be a single human being or the entire universe) is pervaded by a spirit that cannot be rationally explained but can only be intuitively felt.

Continental leaders of the Romantic movement, such as the German philosopher Friedrich Schelling (1775-1854), did not want to abolish scientific research. They did want to change the way research was being done, and they proposed a new type of science called "Nature Philosophy." It is important to distinguish between this nature philosophy and the older "natural philosophy," meaning just physics. The nature philosopher does not analyze phenomena into separate parts or factors which he can measure quantitatively in his laboratory—or at least that is not his primary purpose. Instead, he tries to understand the phenomenon as a whole, and looks for underlying basic principles that govern all phenomena. The Romantic philosophers in Germany regarded Goethe as their greatest scientist as well as their greatest poet, and they pointed with pride to his theory of color, which flatly contradicted Newton's theory. According to Goethe, white light does not consist of a mixture of colors; the colors are produced by the prism acting on the light which is itself pure. In the judgment of all modern physicists, Newton was right and Goethe was wrong, so nature philosophy seems to be a failure if judged by this very important example. However, this is not the whole story.

The general tendency of nature philosophy was speculative, and insofar as it encouraged speculation about ideas which could never be testable by experiment, nature philosophy was strongly condemned by scientists. Indeed, the reaction against nature philosophy was so strong that several scientific discoveries did not receive proper recognition when first announced because they were described in language "contaminated" by philosophical speculation. Among these discoveries was the generalized principle of conservation of energy, which is described in Chapter 10. It is now generally agreed by historians of science that nature philosophy played an important role in the historical origins of this discovery. This is perhaps not surprising, since the principle of conservation of energy confirms the viewpoint of nature philosophy by asserting that all the "forces of nature"—heat, gravity, electricity, magnetism and so forth—are really just different forms or manifestations of one underlying "force" which we now call energy.

It is impossible to say whether the viewpoint of nature philosophy was good or bad for science; it has often merely encouraged futile speculation and disregard for precise measurements, but in one or two very important cases it has led to important discoveries.

Much of the dislike which Romantics (and some modern artists and intellectuals) expressed for science was based on the notion that scientists claimed to be able to find a mechanistic explanation for everything, including the human mind. If everything is explained by Newtonian science, then everything is also determined. Many modern scientists no longer believe this, but scientists in the past have made statements of this kind. For example, the French mathematical physicist Laplace (1749-1827) said:

> We ought then to regard the present state of the universe as the effect of its previous state and as the cause of the one which is to follow. Given for one instant a mind which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—a mind sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.

Even the ancient Roman philosopher Lucretius (100-55 B.C.), who supported the atomic theory in his poem On the Nature of Things, did not go as far as this. In order to preserve some vestige of "free will" in the universe, Lucretius suggested that the atoms might swerve randomly in their paths. This was still unsatisfactory to a Romantic scientist like Erasmus Darwin (grandfather of evolutionist Charles Darwin) who asked:

> Dull atheist, could a giddy dance
>   Of atoms lawless hurl'd
> Construct so wonderful, so wise,
>   So harmonised a world?

The nature philosophers thought they could discredit the Newtonian scientists by forcing them to answer this question; to say "yes," they argued, would be absurd, and to say "no" would be disloyal to their own supposed beliefs. We shall see in this Unit how successful the Newtonians were in explaining the physical world without committing themselves to any definite answer to Erasmus Darwin's question. Instead, they were led to the discovery of immensely powerful and fruitful laws of nature.

Chapter 9 Conservation of Mass and Momentum

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CANADA
5
SABA
RADIO

9.1 Conservation of mass. If the universe is to go on forever, then the stuff of which it is made cannot disappear. That the total amount of material in the universe does not change is really an old idea. The Roman poet Lucretius (first century B.C.) restated a belief held in Greece as early as the fifth century B.C.:

> ...and no force can change the sum of things; for there is no thing outside, either into which any kind of matter can emerge out of the universe or out of which a new supply can arise and burst into the universe and change all the nature of things and alter their motions.
>
> [On the Nature of Things]

Summary 9.1
The idea that the "amount of stuff" in the universe is constant is an old one, but not until the 18th century, when Lavoisier weighed the contents of a closed flask before and after a chemical reaction, did the idea assume a quantitative form as the law of conservation of mass.

Just twenty-two years before Newton's birth, the English philosopher Francis Bacon included the following among his basic principles of modern science:

> There is nothing more true in nature than the twin propositions that "nothing is produced from nothing" and "nothing is reduced to nothing"...the sum total of matter remains unchanged, without increase or diminution.
>
> [Novum Organon (1620), ii, 40]

These quotations illustrate the belief that the amount of physical matter that makes up the universe remains the same—no new matter appears and no old matter disappears. While the form in which matter exists may change, matter in all our ordinary experience appears to be somehow indestructible: if we break up a large boulder into dust and pebbles, we do not change the amount of stone in the universe.

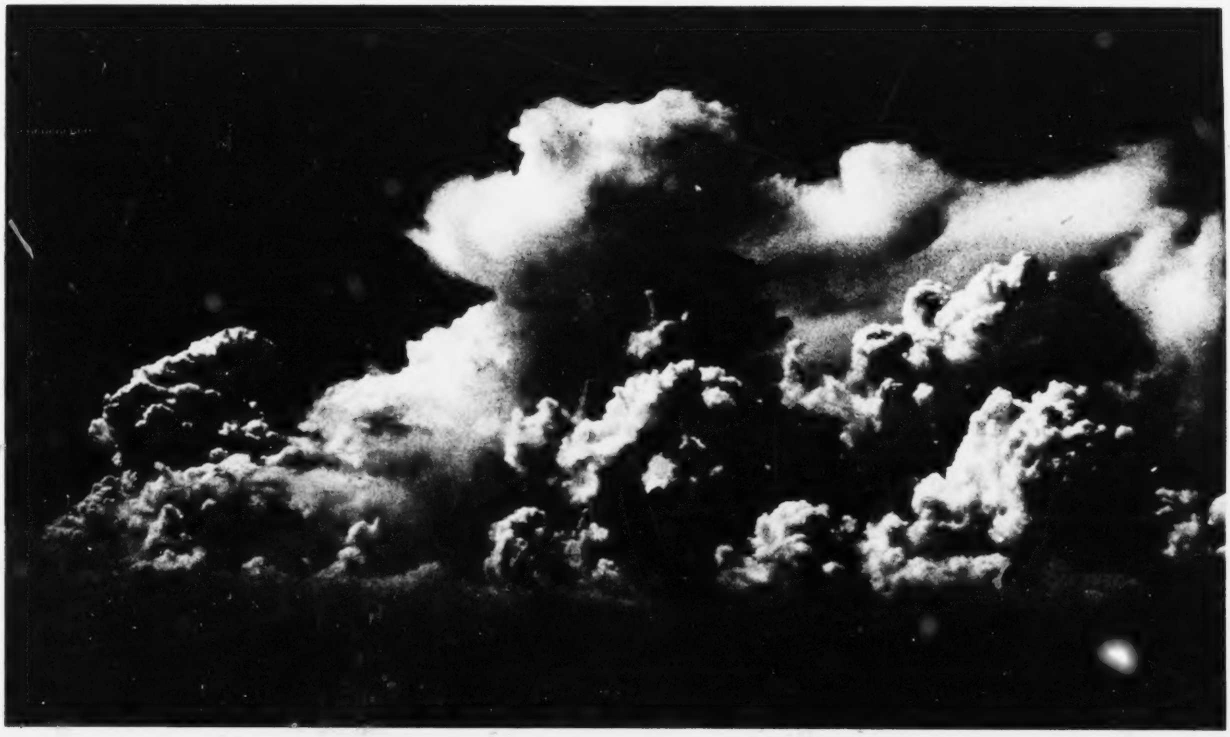
To test the belief that the quantity of matter remains constant, we need to know how to measure that quantity. Scientists recognized several centuries ago that it should not be measured by its volume. For example, if we put water in a container, mark the water level, and then freeze the water, we find that the volume of the ice is larger than the volume of the water we started with. This is true even if we are careful to seal the container so that no water can possibly come in from the outside. Similarly when we compress some gas in a closed container, the volume of the gas decreases, even though none of the gas escapes from the container.

Following Newton, we have come to regard the mass of an object as the appropriate measure of the amount of matter it contains. In our use of Newtonian theory we have been assuming that the mass of an object does not change. But what if we burn the object to ashes or dissolve it in acid? Does its mass remain unchanged even in such chemical reactions?

A sample of gas undergoing a decrease in volume and increase in pressure. (This was one of the earliest high-speed flash photographs.)

A burnt match has a smaller mass than an unburnt one; an iron nail as it rusts increases in mass. Has the mass of these things really changed? Or does something escape from

F17: Elements, compounds, and mixtures.
F18: The perfection of matter
A: Is mass conserved?



In all change, violent or gradual,
the total mass remains constant.

F: Elements, compounds and mixtures.

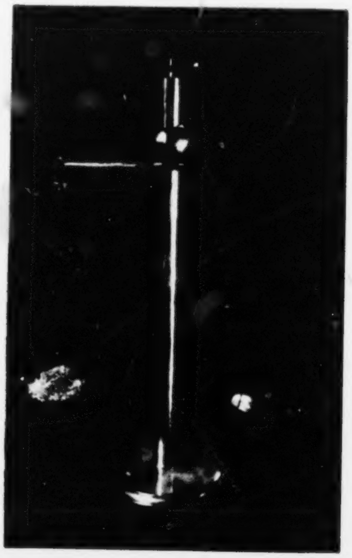
the match, and is something added to the iron of the nail, that will account for the changes in mass? In the eighteenth century there was a strong faith that any changes of mass in chemical reactions could be accounted for by assuming that there is something that escapes or something that enters from outside. Not until the end of the eighteenth century, however, was a sound experimental basis for this faith provided—by Antoine Lavoisier (1743-1794).

Lavoisier caused chemical reactions to occur in closed flasks and carefully weighed the flasks and their contents before and after the reaction. For example, he showed that when iron was burned in a closed flask, the mass of the iron oxide produced was equal to the sum of the masses of the iron and the oxygen used in the reaction. With experimental evidence like this at hand he was able to announce with confidence:

> We may lay it down as an incontestable axiom that in all the operations of art and nature, nothing is created; an equal quantity of matter exists both before and after the experiment,...and nothing takes place beyond changes and modifications in the combinations of these elements. Upon this principle, the whole art of performing chemical experiments depends.
>
> [Traite Elémentaire de Chimie (1789)]

Lavoisier was convinced that if he put some material in a well-sealed bottle and measured its mass, then he could return at any later time and find the same mass regardless of what happened to the material inside the bottle during the interval. Despite changes from solid to liquid or liquid to gas, etc., despite changes of color or consistency, despite even violent chemical reactions of the material inside the bottle, at least one thing would remain unchanged—the mass of what is in the bottle.

In the years after Lavoisier's pioneering work, a vast number of similar experiments were performed with ever increasing accuracy and always with the same result. As far as can be measured with sensitive balances (having an accuracy of better than 0.000001%), mass is conserved in chemical reactions, even when light and heat are allowed to enter or leave the system. Despite changes in location, shape, chemical composition and so forth, the mass of any closed system remains constant. This is the statement of what we shall call the law of conservation of mass.



In some open-air reactions mass seems to decrease, while in others it seems to increase.

Note that although a chemist's sealed glass flask prevents matter from entering or leaving, it does not prevent light and heat from entering or leaving.

The contents of a closed bottle constitute a system. The system is closed or isolated from its surroundings.

A compact way to say that the mass of a system does not change is:

$$\Sigma m = \text{constant}.$$

(The symbol Σ is the Greek letter sigma. It stands for the sum.)

The meaning of the phrase "closed system" is discussed in more detail in Sec. 9.5.

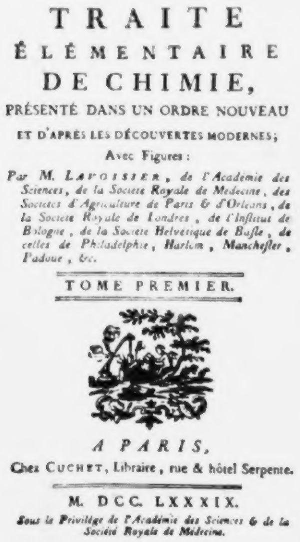
Q1 If 50 cc of alcohol is mixed with 50 cc of water, the mixture amounts to only 98 cc. Is this a contradiction of the law of conservation of mass?

Q2 It is estimated that every year at least 2000 tons of meteoric dust fall on to the earth. The dust is mostly debris

Try these end-of-section questions before going on. Answers are at the very end of the book.

Have students discuss "closed" systems.

he idea of conservation doesn't require the idea of a closed system. If track can be kept of how much of something enters and leaves a system, the conservation of that something can be determined.

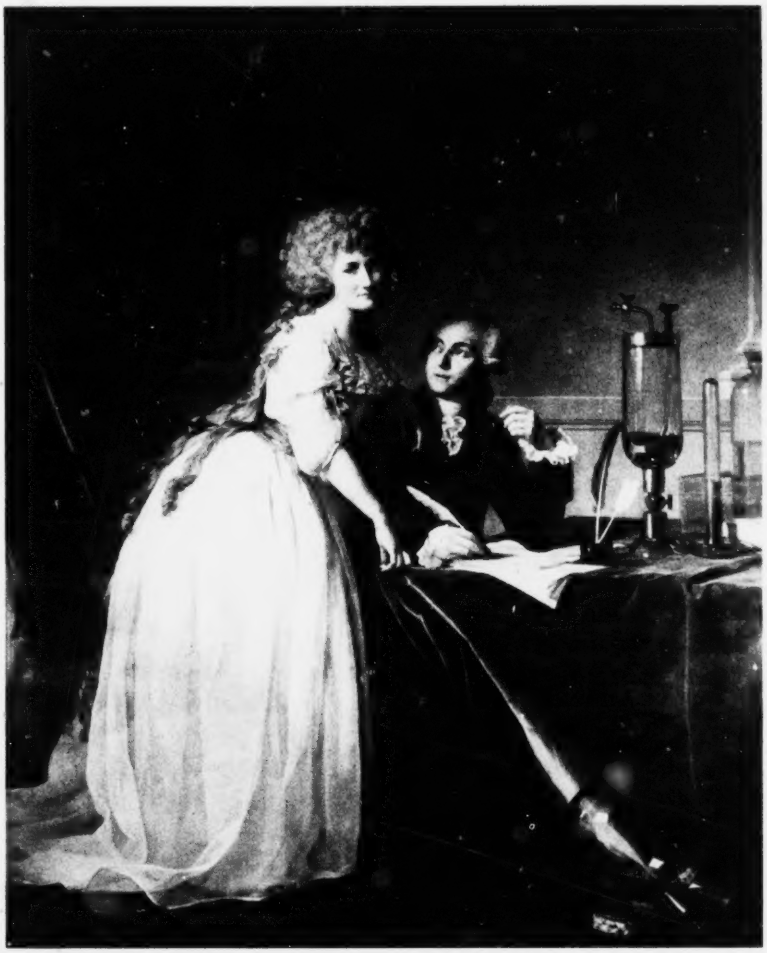


TRAITE
ÉLÉMENTAIRE
DE CHIMIE,
PRÉSENTÉ DANS UN ORDRE NOUVEAU
ET D'APRÈS LES DÉCOUVERTES MODERNES;
Avec Figures:
Par M. Lavoisier, de l'Académie des Sciences, de la Societe Royale de Medecine, des Societes d'Agriculture de Paris & d'Orleans, de la Societe Royale de Londres, de l'Inſtitut de Bologne, de la Societe Helvetique de Baſle, de celles de Philadelphie, Harlem, Mancheſter, Padoue, &c.

TOME PREMIER.

A PARIS,
Chez Cuchet, Libraire, rue & hôtel Serpente.

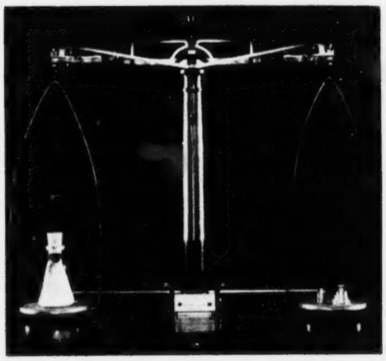
M. DCC. LXXXIX.
Sous le Privilège de l'Académie des Sciences & de la Société Royale de Médecine.



Antoine Laurent Lavoisier (1743-1794) is known as the "father of modern chemistry" because he showed the decisive importance of quantitative measurements, established the principle of conservation of mass in chemical reactions, and helped develop the present system of nomenclature for the chemical elements. He also measured the amount of heat produced by animals, and showed that organic processes such as digestion and respiration are similar to combustion.

To earn money for his scientific research, Lavoisier invested in a private company which collected taxes for the French government. Because the tax collectors were allowed to keep any extra tax which they could get from the public, they acquired a reputation for cheating and became one of the most hated groups in France. Lavoisier was not directly engaged in tax collecting, but he had married the daughter of an important executive of the company, and his association with the company was one of the reasons why Lavoisier was guillotined during the French Revolution.

Madame Lavoisier, who was only fourteen at the time of her marriage, was both beautiful and intelligent. She gave great assistance to her husband by taking notes, translating scientific works from English into French, and making illustrations. About ten years after her husband's execution, she married another scientist, Count Rumford, who is remembered for his experiments which cast doubt on the caloric theory of heat.



Conservation of mass was demonstrated in experiments on chemical reactions in closed flasks. Precision equal-arm balances were used by Lavoisier and other chemists of the time.

This isn't Lavoisier's balance – it was the oldest we could get a photo of.

Students should do some lab work (at least qualitative playing around) with collisions before reading Sec. 9.2.

that was moving in orbits around the sun.

a) Is the earth a closed system as regards the law of conservation of mass?

b) How large would the system including the earth have to be in order to be considered very, very nearly closed?

c) The mass of the earth is about 6 x 10^{21} tons. Do you want to reconsider your answer to part (a)?

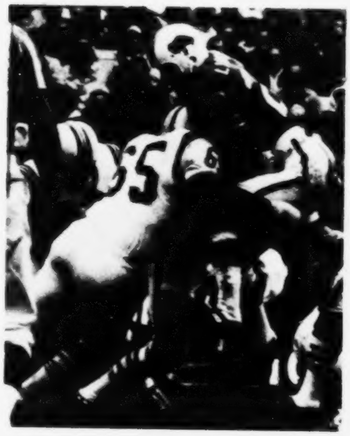
Q3 Which one of the following statements is true?

(a) Lavoisier was the first person to believe that the amount of material stuff in the universe did not change.

(b) Mass is destroyed when heat enters a system.

(c) A closed system was necessary to establish the law of conservation of mass.

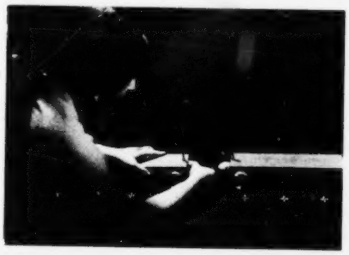
(d) The change in mass of a closed system is constant.



T19: One-dimensional collisions
L18: One-dimensional collisions
L19: Further example of one-dimensional collisions

9.2 Collisions. Looking at moving things in the world around us very easily leads to the conclusion that everything set in motion eventually stops; that every clock, every machine eventually runs down. It would appear that the amount of motion in the universe is decreasing, that the universe, like any machine, is running down.

To seventeenth-century philosophers the idea of a universe that was running down was incompatible with the idea of the perfection of God; surely He would not construct such an imperfect mechanism. It was felt that if the right way could be found to define and measure motion, the quantity of motion in the universe would prove to be constant.

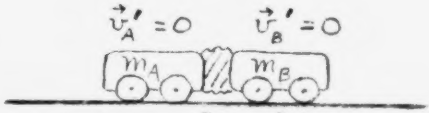


L20: Perfectly inelastic one-dimensional collisions

What is that "right" way to define and measure motion? In what way is motion sustained? What principle is at work which keeps the world machine going? To answer these questions we can look at the results of experiment. In our first experiment, we use two carts on "frictionless" wheels (or better, two dry-ice pucks or two air-track gliders) to which lumps of putty have been attached so that the carts will stick together when they collide. As you can see when you do the experiment, if the masses of the two carts are equal and if the carts are made to approach each other with equal speeds and collide head-on, they stop after the collision: their motion ceases. Is there anything related to the motion which does not change? Indeed there is. If we add the velocity $\vec{v}_A$ of one cart to the velocity $\vec{v}_B$ of the other cart, we find that the vector sum does not change. The vector sum of the velocities of the carts is zero before the collision and it is zero after the collision.

$\vec{v}_A$
m_A
$\vec{v}_B$
m_B

Before: $\vec{v}_A + \vec{v}_B = 0$



After: $\vec{v}_A' + \vec{v}_B' = 0$

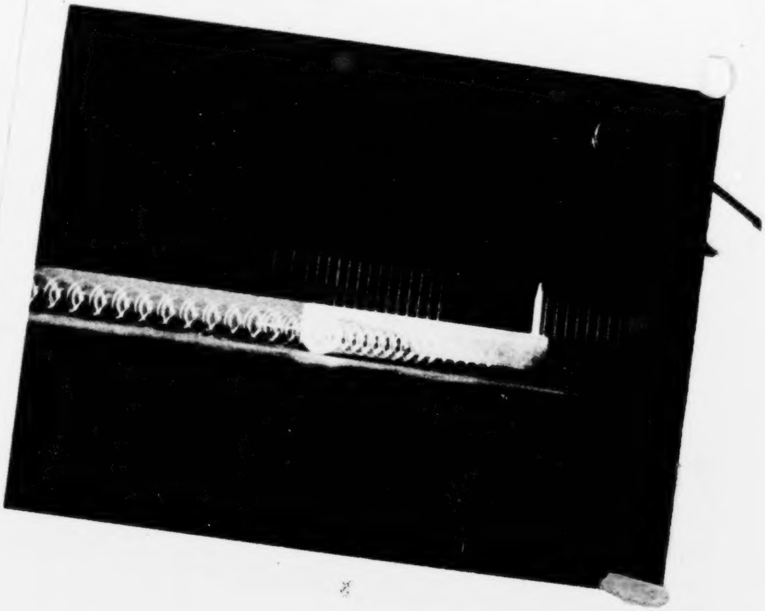
We might wonder whether this "conservation of velocity" holds for all collisions; for we have chosen

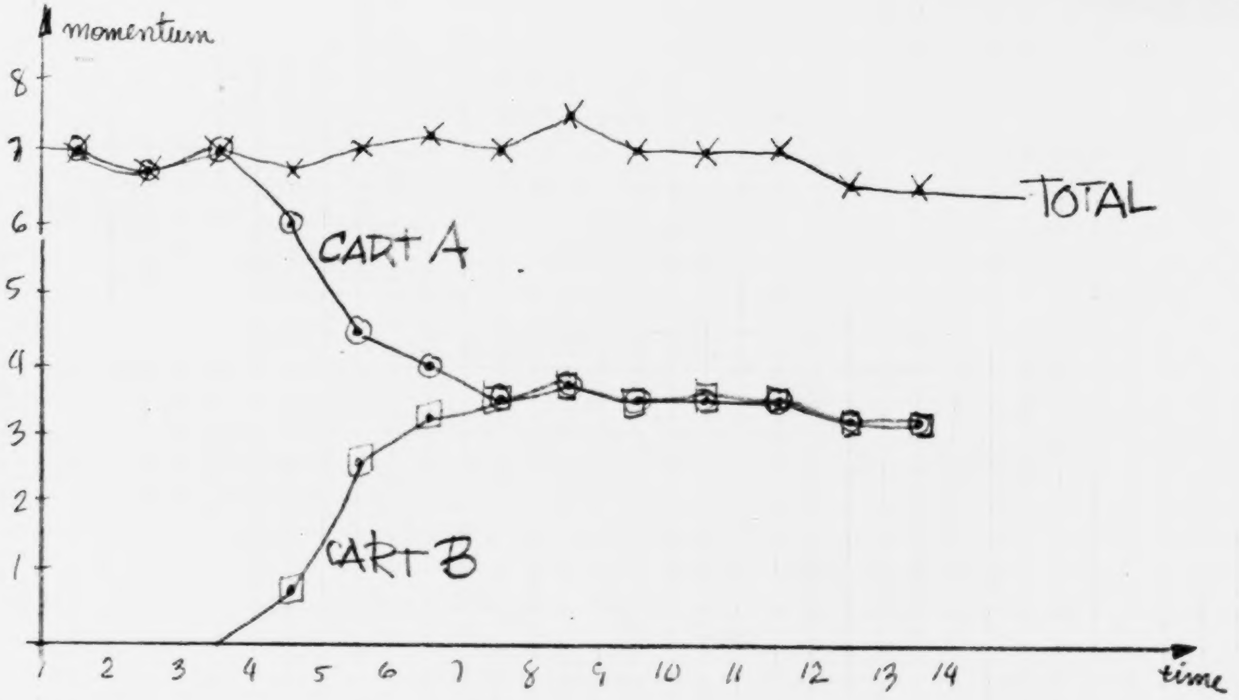
Summary 9.2. The observation that most things eventually stop suggests that the "amount of motion" in the universe is decreasing, an idea repugnant to 17th century philosophers like Descartes, who believed that if "quantity of motion" were properly defined, it would be found to be constant. By considering inelastic collision between two carts, we are led to the discovery that the vector sum of the products of mass and velocity is the same before and after the collision, and therefore may be a definition of "quantity of motion" which vindicates the metaphysical belief that the amount of motion in the universe remains constant.

Inelastic Collision Experiment

Carol Johnson
Tommy Foster
October 26

picture #	Separation cart A	cart B	Total
1			
2	7	–	7
3	6¾	–	6¾
4	7	–	7
5	6	3/4	6¾
6	4½	2½	7
7	4	3¼	7¼
8	3½	3½	7
9	3¾	3¾	7½
10	3½	3½	7
11	3½	3½	7
12	3½	3½	7
13	3¼	3¼	6½
14	3¼	3¼	6½





In this experiment we measured the speeds of two carts that had an inelastic collision, and saw if the total momentum was conserved. We measured speed by measuring the separtion of the pictures of the pencils on the carts. the masses of the carts were 1,057 gm and 1,063 gm. Thay are equal for the accuracy of our ex-

A: Stroboscopic photographs of one-dimensional collisions.

9.2

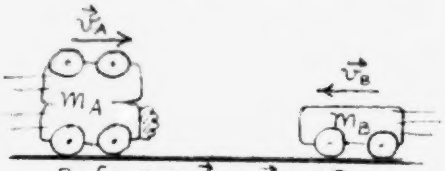
a very special circumstance in the example above: carts with equal masses approaching each other with equal speeds. Suppose we get away from this special situation by making the mass of one of the carts twice the mass of the other cart. We can do this conveniently by putting on top of one cart another cart just like the others. Then let the carts approach each other with equal speeds and collide, as before. This time the carts do not come to rest. What happens, rather, is that there is some motion remaining, and it is in the direction of the initial velocity of the more massive cart. Our earlier guess that the vector sum of the velocities might be conserved in all collisions seems to be wrong. Another example of a collision will confirm this conclusion.

This time let the first cart have twice as much mass as the second, and let the second cart have twice as much speed as the first. When the carts collide head-on and stick together, we find that they stop. The vector sum of the velocities is equal to zero after the collision, but is not equal to zero before the collision.

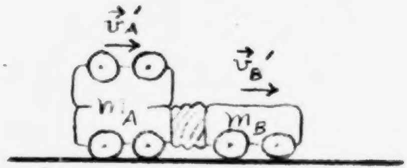
It appears that the definition of quantity of motion must involve the mass of a body as well as its speed if the quantity of motion is to be the same before and after the collision. Descartes had suggested that the proper measure of a body's quantity of motion was the product of its mass and its speed. The examples above, however, show that this is not a conserved quantity. In the first and third collisions, for example, it does not equal zero for either cart before the collision, but after the collision it equals zero for each of them.

But with one very important modification of Descartes' definition of quantity of motion, we can obtain a conserved quantity. Instead of defining "quantity of motion" as the product of mass and speed, we define it, as Newton did, as the product of mass and velocity. On the next page the momentum equations are worked out for the three collisions we have considered. The conclusion is that in all three cases where we have considered head-on collisions between carts, the motion of the two carts before and after the collision is described by this equation:

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A{}' + m_B\vec{v}_B{}' \qquad (9.1)$$

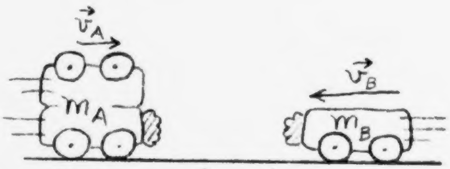


Before : $\vec{v}_A + \vec{v}_B = 0$

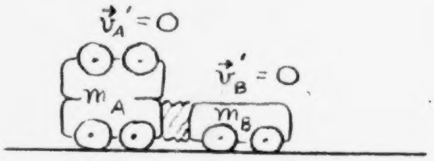


After : $\vec{v}_A{}' + \vec{v}_B{}' \neq 0$

The speed of the stuck-together carts after the collision is one-third the speed of either cart before the collision.



Before : $\vec{v}_A + \vec{v}_B \neq 0$



After : $\vec{v}_A{}' + \vec{v}_B{}' = 0$

A useful demonstration here would be to use momentum carts rolling off inclines. To double the mass of one cart, add another cart. Double mass makes speed increase by $\sqrt{2}$.

Analyses of Three Collisions

In Section 9.2 we discussed three examples of collision between two carts. In each case the carts approached each other head-on, collided, and stuck together. We will show here that in each collision the motion of the carts before and after the collision is described by a general equation namely,

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B \qquad (9.1)$$

where m_A and m_B are the masses of the carts, and $\vec{v}_A$ and $\vec{v}_B$ are their velocities before the collision, and $\vec{v}'_A$ and $\vec{v}'_B$ are their velocities after the collision.

Example 1:

Carts with equal masses and equal speeds (but opposite directions of motion) before the collision. The speed of the stuck-together carts after the collision is zero.

In symbols:

$$m_A = m_B,$$
$$\vec{v}_B = -\vec{v}_A,$$
$$\vec{v}'_A = \vec{v}'_B = 0$$

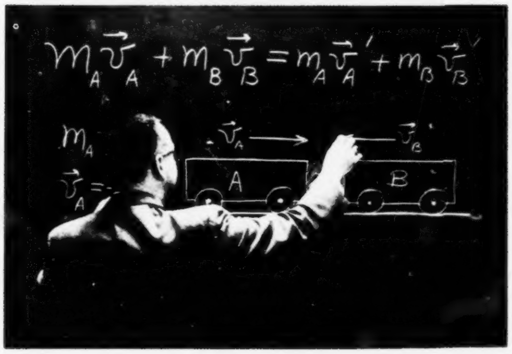
Before the collisions, the sum of the values of mv is given by:

$$\begin{aligned} m_A\vec{v}_A + m_B\vec{v}_B &= m_A\vec{v}_A + m_B(-\vec{v}_A) \\ &= m_A\vec{v}_A - m_B\vec{v}_A \\ &= (m_A - m_B)\vec{v}_A. \end{aligned}$$

Since $m_A = m_B$, the vector sum equals zero. After the collision, the sum is given by:

$$m_A\vec{v}'_A + m_B\vec{v}'_B = m_A(0) + m_B(0) = 0.$$

Thus both before and after the collision, the vector sum of the products of mass and velocity has the same value: zero. That is just what Eq. (9.1) demands; hence it does describe the collision correctly.



$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A' + m_B\vec{v}_B'$
m_A
$\vec{v}_A =$
$\vec{v}_A$
$\vec{v}_B$
A
B

Example 2:

Carts with equal speeds before the collision. The mass of one cart is twice that of the other. The velocity of the stuck-together carts after the collision is found to be 1/3 the original velocity of the more massive cart.

In symbols:

$$m_A = 2m_B,$$
$$\vec{v}_B = -\vec{v}_A,$$
$$\vec{v}'_A = \vec{v}'_B = 1/3\vec{v}_A$$

Before the collision:

$$\begin{aligned} m_A\vec{v}_A + m_B\vec{v}_B &= (2m_B)\vec{v}_A + m_B(-\vec{v}_A) \\ &= m_B\vec{v}_A. \end{aligned}$$

After the collision:

$$\begin{aligned} m_A\vec{v}'_A + m_B\vec{v}'_B &= (2m_B)\frac{\vec{v}_A}{3} + m_B\frac{\vec{v}_A}{3} \\ &= m_B\vec{v}_A \end{aligned}$$

Again Eq. (9.1) describes the collision correctly, since the sum of $m\vec{v}$ is the same before and after the collision.

Example 3:

The mass of one cart is twice that of the other. The speed of the less massive truck is twice that of the more massive truck before the collision. The speed of the stuck-together carts after the collision is found to be zero.

In symbols:

$$m_A = 2m_B,$$
$$\vec{v}_B = -2\vec{v}_A,$$
$$\vec{v}'_A = \vec{v}'_B = 0$$

Before the collision:

$$\begin{aligned} m_A\vec{v}_A + m_B\vec{v}_B &= (2m_B)\vec{v}_A + m_B(-2\vec{v}_A) \\ &= 0 \end{aligned}$$

After the collision:

$$\begin{aligned} m_A\vec{v}'_A + m_B\vec{v}'_B &= (2m_B)(0) + m_B(0) \\ &= 0 \end{aligned}$$

Thus Eq. (9.1) can be used to describe this collision also.

where m_A and m_B are the masses of the carts, $\vec{v}_A$ and $\vec{v}_B$ are their velocities before the collision and $\vec{v}_A{}'$ and $\vec{v}_B{}'$ are their velocities after the collision.

Q4 Why is each of the following not a good measure of quantity of motion?

a) speed
b) velocity
c) the product of mass and speed

Q5 Two carts collide head-on and stick together. In which of the following cases will the carts be at rest after the collision?

	Cart A		Cart B	
	Mass	Speed	Mass	Speed
a)	2 kg	3 m/sec	2 kg	3 m/sec
b)	2 kg	2 m/sec	3 kg	3 m/sec
c)	2 kg	3 m/sec	3 kg	2 m/sec
d)	2 kg	3 m/sec	1 kg	6 m/sec

9.3 Conservation of momentum. Equation (9.1) is the mathematical expression of a conservation law. For the colliding carts that we have considered, it says that the vector sum of the carts' quantities of motion is the same before and after the collision. Because the product of mass and velocity plays such an interesting role, we give it a better name than "quantity of motion"; we call it "momentum."

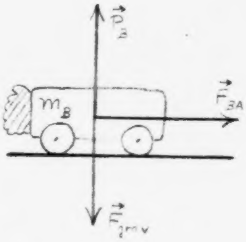
The momentum of a collection or system of objects (for example, the two carts) is the vector sum of the momenta of the objects that make up the system. In each of the collisions that we examined, the momentum of the system was the same before and after the collision. The momentum of the system was conserved.

Although we obtained it by observing collisions between two carts that stuck together when they collided, we shall see that the law of conservation of momentum is a very general law. The momentum of any system is conserved—provided that one condition is met.

To see what that condition is, let us examine the forces acting on the carts. Each cart experiences three forces: a downward (gravitational) pull $\vec{F}_{grav}$ exerted by the earth, an upward push $\vec{P}$ exerted by the floor, and, during the collision, a push $\vec{F}$ exerted by the other cart. The first two forces evidently cancel, since the cart is not accelerating up or down. Thus the net force on each cart is just the force exerted on it by the other cart. (We are assuming that frictional forces, which are also forces exerted on the carts, have been made so small that we can neglect them here. That was the reason for using dry-ice pucks, air-track gliders or carts with "frictionless" wheels.)

In Unit 1, initial and final velocities were represented as $\vec{v}_i$ and $\vec{v}_f$. In Eq. (9.1) they are represented by $\vec{v}$ and $\vec{v}'$ because we now need to add subscripts such as A and B.

Summary 9.3
The product of mass and velocity is called "momentum". The momentum of a collection or system of objects is defined as the vector sum of the momenta of the objects in the system. In the experiments of Sec. 9.2, then, the momentum of the system was conserved. The momentum of any system is conserved, provided that one condition is met; namely, that the net force on the system-as-a-whole is zero. This statement is the law of conservation of momentum. The law of conservation of momentum is perfectly general: no matter what kind of forces the objects in a system exert on one another, or how many objects comprise the system or whether the collision is one-dimensional or two- or three-dimensional or whether the objects actually collide or not — the law of conservation of momentum applies.



Forces on cart B during collision. $\vec{F}_{BA}$ is the force exerted on cart B by cart A. (Cart A is not shown in the figure.)

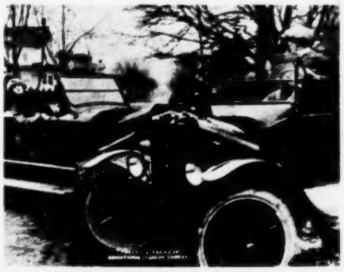
T20: Equal mass two-dimensional collisions
T21: Unequal mass two-dimensional collisions

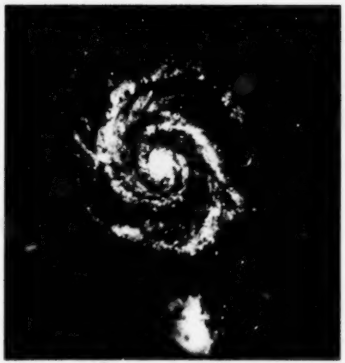
L23: Perfectly inelastic two-dimensional collisions

E23: Collision in two dimensions.

A: Momentum activity
A: Interesting exchange-of-momentum devices.

The sun and the moon also exert forces on each cart, but the forces are so small that we can neglect them.





9.3

The two carts comprise a system of (two) bodies. The force exerted by one cart on the other one is a force exerted by one *part of the system on another part; it is not a force* on the system as a whole. No net force is exerted on either cart by anything outside the system. Therefore the net force on the system as a whole is zero.

This is the condition that must be met in order for the momentum of a system of bodies to stay constant, to be conserved.

If the net force on a system of bodies is zero, the momentum of the system will not change. This is the law of conservation of momentum.

So far we have considered only instances in which two bodies exert forces on each other by direct contact and in which they stick together when they collide. But the remarkable thing about the law of conservation of momentum is how generally it applies.

It is valid no matter what kind of forces the bodies exert on each other: gravitational forces, electrical or magnetic forces, tension in strings, compression in springs, attraction or repulsion—it doesn't matter.

Furthermore it doesn't matter whether the bodies stick together when they collide or whether they bounce apart. They don't even have to touch (as when two strong magnets repel; see Study Guide 9.6).

The law is not restricted to systems of only two objects; there can be any number of objects in the system. Nor is the size of the system important. The law applies to the solar system as well as to an atom. The angle of the collision does not matter either. All the examples we have considered so far have been collisions between two bodies moving along the same straight line; they were "one-dimensional collisions." But if two bodies make a glancing collision rather than a head-on collision, each will move off at some angle to the line of approach. The law of conservation of momentum applies in such two-dimensional collisions also. (Remember that momentum is a vector quantity.) The law of conservation of momentum applies also in three dimensions: the vector sum of the momenta is the same before and after the collision.

On page 20 is a worked-out example to help you become familiar with the law of conservation of momentum. At the end of the chapter is a special page on the analysis of a two-dimensional collision.

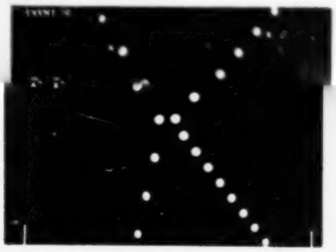
There is also a variety of stroboscopic photographs and

- Note that the momentum of each body in the system may change. It is the momentum of the system which is conserved.

A system on which the net force is zero is called an isolated system. The word was used in a different sense in Section 9.1; there it meant "closed to the passage of matter."

Although conservation of momentum in 2 dimensions is given only a cursory treatment (page 30), the topic can be expanded by use of film loops L21 – L25 and the stroboscopic still photographs in the Student Handbook.

film loops of colliding bodies and exploding objects, which are available for you to analyze. They include collisions and explosions in two dimensions. Analyze as many of them as you can to see whether momentum is conserved. The more of them you analyze, the more strongly you will be convinced that the law of conservation of momentum applies to any isolated system.



One of the stroboscopic photographs that appears full-page in the Student Handbook.

In addition to illustrating the use of the law of conservation of momentum, the worked out example displays a characteristic feature of physics. Again and again, a physics problem is solved by writing down the expression of a general law (for example, $\vec{F}_{net} = m\vec{a}$) and applying it to a specific situation. Both the beginning student and a veteran research physicist find it surprising that one can do this—that a few general laws of physics enable one to solve an almost infinite number of specific individual problems. Often in everyday life people do not work from general explicit laws but rather make intuitive decisions. The way a physicist uses general laws to respond to problems can become, with practice, quite intuitive and automatic also.

All this is being said not to urge that all behavior should be deduced by applying a few general laws in every social encounter, but to explain why the process by which physicists solve problems seems a bit unfamiliar.

Q6 Which of the following has the least momentum? Which has the greatest momentum?

a) a pitched baseball
b) a jet plane in flight
c) a jet plane taxiing toward the terminal

Q7 A girl on ice skates is at rest on a horizontal sheet of smooth ice. After catching a rubber ball thrown horizontally at her, she moves at 2 cm/sec. Give a rough estimate of what her speed would have been

a) if the rubber ball were thrown twice as fast?
b) if the rubber ball had twice the mass?
c) if the girl had twice the mass?
d) if the rubber ball were not caught by the girl, but bounced straight back with no change of speed?

9.4 Momentum and Newton's laws of motion. In Sec. 9.2 we discovered (perhaps "invented" is a better term) the concept of momentum and the law of conservation of momentum by considering the results of experiments with colliding carts. We did not derive the law from any theory. This is the way the law was discovered—as a generalization from experiment.

It is possible to show, however, that the law of conservation of momentum is consistent with Newton's laws of motion. In this section we shall derive the law of conservation of momentum from Newton's laws of motion.

L 28: Recoil

More precisely, $m\vec{v}$ is called "linear momentum".

Physicists often use the symbol $\vec{p}$ for (linear) momentum: $\vec{p} \equiv m\vec{v}$.

Summary 9.4

1. Newton's second law can be rewritten in terms of the change of momentum of the object upon which a net force acts. In this form, Newton's second law, together with Newton's third law, leads to the law of conservation of momentum, which we discovered in Sec. 9.2 and Sec. 9.3 as a generalization from experiment.

2. When written in terms of change of momentum, Newton's second law can be used to treat problems in which the mass of the object being accelerated is not constant.

3. Although the law of conservation of momentum is implicit in Newton's laws of motion, it can be used to solve problems where Newton's laws cannot easily be used; for example, when the forces are not known. Application of Newton's laws, however, provides more detailed information.

Example of the Use of the Conservation of Momentum

Here is an example which illustrates how one can use the law of conservation of momentum.

(a) A polar bear of mass 999.9 kilograms lies sleeping on a horizontal sheet of ice. A hungry hunter fires into him a 0.1 kilogram bullet moving horizontally with a speed of 1000 m/sec. How fast does the dead bear (with the bullet imbedded in him) slide after being hit?

Mass of bullet = m_A = 0.1 kg

Mass of bear = m_B = 999.9 kg

Velocity of bullet before collision $= \vec{v}_A = 1000$ m/sec

Velocity of bear before collision $= \vec{v}_B = 0$

Velocity of bullet after collision $= \vec{v}\,'_A$

Velocity of bear after collision $= \vec{v}\,'_B$

According to the law of conservation of momentum,

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B$$

Since $\vec{v}_B = 0$ and $\vec{v}\,'_A = \vec{v}\,'_B$, the equation becomes

$$m_A\vec{v}_A = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B = (m_A + m_B)\vec{v}\,'_A.$$

$$\text{So } \vec{v}\,'_A = \frac{m_A\vec{v}_A}{(m_A + m_B)} = \frac{(0.1)(1000)\text{ kg m/sec}}{(0.1 + 999.9)\text{ kg}}$$

$$= \frac{(0.1)(1000)}{1000}\text{ m/sec}$$

$$= 0.1\text{ m/sec.}$$

The corpse of the bear slides across the ice with a speed of 10 cm/sec.

(b) Another polar bear, of mass 990 kilograms, wearing a bullet-proof vest of mass 10 kilograms, lies sleeping on the ice. The same hunter fires a bullet at him, as he did in Part (a), but the bullet bounces straight back with almost no change of speed. How fast does the bear slide after being hit by the bullet?

Mass of bullet = m_A = 0.1 kg

Mass of (bear and vest) = m_B = 1000 kg

Velocity of bullet before collision $= \vec{v}_A = 1000$ m/sec

Velocity of (bear and vest) before collision $= \vec{v}_B = 0$

Velocity of bullet after collision $= \vec{v}\,'_A = -1000$ m/sec

Velocity of (bear and vest) after collision $= \vec{v}\,'_B$

The law of conservation of momentum states that

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B.$$

$$(0.1)(1000) + 0 = (0.1)(-1000) + (1000)\vec{v}\,'_B$$

$$100 = -100 + 1000\,\vec{v}\,'_B$$

$$1000\,\vec{v}\,'_B = 200$$

$$\vec{v}\,'_B = 0.2\text{ m/sec.}$$

This time the bear slides along the ice with a speed of 20 cm/sec -twice as fast as his unfortunate comrade. There is a general lesson here. It follows from the law of conservation of momentum that the struck object is given less momentum if it absorbs the bullet than if it reflects it.

Newton's second law is a relation between the net force acting on a body and the mass of the body and its acceleration: $\vec{F}_{net} = m\vec{a}$. We can also write the law in terms of the momentum of the body. If we remember that acceleration is the rate-of-change of velocity, we can write the second law as

$$\vec{F}_{net} = m\,\frac{\Delta\vec{v}}{\Delta t} \qquad (9.3)$$

or

$$\vec{F}_{net}\,\Delta t = m\,\Delta\vec{v}. \qquad (9.4)$$

If the mass of the body is constant, the change in its momentum $\Delta(m\vec{v})$ is the same as its mass times its change in velocity $m(\Delta\vec{v})$. Then we can write Eq. (9.4) as

$$\vec{F}_{net}\,\Delta t = \Delta(m\vec{v}), \qquad (9.5)$$

that is, the product of the net force on a body and the time during which this force acts equals the change in momentum of the body.

If m is a constant,

$$\begin{aligned}\Delta(m\vec{v}) &= m\vec{v}' - m\vec{v}\\ &= m(\vec{v}' - \vec{v})\\ &= m\Delta\vec{v}.\end{aligned}$$

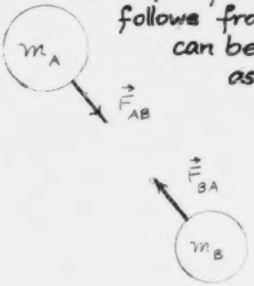
Since $\vec{F}_{net}\,\Delta t = \Delta(m\vec{v})$, $\vec{F}_{net} = \dfrac{\Delta(m\vec{v})}{\Delta t}$ when m is constant.

This gives a new interpretation of net force: it is the time rate-of-change of momentum, when m is constant.

The concept of "impulse" naturally follows from Eq. 9.5, which can be pursued briefly as an optional topic.

Newton's second law, in the form of Eq. (9.5), together with Newton's third law, is enough to enable us to derive the law of conservation of momentum.

Suppose two bodies with masses m_A and m_B exert forces on each other. $\vec{F}_{AB}$ is the force exerted on body A by body B and $\vec{F}_{BA}$ is the force exerted on body B by body A. No other unbalanced force acts on either body. By Newton's third law the forces $\vec{F}_{AB}$ and $\vec{F}_{BA}$ are equal in magnitude and opposite in direction. Each body acts on the other for exactly the same time Δt. Newton's second law, Eq. (9.5), applied to each of the bodies, says



$$\vec{F}_{AB}\,\Delta t = \Delta(m_A\vec{v}_A) \qquad (9.6a)$$

$$\vec{F}_{BA}\,\Delta t = \Delta(m_B\vec{v}_B). \qquad (9.6b)$$

But $\vec{F}_{AB} = -\vec{F}_{BA}$, so the left side of Eq. (9.6a) and the left side of Eq. (9.6b) are equal in magnitude and opposite in direction. Therefore the same must be true of the right sides; that is,

$$\Delta(m_A\vec{v}_A) = -\Delta(m_B\vec{v}_B). \qquad (9.7)$$

Suppose that the masses m_A and m_B are constant. Let $\vec{v}_A$ and $\vec{v}_B$ stand for the velocities of the two bodies at some instant and let $\vec{v}_A{}'$ and $\vec{v}_B{}'$ stand for their velocities at some later instant. Then Eq. (9.7) becomes

$$m_A\vec{v}_A{}' - m_A\vec{v}_A = -(m_B\vec{v}_B{}' - m_B\vec{v}_B). \qquad (9.8)$$

A little rearrangement of Eq. (9.8) leads to

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A{}' + m_B\vec{v}_B{}',$$

As always with equations containing vector qualities, the + and - signs indicate vector addition and vector subtraction.

A: Stroboscopic photographs of two-dimensional collisions

Perhaps you could see even from Eq. (9.7) that conservation of momentum is implied. The change of momentum of one body is accompanied by an equal and opposite change of momentum of the other body. Accordingly the momentum of the system does not change during the time interval Δt.

which we recognize as Eq. (9.1), the law of conservation of momentum. The vector sum of the momenta of the bodies at any instant is the same as the vector sum of the momenta of the bodies after a time interval Δt, provided that during that time interval the only unbalanced force acting on either body is the force exerted on it by the other body.

What we have done here with a system consisting of two bodies, we could do as well with a system consisting of any number of bodies. Study Guide 9.18 shows you how to derive the law of conservation of momentum for a system of three bodies.

Actually Eq. (9.5) is more general than its derivation indicates. We considered a body with constant mass, but in Eq. (9.5) the change of momentum can arise from a change of mass as well as from (or in addition to) a change of velocity.

Are there any situations where the mass of a body changes? As a rocket spews out exhaust gases, its mass decreases. The mass of a train of coal cars increases as it moves under a hopper which drops coal into the cars. In Unit 5 you will learn that any body's mass increases as it moves faster and faster (although noticeably so only at extremely high speeds).

Since $\vec{F}_{net} = m\vec{a}$ expresses Newton's second law for cases where the mass is constant, it may not be so easy to use this form to deal with situations, like those above, where the mass changes; we must use the law in the form of Eq. (9.5). In fact, that is more nearly the way that Newton formulated the law in his Principia.

We have shown that the law of conservation of momentum can be derived from Newton's second and third laws. The law of conservation of momentum is not a new principle of physics but is already implicit in Newton's laws. Nevertheless the law of conservation of momentum enables us to solve many problems which would be impossible or difficult to solve using Newton's laws.

For example, suppose a cannon fires a shell. Although initially at rest, the cannon moves after firing the shell; it recoils. The expanding gases in the cannon barrel push the cannon backward just as hard as they push the shell forward. If we knew the magnitude of the force, we could apply Newton's second law to the cannon and to the shell to find their accelerations, and after a few more steps we could find the speed of the shell and the recoil speed of the cannon. But, in fact, we do not know the magnitude of the force; it is probably not steady and it almost certainly decreases as the shell moves toward the end of the barrel. Hence it would be very difficult to use Newton's laws.

We can use the law of conservation of momentum, however, even if we know nothing about the force (except that the force on the shell is always the same magnitude as the force on the cannon). It tells us that since the momentum of the system (cannon-plus-shell) is zero initially, it will also be zero after the shell is fired. Thus the law of conservation of momentum can be applied to cases where we do not have enough information to apply Newton's laws.

We pay for our ignorance, however. If we could use Newton's laws we would be able to calculate the speed of the shell and the speed of the cannon. But the law of conservation of momentum only makes it possible to calculate the relative speeds of shell and cannon.

Mathematically, the reason we obtain more information when we use Newton's laws is that in that case we have two equations for the two unknown speeds. The law of conservation of momentum provides only one equation for the two unknown speeds.

Q8 a) The five engines of the first stage of the Apollo/Saturn rocket together develop a thrust of 35 million newtons for 150 seconds. How much momentum will they give the rocket?

b) The final speed of the rocket is 6100 miles/hour. Why can you not compute its mass?

Q9 Newton's second law can be written $\vec{F}\Delta t = \Delta(m\vec{v})$. Use the second law to explain the following:

a) When jumping down from a chair, you should bend your knees.

b) Firemen use elastic nets to catch people who jump out of burning buildings.

c) Hammer heads are generally made of steel rather than rubber.

d) The 1968 Pontiac GTO has plastic bumpers which, when deformed, slowly return to their original shape.

Q10 A girl is coasting down a hill on a skate board. Can she make a sharp turn without touching the ground with her foot?

9.5 Isolated systems.

There are similarities in the ways we test and use the conservation laws of mass and linear momentum. In both cases we test the laws by observing systems that have in some sense been isolated or closed off from the rest of the universe.

When testing or using the law of conservation of mass we arrange a system that is closed, so that matter can neither enter nor leave. When testing or using the law of conservation of momentum we set up a system which is isolated in the sense that each body in the system experiences no net force originating from outside the system.

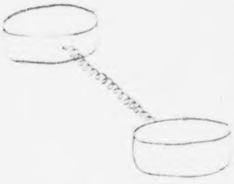
It may be difficult or even impossible actually to isolate a system completely. Consider for example two dry-ice pucks

Summary 9.5

In using the law of conservation of mass, we define an isolated system as one which is closed to the passage of matter. In using the law of conservation of momentum we define an isolated system as one upon which no net force acts. An isolated system in the latter sense is an idealization. Usually there are net forces on the bodies in the system exerted by bodies outside the system, but frequently they are negligible, in which case the system can be treated, for all practical purposes, as isolated.

L24: Scattering of a cluster of objects

L25: Explosion of a cluster of objects



connected by a spring and sliding on a frictionless table. The pucks and spring form a system from which we exclude the table and the earth, whose effects on each puck cancel. For although each puck experiences a downward gravitational force exerted by the earth, the table exerts an equally strong upward push on it. Thus it appears that the net force exerted on each puck by bodies outside the system is zero.

But what about the gravitational attraction of the sun or the moon? Or electric or magnetic forces exerted on the pucks by bodies outside the system? We cannot be sure that all such forces are completely balanced, and therefore the spring and two pucks do not actually form a completely isolated system. Nevertheless, for all real cases, the unbalanced forces usually can be kept negligibly small compared to the forces exerted by the bodies in the system on one another, so that for all practical purposes, the system can be considered isolated.

For example, if two cars are skidding toward a collision, the frictional force exerted by the road on each car may be large, perhaps a few hundred pounds. Nevertheless, it may be considered negligible compared to the immense force (many tons) exerted by each car on the other when they collide; during the collision we can consider the two skidding cars as very nearly an isolated system. If friction is too great to ignore, the law of conservation of momentum still holds, but we must apply it to a system which includes the objects that provide the friction. In the case of the skidding cars, we would have to include the entire earth in the "closed system".

Q11 Which of the following is the way that we define an isolated system when using the law of conservation of momentum?

a) a system in which each object experiences a net force of zero
b) a system in which each object experiences no forces exerted by objects outside the system
c) a system on which the net force is zero
d) a system in which the objects exert no forces on one another

Q12 Explain why the following are not isolated systems.

a) a baseball thrown horizontally
b) an artificial earth satellite
c) the earth and the moon

9.6 Elastic collisions. In 1666, members of the recently-formed Royal Society of London witnessed an experiment at one of the Society's regular meetings.

Two balls made of the same hard wood and of equal size were suspended at the ends of two strings to form two pendulums. When one ball was released from rest at a certain height, it swung down and struck the other, which had been hanging without moving.

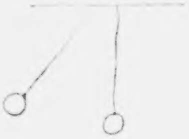
Summary 9.6

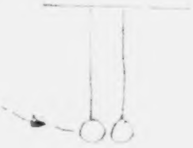
1. A demonstration witnessed by the Royal Society in 1666, in which a pendulum bob collided with another, initially stationary, pendulum bob, raised the question of how to explain the behavior of the bobs. Christiaan Huygens explained the demonstration by showing that in such collisions not only is momentum $m\vec{v}$ conserved in the collision, but also the sum of the products of mass and the square of the speed: mv^2, a quantity which came to be called vis viva.

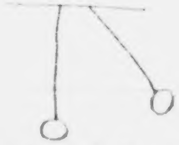
2. Vis viva is conserved only when the colliding objects are hard. Such collisions are called "perfectly elastic" collisions. In general, collisions are not perfectly elastic, although collisions between steel balls or glass marbles are nearly so. Collisions between atoms and between subatomic particles are believed to be perfectly elastic in many circumstances.

L21 : Two-dimensional collisions
L22 : Further examples of two-dimensional collisions

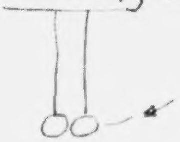
After impact the second ball swung up to nearly the same height as that from which the first had been released and the first became very nearly motionless. When the second ball returned and struck the first, it was now the second ball which came nearly to rest, and the first swung up to almost the same height as that from which it had originally been released. And so the motion continued, back and forth through many swings.

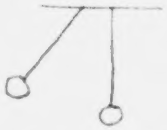






This demonstration aroused considerable interest among members of the Society. In the years immediately following it also generated heated and often confusing arguments. Why did the balls rise each time to nearly the same height? Why was the motion transferred from one ball to the other when they collided? Why didn't the first ball bounce back or continue moving slowly forward? Momentum conservation doesn't prohibit such behavior. Give examples of several outcomes which would satisfy momentum conservation.





In 1668 the membership directed its secretary to write to three men who could be expected to throw light on the whole matter of impacts. The three men were John Wallis, Christopher Wren and Christiaan Huygens. Within a few months, all three men replied. Wallis and Wren had partial answers to explain some of the features of collisions, but Huygens analyzed the problem in complete detail.

His work was performed in 1667, but his reasoning did not become public until 1703, when his works, including "On the Movement of Bodies through Impact," were published posthumously.

Huygens explained the behavior of the pendulums by showing that in such collisions another conservation law is at work, in addition to the law of conservation of momentum. Specifically, he showed that

$$m_A v_A^2 + m_B v_B^2 = m_A v_A'^2 + m_B v_B'^2. \qquad (9.9)$$

In order to calculate the velocity of each of two bodies after they collide, we need two equations, since these are two unknown quantities. The law of conservation of momentum is one of the equations and Eq. (9.9), the law of conservation of vis viva, is the other one.

The quantity mv^2 came to be called *vis viva*. Eq. (9.9), then, is the mathematical expression of the conservation of *vis viva*.

Vis viva is Latin for "living force." Seventeenth century scientists were greatly interested in distinguishing and naming various "forces." They used the word loosely; it meant sometimes a push or a pull (the modern use of the word), sometimes what we now call "momentum" and sometimes what we now call "energy."

Scientists no longer use the term.

We have seen that the momentum of every isolated system of bodies is conserved, no matter what goes on within the system. Is the law of conservation of *vis viva* equally general? Is the *vis viva* of every isolated system conserved? It is easy to see that it is not.

Consider the first example of Sec. 9.2, in which two carts of equal mass approach each other with equal speeds, collide, stick together and stop. The *vis viva* of the system after the collision is

F19 : Elastic collisions and stored energy

$$m_A v_A'^2 + m_B v_B'^2 = m_A(0) + m_B(0) = 0.$$

Before the collision the *vis viva* of the system was $m_A v_A^2 + m_B v_B^2$. Since $m_A v_A^2$ and $m_B v_B^2$ are both positive numbers, their sum cannot possibly equal zero (unless both v_A and v_B are zero, in which case there would be no collision—and not much of a problem). *Vis viva* is not conserved in this collision.

The law of conservation of *vis viva*, then, is not as general as the law of conservation of momentum. If two bodies collide, the *vis viva* may or may not be conserved.

As Huygens pointed out, it is conserved if the colliding bodies are hard so that they do not crumple or smash or dent. We call such bodies "perfectly elastic," and we describe collisions between them as "perfectly elastic collisions." In perfectly elastic collisions both momentum and *vis viva* are conserved.

The colliding pendulums witnessed by the Royal Society were hard enough that vis viva was very nearly conserved.

In Chapter 10 a more accurate definition of "perfectly elastic" will be presented.

In most collisions that we witness, *vis viva* is not conserved; the total *vis viva* after the collision is less than before the collision. Such collisions might be called almost perfectly elastic, or partially elastic, or perfectly inelastic, depending on how much of the *vis viva* is "lost" in the collision. The loss of *vis viva* is greatest when the colliding bodies remain together.

In Chapter 10 it will be shown that in inelastic collisions the vis viva is not really lost.

A collision between steel ball-bearings or glass marbles is almost perfectly elastic, provided that the collision is not so violent as to damage the bodies; the total *vis viva* after the collision might be as much as, say, 98% of its value before the collision. Examples of true perfectly elastic collisions are found only in collisions between atoms or between sub-atomic particles.

Q13 *Vis viva* is conserved
- a) in all collisions.
- b) only when momentum is not conserved.
- c) in some collisions.
- d) when the colliding objects are not too hard.
- e) only in living systems.

Q14 True or false:
Huygens demonstrated an experiment to the members of the Royal Society that aroused much interest.

Q15 In the diagram of the pendula on the last page, why are the strings parallel instead of attached to the same point at the ceiling?

Q16 *Vis viva* is never negative because
- a) it is impossible to draw vectors with negative length.
- b) speed is always greater than zero.
- c) it is proportional to the square of the speed.
- d) scalar quantities are always positive.

9.7 Leibniz and the conservation of *vis viva*. Descartes, believing that the total quantity of motion in the universe did not change, proposed to define the quantity of motion of an object as the product mv of its mass and its speed. As we have seen however, this is not a conserved quantity in all collisions. For example, if two carts with equal masses and equal speeds collide head-on and stick together, they both stop after the collision.

"It is wholly rational to assume that God, since in the creation of matter he imparted different motions to its parts, and preserves all matter in the same way and conditions in which he created it, so he similarly preserves in it the same quantity of motion."

[Descartes, Principles of Philosophy, 1644]

Gottfried Wilhelm Leibniz was aware of the error in Descartes' work on motion. In a letter in 1680 he wrote

> M. Descartes' physics has a great defect; it is that his rules of motion or laws of nature, which are to serve as the basis, are for the most part false. This is demonstrated. And his great principle, that the same quantity of motion is conserved in the world, is an error.

Leibniz' philosophical studies made him sure that something was conserved, however; namely, what he called "force". He described what he meant by "force" in a paper published in 1686 entitled, "A short demonstration of a famous error of Descartes and other learned men, concerning the claimed law of nature according to which God always preserves the same quantity of motion; by which, however, the science of mechanics is totally perverted." He said that

> the force must be evaluated by the quantity of effect it can produce, for example by the height to which it can raise a heavy load... and not by the velocity it can impress on [the load].

If an object is thrown upward, the height y to which it will rise is proportional to the square of its initial speed v:

$$y = \frac{1}{2}v^2/a_g.$$

In this way he was led to identify "force" as *vis viva* mv^2. As he stated in his "Essay in Dynamics on the Laws of Motion," written in 1691,

> Now it is found from reasoning and experience that it is the absolute *vis viva* or the force measured by the violent effect it can produce which is conserved and not at all the quantity of motion.

Leibniz believed that even at the top of its trajectory, a stone that has been thrown upwards still possesses "force"; as it falls back toward the ground, gaining speed, the "force" becomes evident again as *vis viva*.

But, as Huygens had pointed out, *vis viva* is conserved in collisions only if the colliding bodies are hard, that is, only in elastic collisions. In inelastic collisions the total *vis viva* after the collision is less than before the collision. It seems that Leibniz' *vis viva* is no more a conserved quantity than Descartes' quantity of motion.

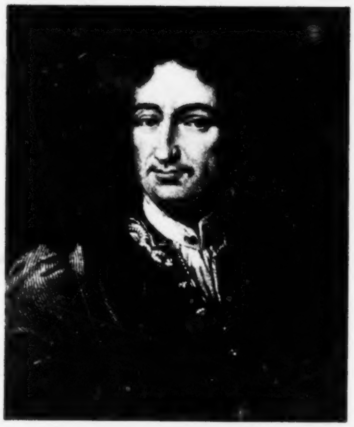
For philosophical reasons, however, Leibniz was convinced that *vis viva* is always conserved. In order to save his conservation law, he was obliged to invent an ingenious but unproven explanation for its apparent disappearance in inelastic collisions. He maintained that in such collisions the *vis viva* is not destroyed "but dissipated among the small parts" of which the colliding bodies are made.

By "small parts" Leibniz did not mean atoms. His contemporaries were speculating about the existence of atoms, and he himself had accepted the idea in his youth, but "reason made me change this opinion."

The question whether quantity of motion mv or *vis viva* mv^2 was the "correct" conserved quantity was much debated by philosophers and scientists in the early eighteenth century. In fact, each theory developed into concepts which are fundamental to modern science. Descartes' mv developed into the modern $m\vec{v}$, which, as we have seen in Sec. 9.3 of this chapter, is always conserved in an isolated system. Leibniz' *vis viva* mv^2 very closely resembles the modern concept of energy of motion, kinetic energy, as we shall see in Chapter 10. Energy, of which energy of motion is only one of many forms, is also a conserved quantity in an isolated system.

Q17 According to Leibniz, Descartes' principle of conservation of quantity of motion was
a) correct, but trivial.
b) another way of expressing the conservation of vis viva.
c) incorrect.
d) correct only in elastic collisions.

Q18 How did Leibniz explain the apparent disappearance of *vis viva* in inelastic collisions?



Leibniz (1646-1716), a contemporary of Newton, was a German philosopher and diplomat, an adviser to Louis XIV of France and Peter the Great of Russia. Independently of Newton he invented the method of mathematical analysis called calculus. A long public dispute resulted between the two great men concerning charges of plagiarism of ideas.

Descartes (1596-1650) was the most important French scientist of the seventeenth century. In addition to his early contribution to the idea of momentum conservation, he is remembered by scientists as the inventor of coordinate systems and the graphical representation of algebraic equations. His system of philosophy, which used the deductive structure of geometry as its model, is still influential. Of a sickly constitution, he did much of his thinking while lying in bed.

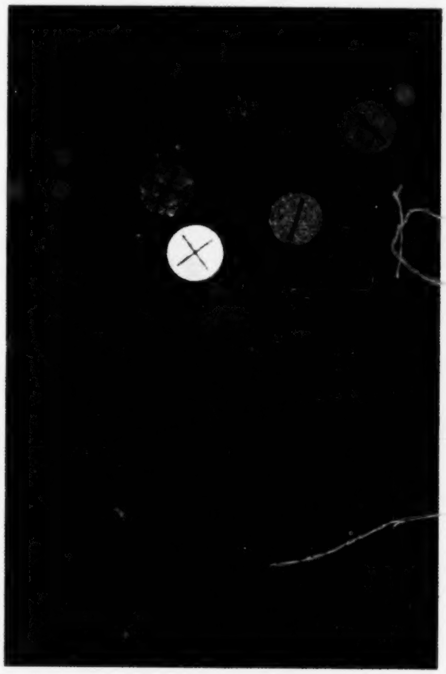
John Wallis (1616-1703) was a professor of mathematics at Oxford. He was later to make one of the earliest careful analyses of vibrations, thereby discovering how partial tones are produced on a stringed instrument. He also introduced the symbol ∞ for infinity.

St. Paul's Cathedral

Christiaan Huygens (1629-1695) was a Dutch physicist. He devised an improved telescope with which he discovered a satellite of Saturn and saw Saturn's rings clearly. He was the first to understand centripetal force, and he invented the pendulum-controlled clock. We shall hear more of Huygens later. Huygens' scientific contributions were major, and his reputation would undoubtedly have been even greater had he not been overshadowed by his contemporary, Newton.

Christopher Wren (1632-1723) was a professor of astronomy at Oxford and was generally considered to be one of the outstanding mathematicians of the day. He is best remembered as the imaginative architect of many impressive buildings in London, including more than fifty churches and cathedrals—Saint Paul's among them.

A Collision in Two Dimensions



The stroboscopic photograph shows a collision between two wooden discs. The discs are on tiny plastic spheres which make their motion nearly frictionless. Body B (marked +) is at rest before the collision. After the collision it moves to the left and Body A (marked -) moves to the right. The mass of Body B is twice the mass of Body A: $m_B = 2m_A$. We will analyze the photograph to see whether momentum was conserved. (Note: the size reduction factor of the photograph and the (constant) stroboscopic flash rate are not given. So long as all velocities are measured in the same units, it does not matter what those units are.)

In this analysis we will measure the distance the discs moved on the photograph, in centimeters. We will use the time between flashes as the unit of time. Before the collision, Body A traveled 36.7 mm in the time between flashes: v_A = 36.7 speed-units. Similarly v_A' = 17.2 speed-units and v_B' = 11.0 speed-units.

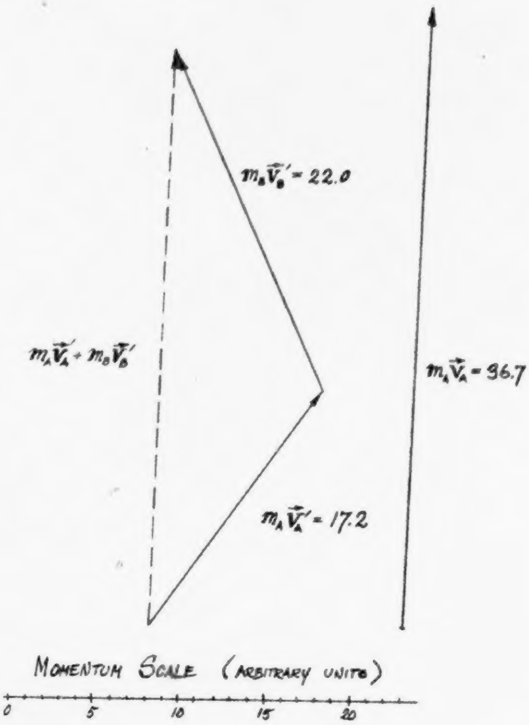
The vector diagram shows the momenta $m_A\vec{v}_A'$ and $m_B\vec{v}_B'$ after the collision; $m_A\vec{v}_A'$ is represented by a vector 17.2 momentum-units long, and $m_B\vec{v}_B'$ by a vector 22.0 momentum-units long. (Remember that $m_B = 2m_A$.)

The dotted line represents the vector sum of $m_A\vec{v}_A'$ and $m_B\vec{v}_B'$; i.e., the total momentum after the collision. Measurement shows it to be 34.0 momentum-units long.

The total momentum before the collision is just $m_A\vec{v}_A$ and is represented by a vector 36.7 momentum-units long. Thus, our measured values of the total momentum before and after the collision differ by only 2.7 momentum-units, a difference of about 7%.

You can verify for yourself that the direction of the total momentum is the same before and after the collision.

Have we then demonstrated that momentum was conserved in the collision? Is the 7% difference likely to be due entirely to experimental inaccuracies, or is there any reason to expect that the total momentum of the two discs after the collision is really a bit less than before the collision?



Study Guide

9.1 Discuss the law of conservation of mass as it applies to the following situations.

a) a satellite "soft-landing" on Venus
b) a rifle firing a bullet
c) the manufacture of styrofoam
d) a person drinking a Coke

Discussion

9.2 Would you expect that in your lifetime, when more accurate balances are built, you will see experiments which show that the law of conservation of mass is not entirely true for chemical reactions?

Yes

9.3 Dayton C. Miller was a renowned experimenter at Case Institute of Technology. He was able to show that two objects placed side by side on a pan balance did not balance two identical objects placed one on top of the other. The reason is that the pull of gravity decreases with distance from the center of the earth. Would Lavoisier have said this experiment contradicted the law of conservation of mass?

No

9.4 A children's toy known as a snake consists of a tiny pill of mercuric thiocyanate. When the pill is ignited, a large, serpent-like foam curls out almost from nothingness. Devise and describe an experiment by which you could test the law of conservation of mass for this demonstration.

Discussion

9.5 Consider the following chemical reaction, which was studied by Landolt in his tests of the law of conservation of mass. A solution of 19.4 g of potassium chromate in 100.0 g of water is mixed with a solution of 33.1 g of lead nitrate in 100.0 g of water. A bright yellow solid precipitate forms and settles to the bottom of the container. When removed from the liquid, this solid is found to have a mass of 32.3 g and is found to have properties different from either of the reactants.

a) What is the mass of the remaining liquid? (Assume the combined mass of all substances in the system is conserved.) 220.2 g
b) If the remaining liquid (after removal of the yellow precipitate) is then heated to 95° C, the water it contains will evaporate, leaving a white solid. What is the mass of this solid? [Assume that the water does not react with anything, either in (a) or in (b).]

20.2g

9.6 If powerful magnets are placed on top of each of two carts, and the magnets are so arranged that like poles face each other as one cart is pushed toward the other, the carts bounce away from each other without actually making contact.

a) In what sense can this be called a collision?
b) Does the law of conservation of momentum apply? Discussion
c) Does the law of conservation of momentum hold for any Yes two times during the interval when the cars are approaching or receding (the first called before and the second called after the "collision")?

Yes

9.7 A freight car of mass 10^5 kg travels at 2.0 m/sec and collides with a motionless freight car of mass 1.5×10^5 kg. The two cars lock and roll together after impact. Find the final velocity of the two cars after collision.

HINTS:

a) Equation (9.1) states 0.8 m/sec

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B$$

What factors in this equation are given in the problem?

b) Rearrange terms to get an expression for $\vec{v}\,'_A$.
c) Find the value of $\vec{v}\,'_A$ $(\vec{v}\,'_A = \vec{v}\,'_B)$.

9.8 From the equation

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

show that the change in momentum of object A is equal and opposite to the change of momentum of object B. Derivation

9.9 Benjamin Franklin, in correspondence with his friend James Bowdoin (founder and first president of the American Academy of Arts and Sciences), objected to the corpuscular theory of light by saying that a particle traveling with such immense speed (3×10^8 m/sec) would have the impact of a 10 kg ball fired from a cannon at 100 m/sec. What mass did Franklin assign to the light particle? 3.33×10^{-6} kg

9.10 In a baseball game, both the bunt and the long outfield fly are sacrifice hits. Contrast the collision processes which make them. Discussion

9.11 In the light of your knowledge of the relationship between momentum and force, comment on reports about unidentified flying objects (UFO) turning sharp corners. Discussion

9.12 A hunter fires a gun horizontally at a target fixed to a hillside. Describe the changes of momentum of the hunter, the bullet, the target and the earth. Is momentum conserved in each case? Discussion

9.13 A girl on skis (mass of 60 kg including skis) reaches the bottom of a hill going 20 m/sec. What is her momentum? She strikes a snowdrift and stops within 3 seconds. What force does the snow exert on the girl? How far does she penetrate the drift?

1200 kg-m/sec
400 N
30 m

9.14 A horizontal conveyor belt is used to transport grain from a bin to a truck. A 50.0-kg bag of grain falls straight from a chute onto the belt every 20 seconds. The velocity of the conveyor belt is 4.0 m/sec.

- a) What is the momentum gained or lost by a bag of grain just as it is placed on the belt? 200 kg-m/sec
- b) What is the average additional force required to drive the belt when carrying grain? 10 N

9.15 The text derives the law of conservation of momentum for two bodies from Newton's third and second laws. Is the principle of the conservation of mass essential to this derivation? If so, where does it enter? Yes

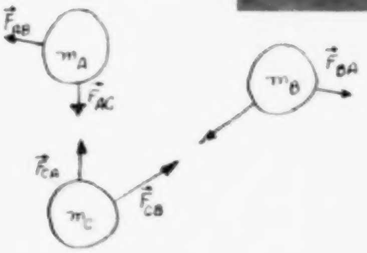
9.16 If mass remains constant, then $\Delta(mv) = m(\Delta v)$. Check this relation for the case where $m = 3$ units, $v' = 6$ units and $v = 4$ units. Derivation

9.17 Equation (9.1), $m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$, is a general equation. For a loaded cannon where the subscripts c and s refer to cannon and shell respectively,

- a) what are the values of v_c and v_s before firing? 0
- b) what is the value of the left hand side of the equation before firing? after firing? 0 $M_c\vec{V}_c + M_s\vec{V}_s$
- c) compare the magnitudes of the momenta of cannon and shell after firing. $M_c\vec{V}_c = -M_s\vec{V}_s$
- d) compare the ratios of the speeds and the masses of shell and cannon after firing. $V_s / V_c = M_c / M_s$
- e) a 10 kg shell has a speed of 1000 m/sec. What is the recoil speed of 1000 kg cannon? 10 m/sec

9.18 Newton's laws of motion lead to the law of conservation of momentum not only for two-body systems, as was shown in Sec. 9.4, but for systems consisting of any number of bodies. In this problem you are asked to repeat the derivation of Sec. 9.4 for a three-body system. Derivation

The figure shows three bodies, with masses m_A, m_B and m_C, exerting forces on one another. The force exerted on body A by body B is $\vec{F}_{AB}$; the force exerted on body C by body A is $\vec{F}_{CA}$, etc.



Using the symbol $\vec{p}$ to represent momentum, we can write Newton's second law as:

$$\vec{F}_{net}\,\Delta t = \Delta(m\vec{v}) = \Delta\vec{p}.$$

Applied to body A, the second law says:

$$(\vec{F}_{AB} + \vec{F}_{AC})\,\Delta t = \Delta\vec{p}_A.$$

a) Copy the last equation above and write corresponding equations for body B and body C.

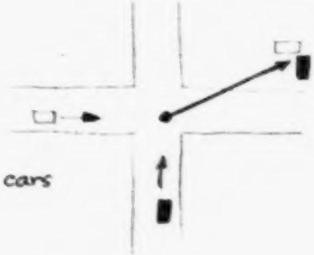
b) According to Newton's third law, $\vec{F}_{AB} = -\vec{F}_{BA}$, $\vec{F}_{AC} = -\vec{F}_{CA}$ and $\vec{F}_{BC} = -\vec{F}_{CB}$. Combine these relations with the three equations of (a) to obtain $\Delta\vec{p}_A + \Delta\vec{p}_B + \Delta\vec{p}_C = 0$.

c) Show that the result of (b) is equivalent to $\vec{p}_A + \vec{p}_B + \vec{p}_C = \vec{p}_A{}' + \vec{p}_B{}' + \vec{p}_C{}'$.

The last equation says that the momentum of the three-body system is constant during the time interval Δt.

9.19 A police report of an accident describes two vehicles colliding (inelastically) at a right-angle intersection. The cars skid to a stop as shown. Suppose the masses of the cars are approximately the same. Which car was traveling faster at collision? What information would you need in order to calculate the speed of the automobiles? Left car (1) speed of one car or (2) mass of cars

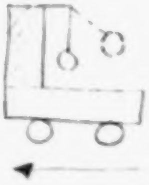
Distance Force



9.20 Two pucks on a frictionless horizontal surface are joined by a spring. Can they be considered an isolated system? How do gravitational forces exerted by the earth affect your answer? What about forces exerted on the planet earth? Do you wish to reconsider your answer above? How big would the system have to be in order to be considered completely isolated? Discussion

9.21 Two balls, one of which has three times the mass of the other, collide head-on, each moving with the same speed. The more massive ball stops, the other rebounds with twice its original speed. Show that both momentum and vis viva are conserved. Derivation

9.22 If both momentum and vis viva are conserved and a ball strikes another three times its mass at rest, what is the velocity of each ball after impact? One-half initial speed in opposite directions



9.23 A student in a physics class, having learned about elastic collisions and conservation laws, decides that he can make a self-propelled car. He proposes to fix a pendulum on a cart, using a "super-ball" as a pendulum bob. He fixes a block to the cart so that when the ball reaches the bottom of the arc, it strikes the block and rebounds elastically. It gives the cart a series of bumps that propel it along.

a) Will his scheme work? (Assume that the "super-ball" is perfectly elastic.) Give reasons for your answer. No

b) What would happen if the cart had an initial velocity in the forward direction? Continues forward

c) What would happen if the cart had an initial velocity in the backward direction? Continues backward

9.24 A billiard ball moving 0.8 m/sec collides with the cushion along the side of the table. The collision is head-on and perfectly elastic. What is the momentum of the ball before impact? After impact? What is the change in momentum? Is momentum conserved? (Pool sharks will say that it depends upon the "English" [spin] that the ball has, but the problem is much simpler if you neglect this condition.)

.8 kg-m/sec forward
.8 kg-m/sec away
1.6 kg-m/sec away
No

9.25 Fill in the blanks:

OBJECT	MASS (kg)	VELOCITY (m/sec)	MOMENTUM kg-m/sec	VIS VIVA kg-m^2/sec^2
baseball	0.14	30.0	4.2 kg-m/sec	1260 kg-m^2/sec^2
hockey puck	0.171 kg	50.0	8.55	427.5 kg-m^2/sec^2
superball	0.050	1.5	0.075 kg-m/sec	0.1125 kg-m^2/sec^2
Corvette	1460	111.0 m/sec	1.62×10^4 kg-m/sec	1.79×10^6
mosquito	10^{-4} kg	0.2 m/sec	2.0×10^{-5}	4.0×10^{-6}

9.26 You have been given a precise technical definition for the word momentum. Look it up in a big dictionary and record its various uses. Can you find anything similar to our definition in these more general meanings? Why was the word momentum selected as the name for quantity of motion?

Discussion

9.27 Descartes defined the quantity of motion of an object as the product of its mass and its speed. Is his quantity of motion conserved as he believed it was? If not, how would you modify his definition so it would be conserved? No, discussion

After the completion of Chapter 9, have students look at the three-dimensional slide of proton scattering (# 2 in the Ealing set) and have class discuss qualitatively and briefly the law of conservation of momentum in three dimensions for such events.

Chapter 10 Energy

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Summary 10.1

In physics, work is defined as the product of the magnitude of the force exerted on an object and the distance through which the object moves while the force is being applied: $W = Fd$.* The significance of work is that it represents an amount of energy transformed from one form to another. If F is the magnitude of the net force on an object, then $Fd = \Delta(\frac{1}{2}mv^2)$, where $\frac{1}{2}mv^2$ is called the "kinetic energy" of the object. The mks unit of work and energy is the "newton-meter," called the "joule".

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*The necessary qualification, that it is the component of the force in the direction of motion — $F_{\parallel} d$, is postponed until section 10.4, pages 44 and 45.

E25: Speed of a bullet
D33: An inelastic collision

10.1 Work and kinetic energy. In everyday language we say that we are "playing" when we are pitching, catching and running on the softball field; when we are sitting at a desk solving physics problems, we are "working." A physicist would disagree. He would say that while studying we are doing very little work, whereas on the softball field we are doing a great deal of work. "Doing work" means something very definite to a physicist: it means "exerting a force on an object while the object moves in the direction of the force." Thus when you throw a softball, you exert a large force on it while it moves forward about a yard; you do a large amount of work. By contrast, turning pages of a textbook requires you to exert only a small force, and the page doesn't move very far; you don't do much work.

Suppose you are employed in a factory to lift boxes from the floor to a conveyor belt at waist height. Both you and a physicist would agree that you are doing work. It seems like common sense to say that if you lift two boxes at once, you do twice as much work as you do if you lift one box. It also seems reasonable to say that if the conveyor belt were twice as high above the floor, you would do twice as much work to lift a box to it. The work you do depends on both the magnitude of the force you must exert on the box and the distance through which the box moves.

Note that the work you do does not depend on how fast you do your job.

The physicist's definition of work is in agreement with these common-sense notions. The work done on an object is defined as the product of the magnitude, F, of the force exerted on the object and the distance d that the object moves while the force is being exerted:

T22: Inelastic two dimensional collisions

$$W = Fd. \qquad (10.1)$$

A more general definition of work is given in Sec. 10.4.

Equation (10.1) is the definition of work only if the force and the distance are in the same direction. A more general definition of work will be given in Sec. 10.4. Also, Equation (10.1) applies only if the force is constant. If the force varies, a suitably defined average force must be used in Equation (10.1).

So far we have not indicated that the concept of work has any use. You probably realize by now, however, that in physics the only concepts that are defined are those which are useful. Work is indeed a useful concept; it is, in fact, crucial to an understanding of the concept of energy.

There are a great many forms of energy. A few of them will be discussed in this chapter. We will define them, in the sense that we will tell how they can be measured and how they can be expressed algebraically. The general concept of energy is very difficult to define; in this course we shall not attempt to do so. On the other hand, to define some particular forms of energy is straightforward, and it is through the concept of work that the definitions can be made.

The significance of the concept of work is that work represents an amount of energy transformed from one form to

L31: A method of measuring energy (nails driven into wood)
L33: Kinetic energy
F20: Energy and work.

another. For example, when you throw a softball (do work on it), you transform chemical energy, which your body obtains from food, into energy of motion. When you lift a stone (do work on it), you transform chemical energy into gravitational potential energy. If the stone is released, the earth pulls it downward (does work on it), and gravitational potential energy is transformed into energy of motion. When the stone strikes the ground, it pushes the ground downward (does work on it), and energy of motion is transformed into heat.

On the following page it is shown how we can use the definition of work (Eq. 10.1), together with Newton's laws of motion, to get an expression for what we have called "energy of motion." It turns out that if F is the magnitude of the net force exerted on an object of mass m while the object moves a distance d in the direction of the force, and if the object is initially at rest, then

$$Fd = \tfrac{1}{2} mv^2 \qquad (10.2)$$

where v is the speed of the object after it has moved the distance d.

The effect of a net force is, of course, acceleration.

We recognize the left side of Eq. (10.2) as the work done on the object by whatever exerted the force. (On the following page it is "you" who do the work on the object.) The work done on the object equals the amount of energy transformed from some form (chemical energy, for example, as on the facing page into energy of motion of the object. So the right side of Eq. (10.2) must be the expression for the energy of motion of the object. Energy of motion is called kinetic energy.

The Greek word "kinetos" means "moving."

The kinetic energy of an object, therefore, is defined as one half the product of its mass and the square of its speed.

$$(KE) = \tfrac{1}{2} mv^2. \qquad (10.3)$$

Equation (10.2) says that the work done on the object equals its final kinetic energy. This is the case if the object was initially at rest, that is, if its initial kinetic energy was zero. More generally the object may already be moving when the net force is applied. In that case the work done on the object equals the increase in its kinetic energy.

$$Fd = \Delta(KE), \qquad (10.4)$$

where $\Delta(KE) = (\tfrac{1}{2} mv^2)_{final} - (\tfrac{1}{2} mv^2)_{initial}$.

The old concept of vis viva was twice the kinetic energy. Until there were units relating work and energy, the factor of $\tfrac{1}{2}$ was not relevant.

Since work is defined as the product of a force and a distance, its units in the mks system are newtons-times-meters. A newton-meter is called a joule. The unit of energy is thus one joule (abbreviated 1 J).

The name of the unit of energy and work commemorates J.P. Joule, a nineteenth century English physicist, famous for his experiments showing that heat is a form of energy (see Sec. 10.7). The word is pronounced like "jewel."

Doing Work on a Sled

Suppose a loaded sled of mass m is initially at rest on the horizontal frictionless surface of ice. You, wearing spiked shoes, exert a constant horizontal force $\vec{F}$ on the sled. The weight of the sled is balanced by the upward push exerted by the surface, so $\vec{F}$ is the net force on the sled. You keep pushing (running faster and faster to keep up with the accelerating sled) until the sled has moved a distance d.

Since the net force $\vec{F}$ is constant, the acceleration of the sled is constant. Two equations that apply to motion from rest with constant acceleration are

$$v = at$$

and

$$d = \frac{1}{2} at^2$$

where a is the acceleration of the body, t is the time interval during which it accelerates (that is, the time interval during which a net force acts on the body), v is the final speed of the body and d is the distance it moves in the time interval t.

According to the first equation

$$t = v/a.$$

If we substitute this expression for t into the second equation, we obtain

$$d = \frac{1}{2} at^2 = \frac{1}{2} a\frac{v^2}{a^2} = \frac{1}{2} \frac{v^2}{a}.$$

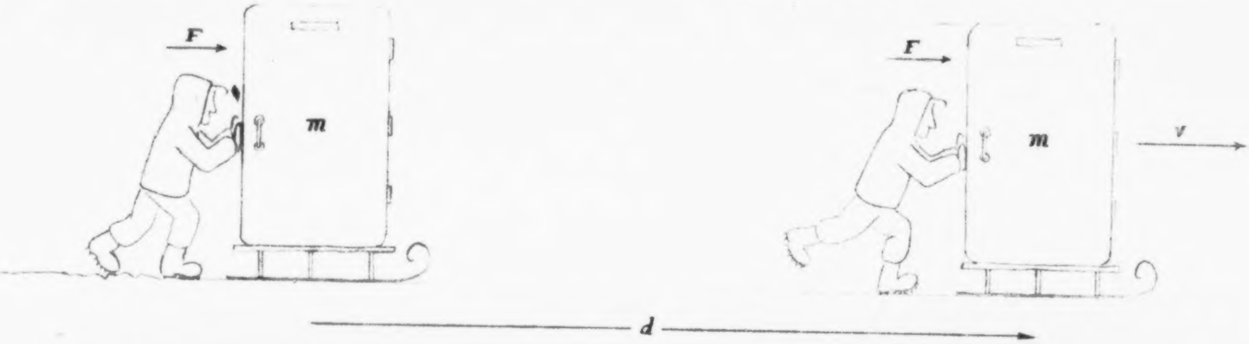
The last equation is a relationship between final speed, (constant) acceleration and distance moved.

The work that you do on the sled is

$$Fd = F\left(\frac{1}{2} \frac{v^2}{a}\right).$$

But, from Newton's second law, F = ma. Therefore,

$$Fd = (ma) \times \left(\frac{1}{2} \frac{v^2}{a}\right) = \frac{1}{2} mv^2.$$



D 34 : Range of a slingshot

Q1 If a Force $\vec{F}$ is exerted on an object while the object moves a distance d in the direction of the force, the work done on the object is

- a) Fd.
- b) $\frac{1}{2}$ Fd^2.
- c) F/d.
- d) F.

Q2 The kinetic energy of a body of mass m moving at a speed v is

- a) mv.
- b) $\frac{1}{2}$mv.
- c) 2 mv^2.
- d) m^{2}v^2.
- e) $\frac{1}{2}$ mv^2.

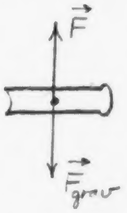
Q3 If you apply 1 joule to lift a $\frac{1}{2}$ kg book, how high will it rise?

In lifting the book, work was done by an agent outside the earth-book system. The increase in energy of that system can subsequently be transformed from one form to another by the forces that parts of the system exert on each other.

10.2 Potential energy. If work is done on an object, its kinetic energy may increase, as we have seen in the previous section. But it can happen that work is done on an object with no increase in its kinetic energy. For example, to lift a book straight up you must do work on the book, even if you lift it at constant speed so that its kinetic energy stays the same. By doing work you are depleting your body's store of chemical energy. But into what form of energy is it being transformed?

It is being transformed into gravitational potential energy. Lifting the book higher and higher increases the gravitational potential energy. When the book has been lifted a distance d, the gravitational potential energy has increased by an amount F_{grav} d, where F_{grav} is the weight of the book.

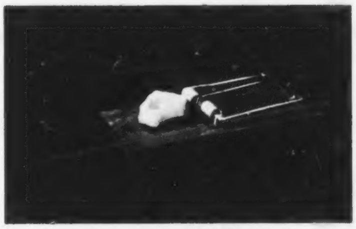
$$\Delta(PE)_{grav} = F_{grav}\, d \qquad (10.5)$$



To lift the book at constant speed, you must exert an upward force $\vec{F}$ equal in magnitude to the weight $\vec{F}_{grav}$ of the book.

Potential energy can be thought of as stored energy. If you allow the book to fall, the gravitational potential energy will decrease and the kinetic energy of the book will increase. When the book reaches the floor, all of the stored gravitational potential energy will have been transformed into kinetic energy.

There are other forms of potential energy also. For example, if you stretch a rubber band or a spring, you increase the elastic potential energy. When you release the rubber band, it can deliver the stored energy to a paper wad in the form of kinetic energy.



A set mouse-trap illustrates elastic potential energy.

In an atom the negatively charged electrons are attracted by the positively charged nucleus. If an electron is pulled away from the nucleus, the electrical potential energy of the atom will increase. If the electron is allowed to be pulled back toward the nucleus, the potential energy will decrease and the electron's kinetic energy will increase.

Chemical energy, to which we have referred frequently, is energy stored in molecules, partly in the form of electrical potential energy of the charged particles comprising the molecules.

T 23 : Slow collisions
L 32 : Gravitational potential energy

Summary 10.2

It is possible to do work on an object without increasing its kinetic energy; for example, by lifting a load at constant speed. The work done represents energy transformed into gravitational potential energy; the increase in gravitational potential energy when an object of weight ma_g is lifted through a vertical distance d is $\Delta(PE)_{grav} = ma_g d$. Potential energy represents stored energy; when the lifted object is allowed to fall, the potential (stored) energy is transformed into kinetic energy.

A : Bouncing ball determination of a_g

E 24: Conservation of mechanical energy

10.3

If two magnets, with north poles facing, are pushed together, the magnetic potential energy will increase. When released, the magnets will move apart, gaining kinetic energy at the expense of potential energy.

The magnetic potential energy does not "belong to" one magnet or the other; it is a property of the system. The same is true of all forms of potential energy. Gravitational potential energy, for example, is a property of the system of the earth and the elevated book.

Q4 If a stone of mass m falls a vertical distance d, the decrease in gravitational potential energy is

- a) md.
- b) ma_g.
- c) ma_gd.
- d) $\frac{1}{2}md^2$.
- e) d.

Q5 When you compress a coil spring you do work on it. The elastic potential energy

- a) disappears.
- b) breaks the spring.
- c) increases.
- d) stays the same.
- e) decreases.

Q6 Two positively charged objects repel one another. To increase the electric potential energy, you must

- a) make the objects move faster.
- b) move one object in a circle around the other object.
- c) attach a rubber band to the objects.
- d) pull the objects farther apart.
- e) push the objects closer together.

10.3 Conservation of mechanical energy. In Sec. 10.1 it was stated that the amount of work done on an object equals the amount of energy transformed from one form to another. Perhaps you realized that this statement implies that the amount of energy does not change, that only its form changes.

If a stone falls, for example, there is a continual transformation of gravitational potential energy into kinetic energy. Over any part of its path the decrease in gravitational potential energy is equal to the increase in kinetic energy. On the other hand, if a stone is thrown upwards, then at any point in its path the increase in gravitational potential energy equals the decrease in kinetic energy. For a stone falling or rising, then

$$\Delta(PE)_{grav} = -\Delta(KE). \qquad (10.6)$$

Equation (10.6) can be written as

$$\Delta(PE)_{grav} + \Delta(KE) = 0, \qquad (10.7)$$

which says that the change in the total energy ($KE + PE_{grav}$) is zero. In other words, the total energy ($KE + PE_{grav}$) remains constant--it is conserved.

The total energy is conserved also when a guitar string is plucked. As the string is pulled away from its unstretched position, the elastic potential energy increases. When the string is released and allowed to return to its unstretched

Summary 10.3

1. When work is done within a closed system, energy is transformed from one form to another, with no change in the amount of energy. For example, as a stone falls, the increase in kinetic energy is accompanied by an equal decrease in gravitational potential energy. Thus the total mechanical energy (KE+PE) is conserved. Mechanical energy is conserved also when elastic potential energy (or any form of potential energy) is transformed into another form or into kinetic energy or vice versa.

Equations (10.6), (10.7), and (10.8) are true only if there is negligible friction.

2. In elastic collisions kinetic energy is transformed into elastic potential energy and then from elastic potential energy into kinetic energy, so that the total kinetic energy after the collision is equal to its value before the collision.

L34: Conservation of energy I — pole vault

L35: Conservation of energy II — aircraft takeoff

L 26 : Speed of rifle bullet – method I ·
L 27 : Speed of rifle bullet – method II

position, the elastic potential energy decreases while the kinetic energy of the string increases. As the string coasts through its unstretched position and becomes stretched in the other direction, its kinetic energy decreases and the elastic potential energy increases. As it vibrates, then, there is a repeated transformation of elastic potential energy into kinetic energy and back again. Over any part of its motion, the decrease or increase in elastic potential energy is accompanied by an equal increase or decrease in kinetic energy:

$$\Delta(PE)_{elas} = -\Delta(KE). \qquad (10.8)$$

Again, at least for short times, the total energy (KE + PE_{elas}) remains constant—it is conserved.

Whenever potential energy is transformed into kinetic energy, or vice versa, or whenever potential energy is transformed into another form of potential energy, the total energy (KE + PE) does not change. This is the law of conservation of mechanical energy.

The law of conservation of mechanical energy is a consequence of the definition of kinetic energy and the definition of change of potential energy, together with Newton's laws of motion. In a sense, the law of conservation of mechanical energy tells us nothing that we do not already know from Newton's laws of motion. However, there are situations where we simply do not have enough information about the forces involved to apply Newton's laws; it is then that the law of conservation of mechanical energy strongly demonstrates its usefulness.

An elastic collision is a good example of a situation where we often cannot apply Newton's laws of motion, because we do not know the force that one object exerts on the other during the collision. But we do know that during the actual collision, kinetic energy of the colliding objects is transformed into elastic potential energy as the objects distort one another. Then all the elastic potential energy is transformed back into kinetic energy, so that when the objects have separated, their total kinetic energy is the same as it was before the collision.

During its contact with a golf club, a golf ball is distorted, as is shown in the high-speed photograph. As the ball recovers its normal spherical shape, elastic potential energy will be transformed into kinetic energy.

It is important to point out that application of Newton's laws, if this were possible, would provide more detailed information; namely, the speed of each of the objects. The law of conservation of mechanical energy gives us only the total kinetic energy of the objects after the collision, not the kinetic energy of each object separately.

Mechanical energy is the concept that Leibniz had been

A : Predicting the range of a bow

groping for in his thinking about the conservation of "force". His *vis viva* differs only by a factor of one half from the modern concept of kinetic energy. He had an idea of gravitational potential energy also, in that he maintained that an object elevated above the ground still possesses "force". He even measured its "force" by the product of its weight and its height above the ground, just as we measure gravitational potential energy.

Q7 As a stone falls
- a) its kinetic energy is conserved.
- b) gravitational potential energy is conserved.
- c) kinetic energy changes into gravitational potential energy.
- d) no work is done on the stone.
- e) there is no change in the total energy.

Q8 In what part of its motion is the kinetic energy of a vibrating guitar string the greatest? When is the elastic potential energy the greatest?

Q9 If a guitarist gives the same amount of elastic potential energy to a bass string and to a treble string, which one will gain more kinetic energy when released? (The mass of a meter of bass string is greater than that of a meter of treble string.)

$W = Fd$ (Eq. 10.1)

Summary 10.4

1. If one carries a load at constant speed horizontally there is no increase in KE or in PE; hence no work is done, despite the fact that a force was exerted on the load while the load moved through a distance. Our previous definition of work (Sec. 10.1) must be modified.
$W = F_{\parallel} d$, where $F_{\parallel}$ represents the component of force parallel to the direction of motion.

If you do not understand what is meant by a "component of force," work through the section on components in the programmed instruction booklet on vectors.

10.4 Forces that do no work. In Sec. 10.1 we defined the work done on an object as the product of the magnitude of the force exerted on the object and the distance through which the object moves while the force is being applied. This definition is satisfactory so long as the object moves in the same direction as the force, as in all the examples we have done so far.

But there are cases where the direction of motion and the direction of the force are not the same. For example, suppose you carry a book horizontally (so that its potential energy does not change), with constant speed (so that its kinetic energy does not change). Since there is no change in the energy of the book, it must be that you did no work on the book. Yet you did exert a force on the book and the book did move through a distance.

The force and the distance, however, were at right angles. You exerted a vertical force on the book—upwards to balance its weight, and the book moved horizontally. We can conclude that if a force is exerted on an object while the object moves at right angles to the direction of the force, the force does no work. In order to include such cases, we must define work more precisely than we did in Sec. 10.1.

The work done on an object is defined as the product of the component of the force on the object in the direction of motion of the object and the distance through which the object moves while the force is being applied:

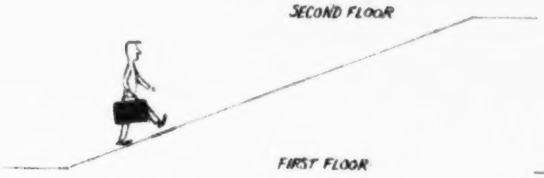
2. Work is done and there is a change in gravitational potential energy only if an object moves vertically. For a given change in height, the work done (the change in gravitational potential energy) is the same no matter what path the object follows.

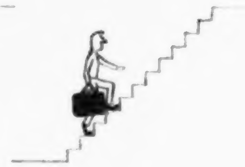
A: Measuring a_g by a whirling weight

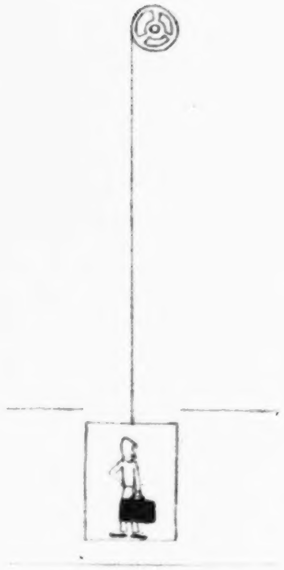
$$W \equiv F_{\parallel} d, \qquad (10.9)$$

where $F_{\parallel}$ stands for the component of force parallel to the direction of motion.

According to Eq. (10.9) no work is done on a suitcase, for example, when it is moved at constant speed along a horizontal path. Work is done on it when it is lifted higher above the ground, increasing its gravitational potential energy. Whether it is lifted at constant speed straight up to the second floor in an elevator, or whether it is carried at constant speed up a flight of stairs or a ramp, there is the same increase in gravitational potential energy and hence the same amount of work is done on the suitcase.







Similarly, whether the suitcase falls out of the second floor window, or tumbles down the stairs, or slides down the ramp, the decrease in the gravitational potential energy is the same; the earth does the same amount of work on it: $W = ma_g d$. If there are no frictional forces, the kinetic energy of the suitcase will be the same when it reaches the first floor, no matter how it got there.

If there are frictional forces on the suitcase or other objects in the system, work has to be done against these by the objects as they move. This work warms up the stairs, the air, the motor, or the body of the man; the change in gravitational potential energy, however, is the same whether there is friction or not.

The change in gravitational potential energy depends only on the change in the vertical position of the suitcase, and not on the path along which it moves. The same is true of other forms of potential energy: change in potential energy depends on the initial and final positions, and not on the path.

Q10 When Tarzan swings from one tree to another on a hanging vine, does the vine do work on him?

Q11 No work is done when
- a) a heavy box is pushed at constant speed along a rough horizontal floor.
- b) a nail is hammered into a board.
- c) there is no component of force parallel to the direction of motion.
- d) there is no component of force perpendicular to the direction of motion.

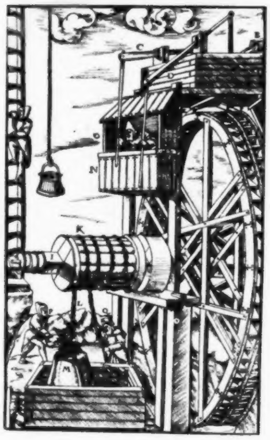
Q12 A suitcase is carried by a porter up a ramp from the ground to the second floor. Another identical suitcase is lifted from the ground to the second floor by an elevator. Compare the increase in gravitational potential energy in the two cases.

E26 : Hotness, thermometers and temperature

10.5 Heat energy and the steam engine. Suppose that a book lying on the table is given a shove and slides along the horizontal surface of the table. If the surface is rough, so that it exerts a frictional force on the book, the book will not keep moving for very long; its kinetic energy gradually disappears. But there is no corresponding increase in potential energy! It appears that in this example mechanical energy is not conserved.

Close examination of the book and the surface of the table, however, would show that they are warmer than before. The disappearance of kinetic energy is accompanied by the appearance of heat. This suggests—but by no means proves—that the kinetic energy of the book was transformed into heat; that is, that heat is one form of energy.

Although scientists today believe that heat is indeed a form of energy, it was not until the middle of the nineteenth century that the view of heat as a form of energy became widely accepted. In Sec. 10.9 we will discuss the reasons for its acceptance at that time, and we will see that one of the reasons was the increased knowledge of heat and work that was gained in the development—for very practical reasons—of the steam engine.



Until about 200 years ago, most work was done by people or animals. Wind and water were exploited also, but generally both were unreliable sources of energy and could not easily be used at the times and places where they were needed. In the eighteenth century there was a great need for an economical method of pumping water out of mines, which otherwise became flooded and had to be abandoned. The steam engine was developed to meet this very practical need.

The steam engine is a device for converting the energy of some kind of fuel (the chemical energy of coal or oil, for example, or the nuclear energy of uranium) into heat energy, and then converting the heat energy into mechanical energy. This mechanical energy can then be used directly to do work, or (as is more common now) can be transformed into electrical energy. In typical twentieth-century industrial societies, most of the energy used in factories and homes is electrical energy. Although waterfalls are used in some part of the country, it is steam engines that generate most of the electrical energy used in the United States today.

The generation and transmission of electrical energy, and its conversion into mechanical energy, will be discussed in Chapter 15. Here we are going to turn our attention to the central link in the chain of energy-conversions, the steam

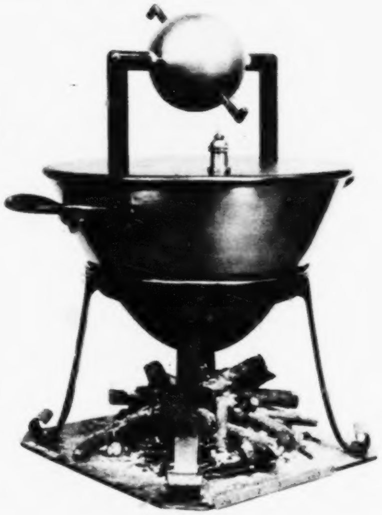
A: Heron's engine
A: Savery's steam pump
A: Small ball-valves (visible action)

engine. As we will see, the development of the steam engine did not take place because of an application of physics to engineering; on the contrary, the engineering analysis of the steam engine led to new discoveries in physics.

Since ancient times it had been known that heat could be used to produce steam, which could then do mechanical work. The "aeolipile," invented by Heron of Alexandria about 100 A.D., worked on the principle of Newton's third law (this principle, of course, had not been announced as a law of physics in the form we now know it). The rotating lawn sprinkler works the same way except that the driving force is due to water pressure rather than steam pressure.

Heron was a Greek inventor and mathematician who lived in Alexandria while it was occupied by the Romans. His books survived through the middle Ages and were circulated widely in Europe during the sixteenth century.

A rotating lawn sprinkler.



A model of Heron's aeolipile. Steam produced in the boiler escapes through the nozzles on the sphere, causing it to rotate.

Heron's aeolipile was a toy, meant to entertain rather than do any useful work. Perhaps the most "useful" application of steam to do work in the ancient world was a device invented by Heron to "magically" open a door in a temple when a fire was built on the altar. Not until late in the eighteenth century, however, were commercially successful steam engines invented.

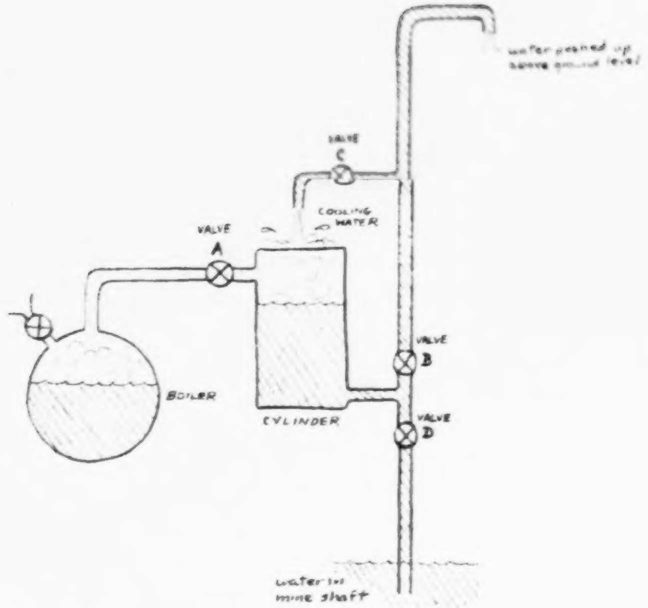
Today we would say that a steam engine uses up its supply of heat energy to do work; that is, it converts heat energy into mechanical energy. But many inventors in the eighteenth and nineteenth centuries did not think of heat in this way. They regarded heat as a substance which could be used over and over again to do work, without being used up itself. The fact that these early inventors held ideas about heat which we do not now accept did not prevent them from inventing engines that actually worked. They did not have to learn the correct laws of physics before they could be successful engineers. In fact the sequence of events was just the opposite: steam engines were developed first by practical men who cared more about making money than they did about science. Later on, men who had both a practical knowledge of what would work as well as a curiosity about how it worked, made new discoveries in physics.

Summary 10.5

1. A book sliding on a rough surface is slowed down by the frictional force exerted on it by the surface; its kinetic energy decreases, but with no corresponding increase in potential energy. At the same time the table and book become warmer, suggesting that KE has been transformed into heat.

2. Since the mid-nineteenth century, it has been accepted that heat is a form of energy. Experimental evidence that heat is a form of energy will be described in Sec. 10.9. A great deal of information about heat and work was obtained in the eighteenth and nineteenth centuries by the development of steam engines to serve very practical purposes. 3. The first commercially successful steam engines were those invented by Savery and Newcomen in the early eighteenth century.

10.5



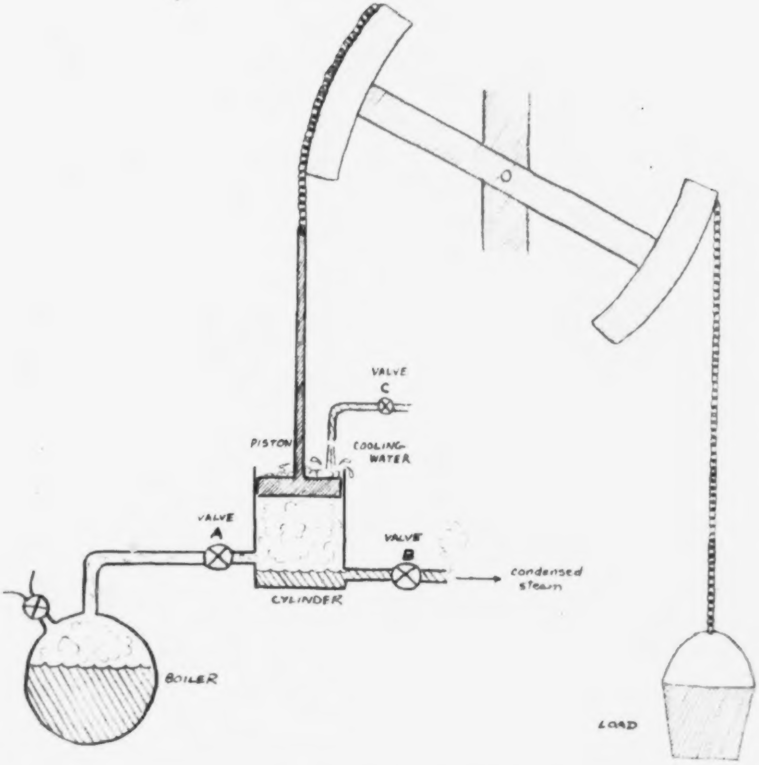
The first commercially successful steam engine was invented by Thomas Savery (1650-1715), an English military engineer, to pump water from mine shafts.

In the Savery engine the water in the mine shaft is connected by a pipe and valve D to a chamber called the cylinder. With valve D closed and valve B open, high-pressure steam from the boiler is admitted to the cylinder through valve A, forcing the water out of the cylinder. Then valve A and valve B are closed and valve D is opened, allowing free access of the water in the mine shaft to the cylinder.

When valve C is opened, cold water pours over the cylinder, cooling the steam in the cylinder and causing it to condense. Since water occupies a much smaller volume than the same mass of steam, a partial vacuum is formed in the cylinder, so that atmospheric pressure forces water from the mine shaft up the pipe and into the cylinder.

The same process, started by closing valve D and opening valves A and B, can be repeated over and over. The engine is in effect a pump, moving water from the mine shaft to the cylinder and, in another step, pushing it from the cylinder to the ground above.

A serious disadvantage of the Savery engine was its use of high-pressure steam, with the attendant risk of boiler or cylinder explosions. This defect was remedied by Thomas Newcomen (1663-1729), another Englishman, who invented an engine which used steam at atmospheric pressure.



In the original Newcomen engine the load was water being lifted from a mine shaft.

In the Newcomen engine there is a working beam with the load to be lifted on one side and a piston in a cylinder on the other side. The working beam is balanced in such a way that when the cylinder is filled with steam at atmospheric pressure, the weight of the load moves the piston to the upper end of the cylinder. While the piston is coming to this position, valve A is open and valve B is closed.

When the piston has reached its highest position, valve A is closed and valves B and C opened. Cooling water flows over the cylinder and the steam condenses, making a partial vacuum in the cylinder, so that atmospheric pressure pushes the piston down. When the piston reaches the bottom of the cylinder, valves B and C are closed, valve A is opened, and steam reenters the cylinder. The combination of load and steam pressure forces the piston to the top of the cylinder, ready to start the cycle of operations again. Originally it was necessary for someone to open and close the valves at the proper times in the cycle, but later a method was devised for doing this automatically, using the rhythm and some of the energy of the moving parts of the engine itself to control the sequence of operations.

The modern term for this reinvestment of information and energy is "feedback."

In the words of Erasmus Darwin,● the engine

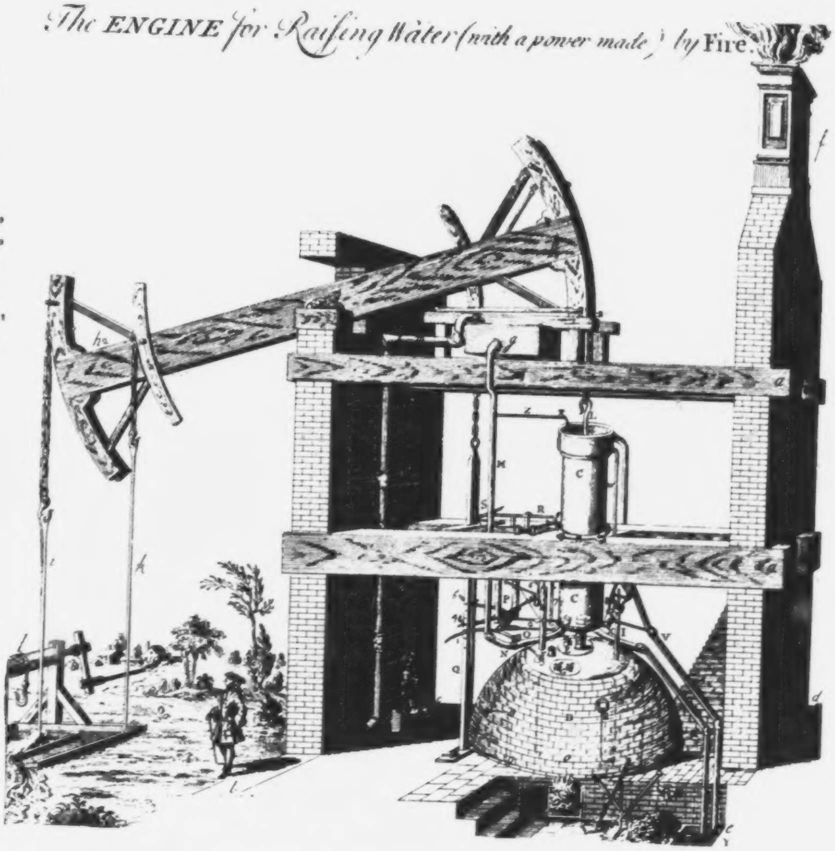
"Bade with cold streams, the quick expansion stop,
And sunk the immense of vapour to a drop
Press'd by the ponderous air the Piston falls
Resistless, sliding through its iron walls;
Quick moves the balanced beam, of giant-birth
Wields his large limbs, and nodding shakes the earth."

●Erasmus Darwin (1731-1802) was an English doctor who wrote poems about science. His grandson, Charles Darwin, proposed the theory of evolution by natural selection in 1859.

10 5

The Newcomen engine was widely used in Britain and other European countries throughout the eighteenth century. By modern standards it was not a very good engine; it burned a large amount of coal but did only a small amount of work at a jerky, slow rate. Nevertheless, because of the demand for machines to pump water from mines, there was a good market even for such an uneconomical steam engine as Newcomen's.

In July 1698 Savery was granted a patent for "A new invention for raiseing of water and occasioning motion to all sorts of mill work by the impellent force of fire, which will be of great use and advantage for drayning mines, serveing townes with water, and for the working of all sorts of mills where they have not the benefitt of water nor constant windes." The patent was good for 35 years and prevented Newcomen from making much money from his superior engine.


The ENGINE for Raising Water (with a power made) by Fire.

Q13 When a book slides to a stop on the horizontal rough surface of a table

- a) the kinetic energy of the book is transformed into potential energy.
- b) heat is transformed into mechanical energy.
- c) the kinetic energy of the book is transformed into heat energy.
- d) the momentum of the book is conserved.

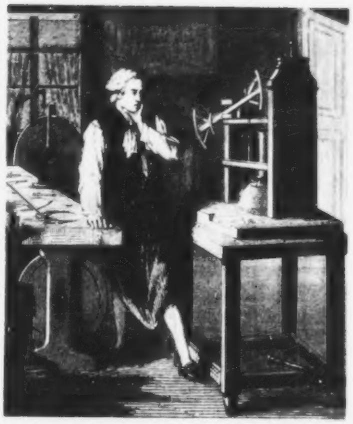
Q14 True or false:
The invention of the steam engine depended strongly on theoretical developments in the physics of heat.

Q15 What economic difficulties did Newcomen encounter in attempting to make his engine a commercial success?

T24: The Watt engine

10.6

10.6 James Watt and the Industrial Revolution. A greatly improved steam engine originated in the work of a Scotsman, James Watt (1736-1819). Watt's father was a carpenter who had a successful business selling equipment to ships. James was sickly most of his life and gained most of his early education at home. He spent much of his childhood in his father's attic workshop where he developed considerable skill in using tools. He wanted to become an instrument maker and went to London to learn the trade. Upon his return to Scotland in 1757, he obtained a position as instrument maker at the University of Glasgow.



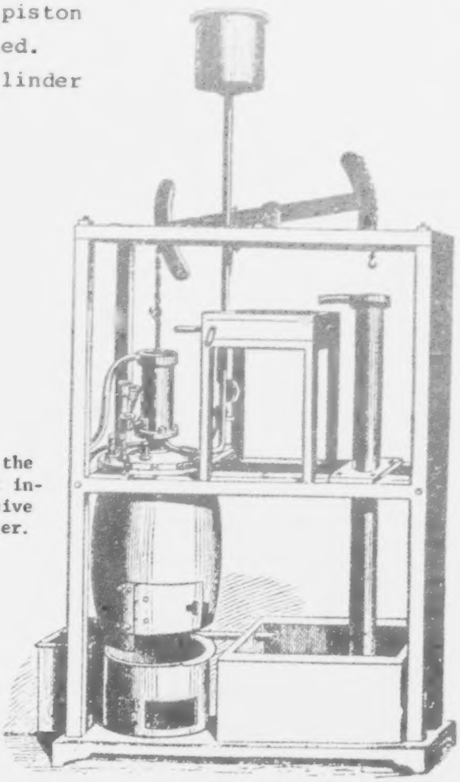
Watt in his workshop contemplating a Newcomen engine. (A romanticized engraving from a nineteenth century volume on technology.)

In the winter of 1763-64 Watt was asked to repair a small model of Newcomen's engine that was used in classes at the university. In familiarizing himself with the model, he observed that Newcomen's engine wasted most of its heat in warming up the walls of its cylinder, which were then cooled down again every time the cold water was injected into the cylinder to condense the steam. This represented a waste of fuel because much of the steam was doing nothing but heating the cylinder walls and then condensing there without doing any work.

This defect was more apparent in the small model than in the full-size engine, because the surface area of a small cylinder is larger compared to its volume.

Early in 1765 Watt saw how this defect could be remedied. He devised a new type of steam engine in which the steam in the cylinder, after having done its work of pushing the piston back, was admitted to a separate container to be condensed. The condenser could be kept cool all the time and the cylinder could be kept hot all the time.

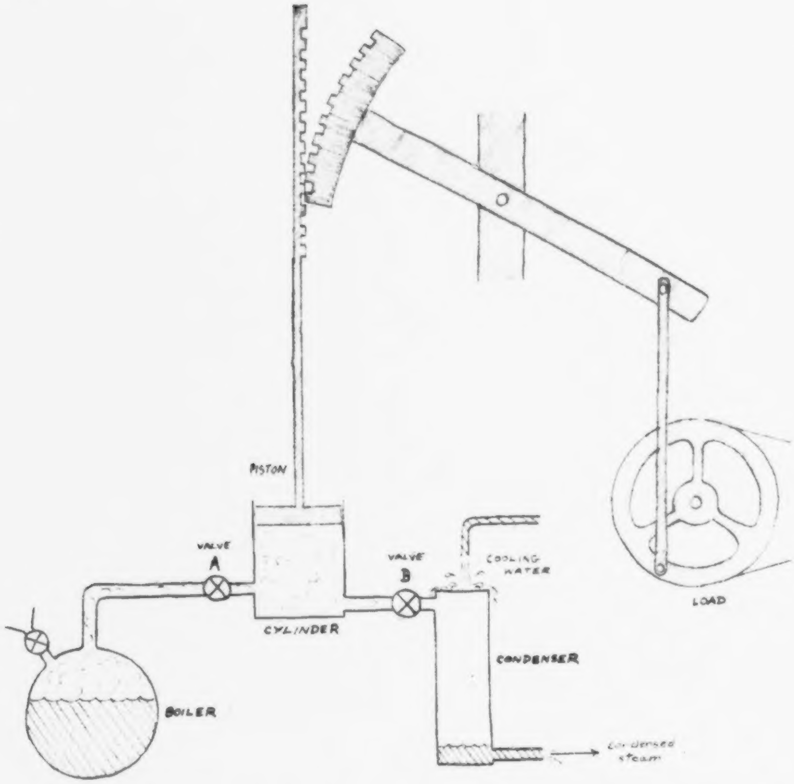
James Watt



The actual model of the Newcomen engine that inspired Watt to conceive the separate condenser.

The London guilds would not accept "foreigners" as apprentices. (A Scotsman was considered a foreigner in England.) When Watt found a master instrument-maker who would overlook this rule, he had to agree to pay a fee of 20 guineas (which equalled about $100) and work without wages for a year.

10.6



In the diagram above of Watt's engine, if valve A is open and valve B is closed, steam enters the cylinder at a pressure higher than atmospheric pressure, and pushes the piston upward against the load. When the piston nears the top of the cylinder, valve A is closed to shut off the steam supply. At the same time valve B is opened, so that steam leaves the cylinder and enters the condenser. As the condenser is kept cool by water flowing over it, the steam condenses. This sets up a difference in pressure between the cylinder and the condenser and maintains the flow of steam in the direction toward the condenser. As steam leaves the cylinder the pressure there decreases and the load (plus atmospheric pressure) pushes the piston down. When the piston reaches the bottom of the cylinder, valve B is closed and valve A is opened, admitting steam into the cylinder and starting the next cycle of operations.

Although Watt's invention of the separate condenser might seem to be only a small step in the development of steam engines, it turned out to be a decisive one. The waste of heat was cut down so much by keeping the cylinder always hot that Watt's engine could do more than twice as much work as Newcomen's engine with the same amount of fuel. As a result of

A: Measuring the sun's power
A: Crooke's radiometer

Summary 10.6

1. The invention of the seperate condenser by Watt in 1765 greatly improved the steam engine, allowing it to do much more work than Newcomen's engine for the same amount of fuel.

2. The power of an engine is defined as the rate at which it can do work. The duty of an engine is a measure of how much work it can do using a given supply of fuel.

3. The steam engine was an important factor in the Industrial Revolution, which transformed the economic and social structure of Western society.

this saving in fuel cost, Watt was able to make a fortune by selling or renting his engines to mine owners for the purpose of pumping water.

The fee that Watt charged for the use of his engines depended on their power. Power is defined as the rate of doing work (or the rate at which energy is transformed from one form to another). The mks unit of power is the joule-per-second, which now is appropriately called one watt.

Matthew Boulton (Watt's business partner) proclaimed to Boswell (the famous biographer of Samuel Johnson): "I sell here, Sir, what all the world desires to have: POWER!"

$$1 \text{ watt} \equiv 1 \text{ joule/sec.}$$

James Watt expressed the power of his engines in different units. He found that a strong work horse could lift a 150-pound weight almost four feet in a second; in other words it could do almost 600 foot-pounds of work per second (more precisely 550 foot-pounds per second). Watt defined this as a convenient unit for expressing the power of his engines, the horsepower.

The foot-pound is a unit of work that we shall not use in this course. One foot-pound is defined as the work done when a force of one pound is exerted on an object while the object moves a distance of one foot.

Typical power ratings in horsepower

Man working a pump	0.036
Man turning a crank	0.06
Overshot waterwheel	3
Post windmill	4
Turret windmill	10
Savery steam engine (1702)	1
Newcomen engine (1732)	12
Smeaton's Long Benton engine (1772)	40
Watt engine from Soho (1778)	14
Cornish engine for London water-works (1837)	135
Corliss Philadelphia Exhibition engine (1876)	2500
Electric power station engines (1900)	1000

Source: R. J. Forbes, in C. Singer et al., History of Technology

1 horsepower = 746 watts

Saturn rocket (1st stage) $>10^{10}$ horsepower.

Another term which is used to describe the economic value of a steam engine is its duty. The duty of a steam engine is defined as the distance in feet that the engine can lift a load of one million pounds, using one bushel (84 pounds) of coal as fuel.

Duty of Steam Engines

Date	Name	Duty
1718	Newcomen	4.3
1767	Smeaton	7.4
1774	Smeaton	12.5
1775	Watt	24.0
1792	Watt	39.0
1816	Woolf	68.0
1828	Improved Cornish engine	104.0
1834	Improved Cornish engine	149.0
1878	Corliss	150.0
1906	Triple expansion engine	203.0

Source: H. W. Dickinson, Short History of the Steam Engine

The modern concept of efficiency is related to duty. The efficiency of an engine is defined as the ratio of the work it can do to the amount of energy supplied to it. Efficiency cannot exceed 100%.

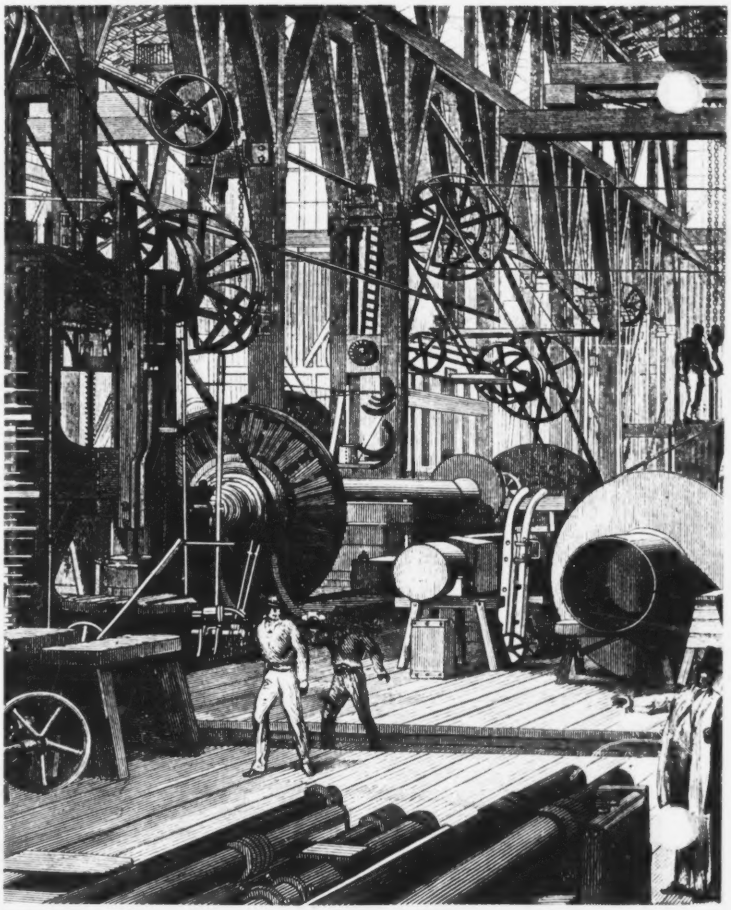
The idea of percentage efficiency wasn't possible until heat and work could be expressed in the same units.

10.6

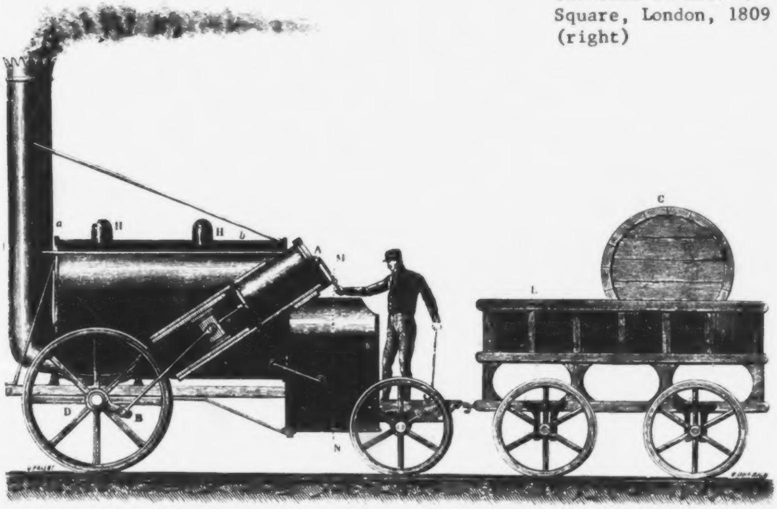
Both power and duty are useful measures of the value of an engine. Its duty tells how much work the engine can do when it uses a given amount of fuel; its power tells how fast the engine can do work.

Watt's invention of the steam engine with separate condenser, so superior to Newcomen's engine, stimulated the development of engines that could do many other kinds of jobs—running machines in factories, driving railway locomotives, steamboats, and so forth. It gave a powerful stimulus to the growth of industry in Europe and America, and thereby helped transform the economic and social structure of Western civilization.

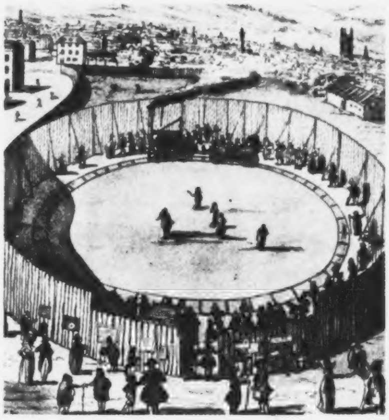
The Industrial Revolution, based on the development of engines and machines for mass production of consumer goods, greatly raised the average standard of living in Western Europe and the United States. Nowadays it is difficult to imagine what life would be like without all the things produced by industry. But not all the effects of industrialization have been beneficial. The nineteenth-century factory system provided an opportunity for some greedy and unscrupulous employers to exploit the workers. These employers made huge profits, while

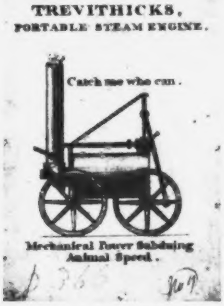


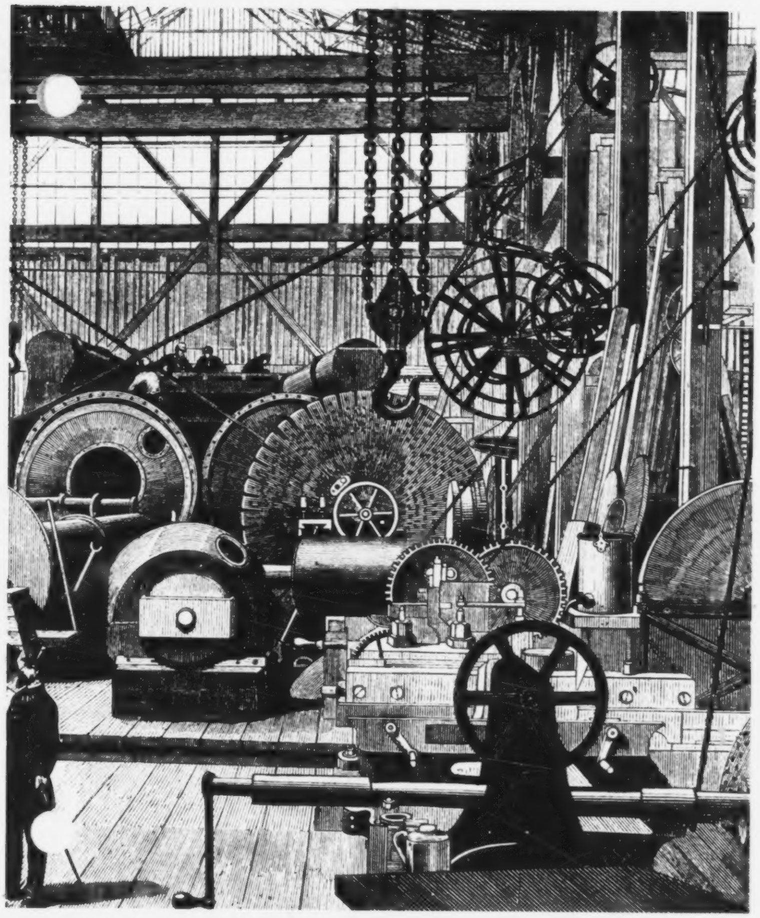
Stephenson's "Rocket" locomotive (below)



Richard Trevithick's railroad at Euston Square, London, 1809 (right)

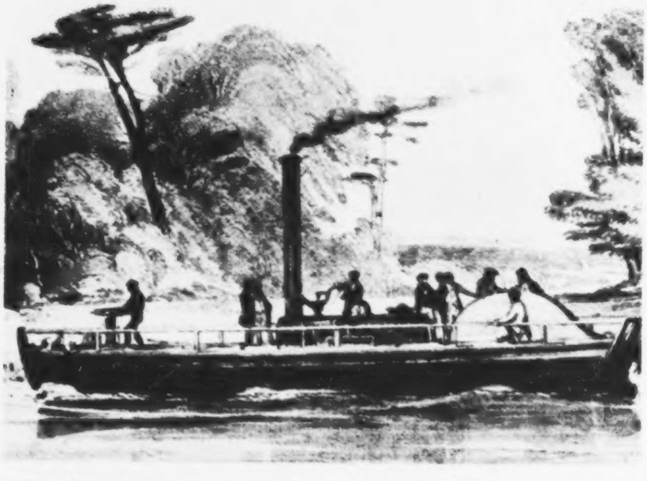



TREVITHICKS,
PORTABLE STEAM ENGINE.
Catch me who can.
Mechanical Power Subduing Animal Speed.



they kept employees and their families on the verge of starvation. This situation, which was especially bad in England early in the nineteenth century, led to demands for reform through government action and legislation. The worst excesses were eventually eliminated.

As more and more people left the farms to work in factories, the conflict between the working class, made up of employees, and the middle class, made up of employers and professional men, became more intense. At the same time, the artists and literary intellectuals began to attack the materialistic tendencies of their society, which was increasingly dominated by commerce and machinery. In some cases they became so fearful of technology that they confused science itself with technical applications and denounced both while refusing to learn anything about them. William Blake asked, sarcastically, "And was Jerusalem builded here/ Among these dark Satanic mills?" John Keats was complaining about science when he asked: "Do not all charms fly/ At the mere touch of cold philosophy?" But not all poets were hostile to science, and even the mystical "nature-philosophy" of the Romantic movement had something to contribute to physics.



The "Charlotte Dundas," the first practical steamboat, built by William Symington, an engineer who had patented his own improved steam engine. It was tried out on the Forth and Clyde Canal in 1801.

See "The Steam Engine Comes of Age" in the Project Physics Reader 3.

10.6

Although steam engines are no longer widely used as direct sources of power in industry and transportation, they have by no means disappeared. The steam turbine, invented by the English engineer Charles Parsons in 1884, has now largely replaced the older kinds of steam engines, and is now used as the major source of energy in most electric-power stations. The basic principle of the Parsons turbine is simpler than that of the Newcomen and Watt engines: a jet of steam at high pressures strikes the blades of a rotor and makes it go around at high speed.

A description of the type of steam turbine now in operation at power stations in the United States shows the change of scale since Heron's toy:

> The boiler at this station [in Brooklyn, New York] is as tall as a 14-story building. It weighs 3,000 tons, more than a U.S. Navy destroyer. It heats steam to a temperature of 1,050° F and to a pressure of 1,500 pounds per square inch. It generates more than 1,300,000 pounds of steam an hour. This steam runs a turbine to make 150,000 kilowatts of electricity, enough to supply all the homes in a city the size of Houston, Texas. The boiler burns 60 tons (about one carload) of coal an hour.

The change in tools symbolizes the change in scale of industrial operations between the nineteenth century and ours.

The 14-story boiler does not rest on the ground. It hangs—all 3,000 tons of it—from a steel framework. Some boilers are even bigger—as tall as the Statue of Liberty—and will make over 3,000,000 pounds of steam in one hour. This steam spins a turbine that will make 450,000 kilowatts of electricity—all of the residential needs for a city of over 4,000,000 people!

[The Story of Steam (1957), Babcock and Wilcox Co.]

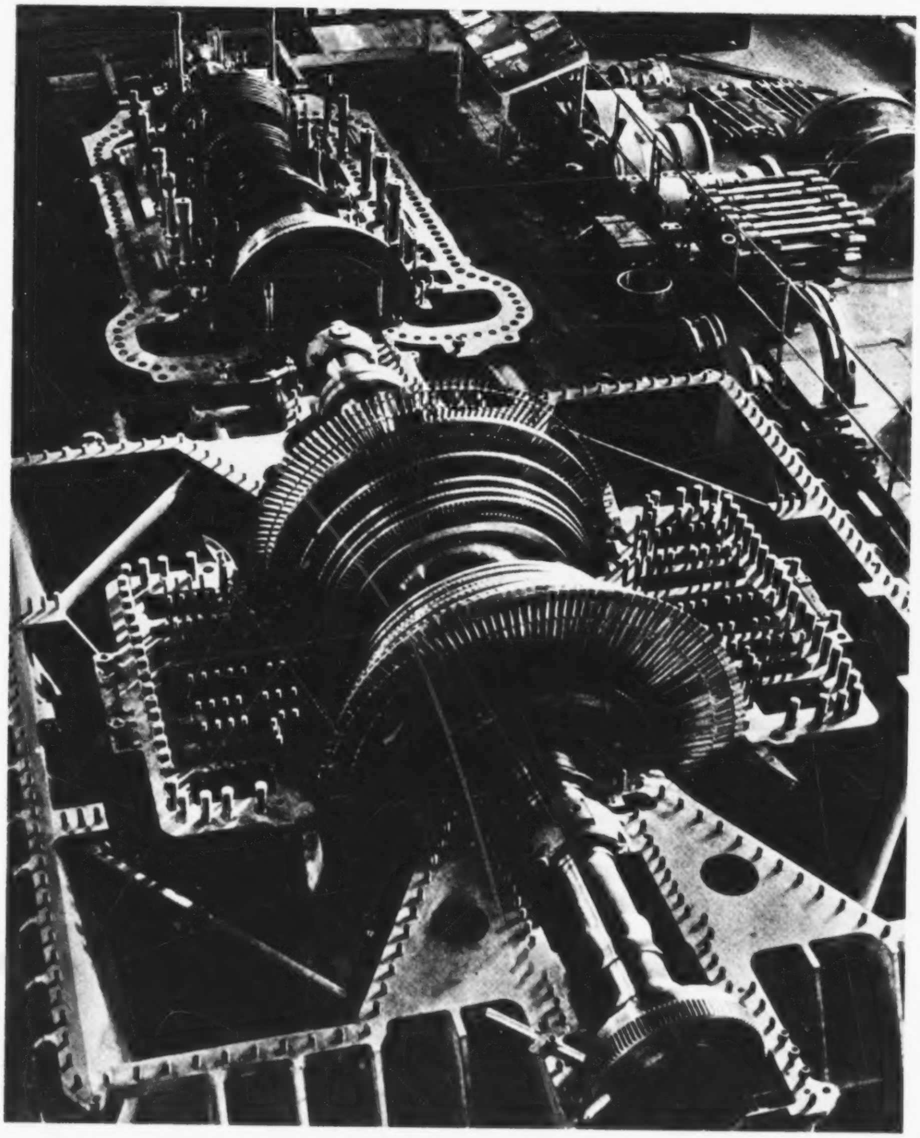
Q16 The purpose of the separate condenser in Watt's steam engine is

a) to save the water so it can be used again.
b) to save the heat so it can be used again.
c) to save fuel by avoiding repeated heating and cooling of the cylinder walls.
d) to keep the steam pressure as low as possible.
e) to make the engine more compact.

Q17 Engine A produces more power than Engine B, but its efficiency is less. This means

a) A is a bigger engine than B.
b) A does more work with the same amount of fuel, but more slowly than B.
c) A does less work with the same amount of fuel, but faster than B.
d) A does more work with the same amount of fuel and faster than B.
e) A does less work with the same amount of fuel and more slowly than B.

A modern steam turbine for use in a city electric power plant, opened to show the blades of the rotor.



Summary 10.7
1. According to the caloric theory of heat, heat is a substance (called "caloric") which can do work when it passes from a high temperature to a low temperature, without change in the amount of caloric. Among others, J.P. Joule did not accept the caloric theory; he believed that heat was a form of energy. To substantiate his belief, he performed a long series of experiments to show that when heat is produced by doing work, the same decrease in mechanical energy always produces the same amount of heat. 2. In his most famous experiments, falling weights caused a paddle-wheel to turn, which stirred water to increase the temperature of the water. 3. It is accepted now that heat is a form of energy, and that one kilocalorie equals 4184 joules.

10.7 *The experiments of Joule*. In the steam engine a certain amount of heat is used to do a certain amount of work. What happens to the heat after it has done the work?

The answer commonly given early in the nineteenth century by most scientists and engineers was that heat can do work if it passes, for example, from steam at high temperature to water at low temperature, but that the amount of heat remains constant. Heat was considered to be a *substance*, called "caloric," and the total amount of caloric in the universe was thought to be conserved.

According to the caloric theory of heat, the way in which heat can do work is analogous to the way that water can do work if it falls from a high level to a low level, with the total amount of water remaining the same. The explanation was accepted because it seemed plausible, even though no measurements had been made of the amount of heat before and after it did work.

There were some dissenting voices; some people favored the view that heat was a form of energy. One of the scientists who held this view was the English physicist James Prescott Joule (1818-1899).

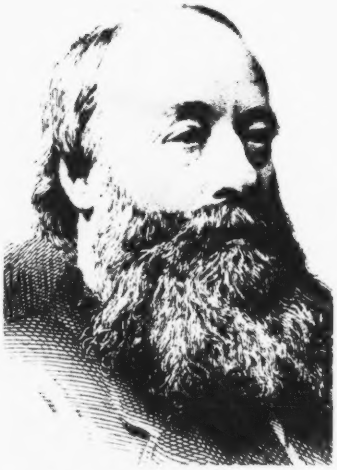
During the 1840's Joule conducted a long series of experiments designed to show that heat is a form of energy. Joule reasoned that if it could be demonstrated in a variety of different experiments that the same decrease in mechanical energy always resulted in the appearance of the same amount of heat, that would mean that heat is a form of energy.

For one of his early experiments he constructed a simple electric generator, which was driven by a falling weight. The electric current that was generated heated a wire immersed in water. From the distance the weight descended, he could calculate the decrease in gravitational potential energy, while the mass of the water and its temperature rise gave him the corresponding amount of heat produced. In another experiment he compressed gas in a bottle immersed in water, measuring the amount of work done to compress the gas and measuring the amount of heat delivered to the water.

His most famous experiments were performed with an apparatus in which descending weights caused a paddle-wheel to turn in a container of water. He repeated this experiment many times, constantly improving the apparatus and refining his analysis of the data. In the end he was taking very great care to insulate the container so that heat was not lost to the room; he was measuring the temperature-rise to a fraction of a degree, and in his analysis he was allowing for the

E 27: Calorimetry

A: Young's modulus



James Prescott Joule

Joule was the son of a wealthy Manchester brewer. He did his research at home and it is said to have been motivated by the desire to develop more efficient engines for the family brewery.

Electrical technology was still a new field in Joule's day.

A familiar example of work on a gas increasing its temperature is provided by pumping up a bicycle tire by hand.

small amount of kinetic energy the descending weights had when they reached the floor.

Joule published his results in 1849. He reported

> 1st. That the quantity of heat produced by the friction of bodies, whether solid or liquid, is always proportional to the quantity of [energy] expended. And 2nd. That the quantity of heat capable of increasing the temperature of a pound of water . . . by 1° Fahr. requires for its evolution the expenditure of a mechanical [energy] represented by the fall of 772 lb. through the distance of one foot.

Joule used whale oil and mercury in place of water in some of his paddle-wheel experiments.

Joule used the word "force" instead of "energy". The scientific vocabulary was still being formed.

The first statement is the evidence that heat is a form of energy. The second statement tells the value of the ratio between the unit of mechanical energy (Joule used the foot-pound) and the unit of heat (Joule used the British Thermal Unit, BTU).

In fact, even today in new fields of science new words are being coined and existing words given new precise technical meanings. For example, spin, parity, strangeness, isotopic spin.

In the mks system the unit of heat is the kilocalorie and the unit of mechanical energy is the joule. Joule's results are equivalent to the statement that 1 kilocalorie equals 4,150 joules. Joule's paddle-wheel experiment, as well as other basically similar ones, has since been performed with great precision. The currently accepted value for the "mechanical equivalent of heat" is

$$1 \text{ kilocalorie} = 4184 \text{ joules}$$

Units for expressing amount of heat were devised before it was realized that heat is a form of energy. The BTU is the amount of heat that must be added to a pound of water to increase its temperature by 1 Fahrenheit degree. The kilocalorie is the amount of heat that must be added to a kilogram of water to increase its temperature by 1 Celsius (or "Centigrade") degree. In fact, the BTU and the kilocalorie, together with the foot-pound and the joule, are all different units of the same thing: energy.

It is generally accepted now that heat is not a substance, but a form of energy. In particular, it is one of the forms of internal energy, that is, energy associated with the molecules and atoms of which matter is composed. This view of heat will be treated in detail in Chapter 11.

In an inelastic collision, some (or all) of the kinetic energy of the colliding objects is transformed into internal energy, so that after the collision the objects have less kinetic energy than before. (Leibniz had expressed somewhat the same idea when he said that the *vis viva* was "dissipated among the small parts" of the colliding objects.)

Most scientists believe that in Joule's paddlewheel experiment it is incorrect to say that heat was added to the water. Rather one should say that mechanical energy was transformed into internal energy of the water. The word "heat" should be used only to refer to internal energy which is being directly transferred from a hot substance to a cold substance, for example, from the hot gas of the flame of a stove to the pan of soup standing on it. Many scientists are often careless in making this distinction. We shall not insist on making the distinction in this course.

Q18 According to the caloric theory of heat, caloric
- a) is a form of water.
- b) can do work when it passes between two objects at the same temperature.
- c) is another name for temperature.
- d) is produced by steam engines.
- e) is a substance that is conserved.

Q19 The kilocalorie is
- a) a unit of temperature.
- b) a unit of energy.
- c) the same as a BTU.
- d) equal to 772 foot-pounds.
- e) an amount of water.

Q20 In Joule's paddle-wheel experiment, was all the gravitational potential energy used to heat the water?

10.8 Energy in biological systems. All living things need a supply of energy to maintain life and to carry on their normal activities. Human beings are no exception; we, like all animals, depend on the food we eat to supply us with energy. Human beings are omnivores; they eat both animal and plant materials. Some animals are herbivores, eating only plants, while others are carnivores, eating only animal flesh.

Food provides not only energy, but also material for building and repairing tissue, as well as small quantities of substances necessary for the complex chemical reactions that occur in the body.

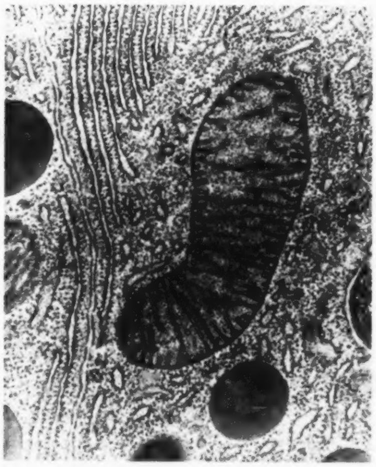
Ultimately, however, all animals, even carnivores, obtain their food energy from plant material. The animal eaten by a lion, for example, has previously dined on plant material (or perhaps on another animal which had eaten plants).

Green plants obtain energy from sunlight. Some of the energy is used to enable the plant to perform the functions of life, but much of it is used to make carbohydrates out of water (H_2O) and carbon dioxide (CO_2). The energy used to synthesize carbohydrates is not lost; it is stored in the carbohydrate molecules in the form of chemical energy.

Carbohydrates are large molecules made of carbon, hydrogen, and oxygen. A simple example is the sugar glucose, the chemical formula for which is $C_6H_{12}O_6$.

The process by which plants synthesize carbohydrates is called photosynthesis. It is still not completely understood; research in the field is continuing. It is known that the synthesis takes place in a large number of small steps, and many of the steps are well understood. When man learns how to photosynthesize carbohydrates without plants, he may be able economically to produce food for the rapidly increasing population of the world. The overall process of producing carbohydrates (the sugar glucose, for example) by photosynthesis can be represented as follows:

carbon dioxide + water + energy ⟶ glucose + oxygen



Electron micrograph of an energy-converting mitochondrion in a bat cell (200,000 times actual size).

The energy stored in the glucose molecules is used by the animal which eats them to maintain its body temperature, to keep its heart, lungs, and other organs operating, to enable various chemical reactions to occur in the body and to do work on external objects. The process whereby the stored energy is made available is complex. It takes place mostly in tiny bodies called mitochondria which are found in all cells. Each mitochondrion contains catalysts (called enzymes), which, in a series of about ten steps, split the glucose molecules into simpler molecules. In another sequence of reactions these molecules are oxidized, releasing most of the stored energy, and forming carbon dioxide and water.

Most — or perhaps all — of the oxygen which now composes about 20% of the earth's atmosphere is believed to be the product of photosynthesis by green plants.

The released energy is used to change a molecule called adenosine di-phosphate (ADP) into adenosine tri-phosphate (ATP), a process which requires energy. In short, the chemical energy originally stored in the glucose molecule is eventually stored as chemical energy in ATP molecules.

The ATP molecules pass out of the mitochondrion into the body of the cell (the cytoplasm). Wherever in the cell energy is needed, it can be supplied by an ATP molecule, which releases its stored energy and changes to ADP. Later, in the mitochondrian, the ADP will be converted again to energy-rich ATP.

In fact only slightly more than half of the chemical energy in the glucose molecule is stored in ATP molecules. The rest of the energy appears as heat energy and as chemical energy in the water and carbon dioxide molecules, which are exhaled.

The overall result of this process is that glucose, in the presence of oxygen, is broken into carbon dioxide and water, with the release of energy:

glucose + oxygen → carbon dioxide + water + energy.

Animals and plants are mutually necessary. The water and carbon dioxide exhaled by animals are used by plants to synthesize carbohydrates, releasing oxygen in the process. The oxygen is used by animals to oxidize the carbohydrates. The energy "used up" by animals (and to a lesser extent, by plants) is continually replenished by energy from sunlight.

Proteins and fats are used to build and replenish tissue, enzymes and padding for delicate organs. They also can be used to provide energy. Both proteins and fats enter into chemical reactions which result in the formation of the same molecules as the splitting carbohydrates; from that point the energy-releasing process is the same as in the case of carbohydrates.

Carbohydrates alone can supply the energy needed to live. But the building blocks (amino acids) from which body tissues and essential enzymes are made can be obtained only from proteins.

The human body can be regarded as resembling in some respects an engine. Just as a steam engine uses chemical energy stored in coal or oil as fuel, the body uses chemical energy stored in food. In both cases the fuel is oxidized to release its stored energy, violently in the steam engine and gently, in small steps, in the body. In both the steam engine and the body, some of the input energy is used to do work and the rest is used up internally and lost as heat.

Some foods supply more energy than others. The energy stored in food is usually measured in kilocalories, but it could just as well be measured in joules—or even in foot-pounds or British Thermal Units. The table below gives the energy content of some foods.

The chemical energy stored in food can be determined by burning the food in a closed container immersed in water and measuring the temperature rise of the water.

Food	Energy Content
Beef (Hamburger)	3880 kilocalories per kg
Whole Milk	730
Sweet Corn	910
White Rice	1270
Potatoes (boiled, peeled)	890
Wheat (whole meal)	770

Adapted from U.S. Dept. Agric., Agriculture Handbook No. 8, June 1950.

The calories referred to in diets are kilocalories.

Summary 10.8
1. Green plants use the energy of sunlight to produce sugar and oxygen from carbon dioxide and water; the solar energy is stored in the sugar molecules. Animals release the stored energy when they oxidize the sugar to carbon dioxide and water.

10.8

If the temperature of the body changes by a few degrees, it seriously affects the rate at which important chemical reactions go on in the body.

A large part of the energy you obtain from food is used to keep your body's "machinery" running and to maintain your body's temperature at the correct value. Even when asleep your body uses about one kilocalorie every minute! This amount of energy is needed just to keep alive.

To do work you need a supply of extra energy—and only a fraction of it can be used to do work. The rest is wasted as heat; like any engine, the human body is not 100% efficient. The efficiency of the body when it does work varies with the job and the physical condition and skill of the worker, but probably in no case does it exceed 25%.

In the table below are estimates of the rate at which a healthy college student uses energy in various activities. They were made by measuring the amount of carbon dioxide exhaled, so they show the total amount of food energy use, including the amount necessary just to keep the body functioning.

Activity	Rate of using food energy
Sleeping	1.0 to 1.3 kilocalories per min
Lying down	1.3 to 1.6
Sitting still	1.6 to 1.9
Standing	1.9 to 2.1
Walking	3.8
Running fast	8 to 12
Swimming	9

According to the data in the table, if a healthy college student did nothing but sleep for eight hours a day and lie quietly the rest of the time, he would still need at least 1700 kilocalories of energy each day. There are countries where large numbers of people exist on less than 1700 kilocalories a day. In India, for example, the average is 1630 kilocalories a day.* (In the United States the average is 3100 kilocalories a day.*) About half the population of India is on the very brink of starvation and vast numbers of other people in the world are similarly close to that line.

According to the Statistical Office and Population Division of the United Nations, the total food production of all land and water areas of the earth in 1950 added up to 5760 billion kilocalories a day. The world population in 1950 was about 2.4 billion. If the available food had been equally distributed among all the earth's inhabitants, each would have had 2400 kilocalories a day, only slightly over the minimum required for life.

In 1963 it was estimated that the population of the world would double in the next 35 years, so that by the year 2000,

2. Energy stored in foods is usually measured in kilocalories. Most of the energy released from food is used to keep the body warm and to keep its machinery running; no more than about a quarter of it is used to enable the body to do work on external objects; the efficiency of the human body, therefore, is at most about 25%.

3. In heavily populated, relatively poor countries the average daily caloric intake from food is only slightly greater than the amount needed just to keep the body functioning; little is left over for doing work on external objects. Supplying adequate food for the increasing population of the world is a serious problem.

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*U.N. Yearbook of National Accounts Statistics, 1964.

The physics of energy transformations in biological processes is one example of a lively interdisciplinary field, namely bio-physics, where physics, biology, chemistry and nutrition all enter. Another connection to physics is provided by the social, political and moral problems of an inadequate world food supply; here, too, many physicists are trying to contribute to solutions through work on aspects using their special competence.

it will be 6 billion. Furthermore the rate at which the population is increasing is itself increasing! The problem of supplying food energy for the world's hungry is rapidly becoming one of the most important problems mankind has ever faced.

Q21 Animals obtain the energy they need from food, but plants
- a) obtain energy from sunlight.
- b) obtain energy from water and carbon dioxide.
- c) obtain energy from seeds.
- d) do not need any supply of energy.

Q22 The human body has an efficiency of about 20%. This means that
- a) only one-fifth of the food you eat is digested.
- b) four-fifths of the energy you obtain from food is destroyed.
- c) one-fifth of the energy you obtain from food is used to run the "machinery" of the body.
- d) you should spend 80% of each day lying quietly without working.
- e) one-fifth of the energy you obtain from food is used to enable your body to do work on external objects.

Supplying enough food energy is only part of the problem. Protein must also be supplied. Plants are in general less rich sources of protein than animals. But to produce 50 lbs. of beef requires an acre of grassland for a year; the same acre of land could produce nearly 4000 lbs. of wheat in a year.

"The Repast of the Lion" by Henri Rousseau

Henri Rousseau

The Metropolitan Museum of Art

10.9 The law of conservation of energy. In Sec. 10.3 we introduced the law of conservation of mechanical energy, which can be used only in situations in which no mechanical energy is transformed into heat energy or vice-versa. Some physicists thought there were other forms of energy in addition to kinetic energy and potential energy, but they did not know how to measure them quantitatively.

Early in the nineteenth century developments in science, engineering and philosophy suggested that all forms of energy (including heat) could be transformed into one another and that the total amount of energy in the universe is constant.

The newly developing science of electricity and magnetism, for example, showed that electricity was related to a number of other phenomena. Volta's invention of the battery in 1800 showed that electricity could be produced in chemical reactions. It was soon found that electric currents could produce heat and light. Oersted discovered in 1820 that an electric current produces magnetic effects. In 1822, Seebeck discovered that if heat is applied to the junction between two metals, an electric current is set up, and in 1831, Faraday discovered electromagnetic induction: a magnet moved near a coil of wire produced an electric current in the wire.

To some speculative minds these discoveries indicated a unity of the phenomena of nature and suggested that they were all the result of the same basic "force". This vague, imprecisely formulated idea bore fruit in the form of the law of conservation of energy: all the phenomena of nature are examples of the transformation from one form to another, without change of quantity, of the same basic thing: energy.

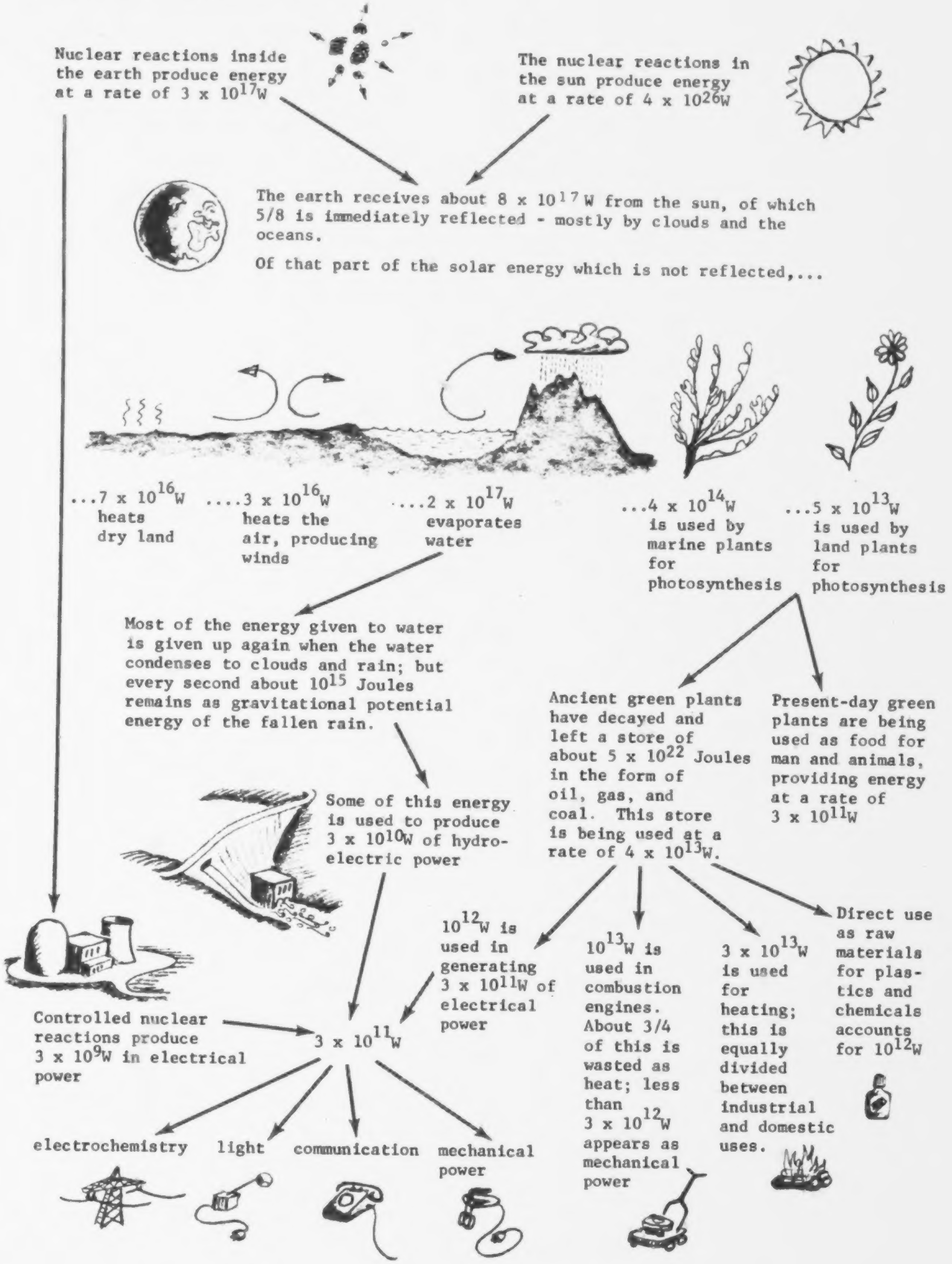
Energy is not a thing, nor is it a substance. The structure of the English language (and of most European languages) forces us to say that a bullet "has" kinetic energy or that a stone "has" gravitational potential energy. It would be more accurate to say that the bullet or the stone is in an energetic state or an energetic condition.

The invention and use of steam engines played a role in the establishment of the law of conservation of energy by showing how to measure changes of energy. Practically from the beginning of their application, steam engines were evaluated by their duty, that is, by how heavy a load they could lift and how high they could lift it when they consumed a certain supply of fuel. In other words, the criterion was how much work an engine could do for the price of a bushel of coal: a very practical consideration which is typical of the engineering tradition in which the steam engine was developed.

The concept of work began to be used in general as a measure of the amount of energy transformed from one form to another (even if the actual words "work" and "energy" were not used) and thus made possible quantitative statements about the transformation of energy. For example, Joule used the work

L29: Colliding freight cars
L30: Dynamics of a billiard ball
L4: A matter of relative motion

Energy Conservation on Earth



Nuclear reactions inside the earth produce energy at a rate of 3×10^{17}W

The nuclear reactions in the sun produce energy at a rate of 4×10^{26}W

The earth receives about 8×10^{17} W from the sun, of which 5/8 is immediately reflected - mostly by clouds and the oceans.

Of that part of the solar energy which is not reflected,...

...7×10^{16}W heats dry land

....3×10^{16}W heats the air, producing winds

....2×10^{17}W evaporates water

...4×10^{14}W is used by marine plants for photosynthesis

...5×10^{13}W is used by land plants for photosynthesis

Most of the energy given to water is given up again when the water condenses to clouds and rain; but every second about 10^{15} Joules remains as gravitational potential energy of the fallen rain.

Some of this energy is used to produce 3×10^{10}W of hydro-electric power

Ancient green plants have decayed and left a store of about 5×10^{22} Joules in the form of oil, gas, and coal. This store is being used at a rate of 4×10^{13}W.

Present-day green plants are being used as food for man and animals, providing energy at a rate of 3×10^{11}W

10^{12}W is used in generating 3×10^{11}W of electrical power

10^{13}W is used in combustion engines. About 3/4 of this is wasted as heat; less than 3×10^{12}W appears as mechanical power

3×10^{13}W is used for heating; this is equally divided between industrial and domestic uses.

Direct use as raw materials for plastics and chemicals accounts for 10^{12}W

Controlled nuclear reactions produce 3×10^{9}W in electrical power

3×10^{11}W

electrochemistry

light

communication

mechanical power

F21: Conservation of energy
R3: Energy

done by the descending weights as a measure of the amount of gravitational potential energy transformed into heat energy.

In 1843, on the basis of his early experiments, which were relatively crude compared to his later ones, Joule expressed the conviction that the expenditure of a certain amount of mechanical energy always resulted in the appearance of a definite amount of heat, and that this was an indication of the conservation of what we now call energy. Joule said that he was

Joule used "force" to mean what we now call "energy".

> satisfied that the grand agents of nature are by the Creator's fiat indestructible; and that, wherever mechanical force [energy] is expended, an exact equivalent of heat is always obtained.

Joule began his long series of experiments by investigating the duty of electric motors. In this case duty was measured by the work the motor could do when a certain amount of zinc was used up in the battery that ran the motor. Joule's interest was to see whether motors could be made economically competitive with steam engines.

This said, Joule got on with his labors in the laboratory; he was basically a practical man and had little time for theological or metaphysical speculation. Meanwhile, other people were reaching the same conclusion—that the total amount of energy in the universe is constant—on the basis of speculative arguments.

A year before Joule's remark, for example, Julius Robert Mayer, a German physician, had proposed the law of conservation of energy. Unlike Joule, he had done no quantitative experiments, although he had observed physiological processes involving heat and respiration. He used published data on the thermal properties of air to calculate the mechanical equivalent of heat, and obtained about the same value that Joule did.

Julius Robert Mayer

Mayer had been influenced strongly by the German philosophical school now known as Naturphilosophie or "nature-philosophy", which flourished in Germany during the late eighteenth and early nineteenth centuries. Its most influential leaders were Johann Wolfgang von Goethe (1749-1832) and Friedrich Wilhelm Joseph von Schelling (1775-1854). Neither of these men is known today as a scientist. Goethe is frequently regarded as Germany's greatest poet and dramatist, while Schelling is considered only a minor philosopher. Nevertheless, both of them had great influence on the generation of German scientists educated at the beginning of the nineteenth century. The nature-philosophers were closely associated with the Romantic movement in literature, art and music, which was a reaction to what they regarded as the sterility and ethical indifference of the mechanistic world view of Descartes and Newton.

Wordsworth, Keats, Coleridge, Byron and Shelley were leaders of the Romantic movement in English literature; Schubert, Schumann and Berlioz were Romantic composers; Delacroix was a Romantic painter.

To the nature-philosophers the idea that nature is just a machine made of dead matter in motion was not merely dull, but actually repulsive. They could not believe that the

A: Rubber band engines

richness of natural phenomena, including Man, with his intellect, his emotions and his aspirations, could be the result of nothing but blind, random motions of particles.

The nature-philosophers believed that the methods of mechanistic science—analyzing nature, mathematizing her into abstraction, torturing her with experiments—gave a totally wrong idea of nature, just as dissecting and measuring a dead flower gives no understanding of the loveliness of a wild rose.

They claimed that they could understand nature as it really is, by direct observation, without complicated and artificial apparatus, just by using their feelings and intuitions. Goethe and Schelling were both very much interested in science and thought that their philosophy could serve as a guide to the discovery of the inner meaning of nature. For Goethe the goal was

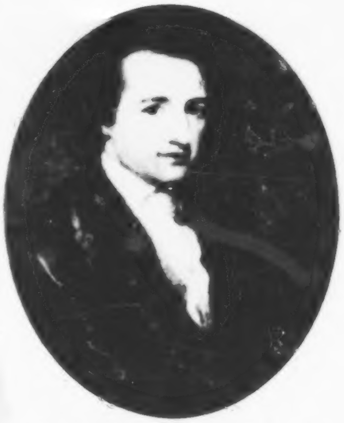
> "That I may detect the inmost force which binds the world, and guides its course."

At first glance it would seem that nature-philosophy had little to do with the law of conservation of energy; that law is practical and quantitative, whereas nature-philosophers tended to be speculative and qualitative. In their insistence on searching for the underlying reality of nature, however, the nature-philosophers did influence the discovery of the law of conservation of energy. They believed that the various forces of nature—gravity, electricity, magnetism, etc.—are not really separate from one another, but are simply different manifestations of one basic force. By encouraging scientists to look for connections between different forces (or, in modern terms, between different forms of energy), nature-philosophy stimulated the experiments and theories that led to the law of conservation of energy.

By the time the law was established and generally accepted, however, nature-philosophy was no longer popular. Those scientists who had previously been influenced by it, including Mayer, were now strongly opposed to it. The initial response of some hard-headed scientists to the law of conservation of energy was colored by their distrust of speculative nature-philosophy. For example, William Barton Rogers, founder of the Massachusetts Institute of Technology, wrote home from Europe to his brother in 1858:

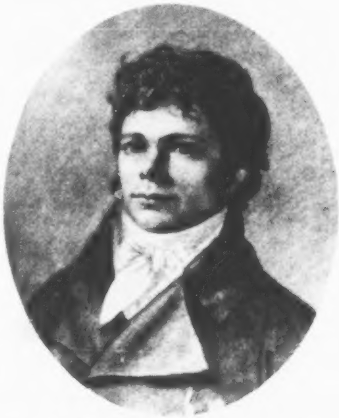
> "To me it seems as if many of those who are discussing this question of the conservation of force are plunging into the fog of mysticism."

However, the law of conservation of energy was so quickly and



Johann Wolfgang von Goethe

Goethe thought that his work on color theory (which most modern scientists consider useless) exceeded in importance all his poems and plays.



Friedrich von Schelling

Oersted's discovery of the connection between electricity and magnetism, which he himself said had been motivated by his belief in Schelling's philosophy, was described by later scientists as accidental.

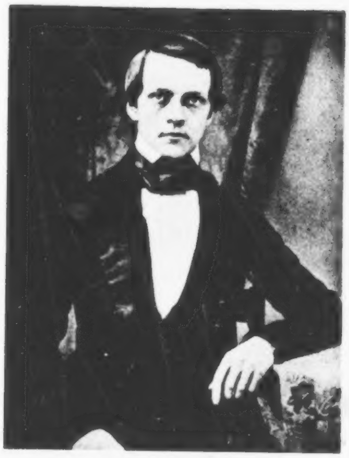
A: Energy analysis of a pendulum swing

successfully put to use in physics that its philosophical origins were soon forgotten.

See "The Seven Images of Science" in Project Physics Reader 3.

In their day-to-day work in laboratories and studies, many scientists stay close to experiments and mathematical calculations. However when truly major advances in science are made, such as the establishment of the law of conservation of energy, philosophical speculation often plays an important role. Creative work in science, as in any field, requires daring and imaginative minds.

Mayer and Joule were only two of at least a dozen people who, between 1832 and 1854, proposed in some form the idea that energy is conserved. Some expressed the idea obscurely; others, quite clearly. Some arrived at their belief primarily from philosophical considerations; others from a practical concern with engines and machines, or from laboratory investigations of energy conversion; still others from a combination of factors.



Hermann von Helmholtz

The wide acceptance of the law of conservation of energy owes much to a paper published in 1847 (two years before Joule published the results of his most precise experiments) by the young German physician and physicist, Hermann von Helmholtz and entitled "On the Conservation of Force." In the paper Helmholtz boldly asserted the idea that others were vaguely expressing; namely, "that it is impossible to create a lasting motive force out of nothing." The idea was even more clearly expressed many years later in one of Helmholtz's popular lectures:

> We arrive at the conclusion that Nature as a whole possesses a store of force [energy] which cannot in any way be either increased or diminished, and that, therefore, the quantity of force [energy] in Nature is just as eternal and unalterable as the quantity of matter. Expressed in this form, I have named the general law 'The Principle of the Conservation of force [energy].'

Helmholtz's paper, "Zur Erhaltung der Kraft," was tightly reasoned and mathematically sophisticated. It related the law of conservation of energy to the established principles of Newtonian mechanics and thereby helped make the law scientifically respectable.

Any machine or engine that does work—provides energy—can do so only by drawing from some source of energy. The machine cannot supply more energy than it obtains from the source, and when the source is depleted, the machine will stop working. Machines and engines can only transform energy; they cannot create it or destroy it.

An isolated system in this case is one which does not exchange heat with its surroundings, which does no work and on which no work is done.
The internal energy of an isolated system remains constant.

A collection of bodies moving and colliding and exerting forces on one another makes up a system. The total kinetic energy and potential energy of the bodies in the system is the internal energy of the system. If objects outside the system do work on the bodies in the system, the internal energy of the system will increase. The internal energy will

The internal energy of a system may be constant even if it is not an isolated system, provided that the net heat H added to the system and the net work W done on the system are both zero.

increase also if heat is transferred from the surroundings to the system. On the other hand, if the system does work on objects outside the system, or if heat is transferred from the system to its surroundings, the internal energy will decrease. Transferring heat and doing work are the two ways of changing the internal energy of the system. These ideas can be summarized by an equation. Let H stand for (heat added to system - heat lost by system); let W stand for (work done on system - work done by system); and let ΔU stand for the increase in internal energy of the system. Then

$$H + W = \Delta U. \qquad (10.10)$$

This statement of the law of conservation of energy, emphasizing the equivalence of heat and work, is called the first law of thermodynamics.

Thermodynamics is the study of the relation between heat and mechanical energy.

Since the middle of the nineteenth century the law of conservation of energy has become one of the most fundamental laws of science. We shall encounter it again and again in this course: in the study of electricity and magnetism, in the study of the structure of atoms, and in the study of nuclear physics.

See "Energy" in Project Physics Reader 3

The principle of conservation of energy has been so successful and is now so firmly believed that most physicists find it almost inconceivable that any new phenomenon will be found that will disprove it. Whenever energy seems to appear or disappear in a system, without being accounted for by changes in known forms of energy, physicists naturally prefer to assume that some unknown kind of energy is involved, rather than accept the possibility that energy is not conserved. We have already mentioned one example of this attitude: the concept of "internal energy" was introduced in order to preserve the validity of the principle of conservation of energy in the case of inelastic collisions and frictional processes. In this case, the physicist's faith in energy conservation was justified, because other evidence showed that internal energy is a useful concept, and that it changes by just the right amount to compensate for changes in external energy.

See "Scientific Cranks" in Project Physics Reader 3.

Another recent example is the "invention" of the neutrino by Wolfgang Pauli in 1933. Experiments had suggested that energy disappeared in certain nuclear reactions; but Pauli proposed that a tiny particle, the neutrino, was produced in these reactions and, unnoticed, carried off some of the energy. Physicists believed in the neutrino theory for more than twenty years even though neutrinos could not be directly found by any method. Finally, in 1956, neutrinos were detected, in

Summary 10.9

1. In the early nineteenth century, developments in the science of electricity, knowledge being gained from the increasing application of steam engines, and the influence of the German philosophical movement called "nature philosophy" combined to encourage the idea that the law of conservation of mechanical energy could be extended to a more general law of conservation of all forms of energy, including heat.

2. The law of conservation of energy, formulated by Mayer, Joule and the others, was most clearly stated by Helmholtz, who maintained that although it may change its form, the total amount of energy in the universe remains constant.

3. Other forms of potential energy are elastic potential energy, electric potential energy and magnetic potential energy.

an experiment using the large flow of neutrinos that comes out of a *nuclear reactor. (The experiment could not have been* done in 1933, since no nuclear reactors existed until nearly a decade later.) Again, the faith of physicists in the law of conservation of energy turned out to be justified.

We believe this will probably always be the case: any apparent exceptions to the law of conservation of energy will sooner or later turn out to be understandable in a way which does not force us to give up the law. At most, they may force us to postulate new forms of energy so that the law will become even more generally applicable and powerful.

The French mathematician and philosopher Henri Poincaré expressed this idea in 1903 in his book Science and Hypothesis.

> Since we can not give a general definition of energy, the principle of conservation of energy signifies simply that there is something which remains constant. Indeed, no matter what new notions future experiences will give us of the world, we are sure in advance that there will be something which will remain constant and which we shall be able to call energy.

Q23 The significance of German nature philosophy in the history of science is that

a) it was the most extreme form of the mechanistic viewpoint.
b) it was a reaction against excessive speculation.
c) it stimulated speculation about the unity of natural forces.

Q24 Discoveries in electricity and magnetism early in the nineteenth century contributed to the discovery of the law of conservation of energy because

a) they attracted attention to conversions of energy from one form to another.
b) they made it possible to produce more energy at less cost.
c) they revealed what happened to the energy that was apparently lost in steam engines.
d) they made it possible to transmit energy over long distances.

Q25 The development of steam engines helped the discovery of the law of conservation of energy because

a) steam engines produce a large amount of energy.
b) the caloric theory could not explain how steam engines worked.
c) steam engines used up so much water that other sources of energy had to be found.
d) the concept of work was developed in order to compare the economic value of different engines.

Q26 According to the first law of thermodynamics

a) the net heat added to a system is always conserved.
b) the net heat added to a system always equals the net work done by the system.
c) energy input equals energy output if the internal energy of the system does not change.
d) the internal energy of a system is always conserved.

Q27 Both Mayer and Joule helped establish the law of conservation of energy. Compare their approaches to the question.

The ultimate source of most of the energy we use is the sun. Water warmed by the sun evaporates to form clouds from which the rain falls to replenish the rivers that drive the hydroelectric generators. Winds, that pushed Phoenician ships to the Pillars of Hercules and beyond, that made possible Columbus' voyage and Magellan's circumnavigation of the world, are produced when the sun warms parts of the earth's atmosphere.

The energy we obtain from food was once solar energy, locked into molecules in green plants by the complex process called photosynthesis, and released in our bodies by the process called respiration.

Coal and oil, still our major sources of industrial energy, are fossilized remains of plants and animals, with energy from sunlight still locked within their molecules.

Study Guide

10.1 An electron of mass 9.1×10^{-31} kg is traveling 2×10^6 meters per second toward the screen of a television set. What is its kinetic energy? How many electrons like this one would it take to make up a joule of energy? 1.82×10^{-18} j 5.5×10^{17} electrons

10.2 Estimate the kinetic energy of each of the following:

a) a pitched baseball 67.5 j
b) a jet plane 4.5×10^9 j
c) a sprinter in a 100-yard dash 3750 j
d) the earth in its motion around the sun 2.7×10^{33} j

10.3 As a home experiment, hang weights on a rubber band and measure its elongation. Plot the force vs. stretch on graph paper. How could you measure the stored energy? Discussion

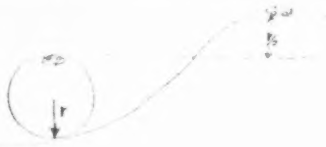
10.4 A penny has a mass of about 3.0 grams and is about 1.5 millimeters thick. You have 50 pennies which you pile one above the other.

a) How much more gravitational potential energy has the top penny than the bottom one? .00225 j
b) How much more have all 50 pennies together than the bottom one? .056 j

10.5 Discuss the following statement: All the chemical energy of the gasoline used in your family automobile is used only to heat up the car, the road, and the air. Discussion

10.6 A 200-kilogram iceboat is supported by a smooth surface of a frozen lake. The wind exerts on the boat a constant force of 1000 newtons, while the boat moves 900 meters. Assume that frictional forces are negligible, and that the boat starts from rest. Find the speed after 900 meters by each of the following methods:

a) Use Newton's second law to find the acceleration of the boat. How long does it take to move 900 meters? How fast will it be moving then? 5 m/sec^2 19 sec   95 m/sec
b) Find the final speed of the boat by equating the work done on it by the wind and the increase in its kinetic energy. Compare your result with your answer in (a). 95 m/sec



10.7 The figure shows a model of a carnival "loop-the-loop." A car starting from a platform above the top of the loop coasts down and around the loop without falling off the track. Show that, to successfully traverse the loop, the car need start no higher than one-half a radius above the top of the loop. Neglect frictional forces.

HINT: The centripetal force required at the top of the loop must be greater than the weight of the car. Derivation

10.8 A cardboard tube a meter long closed at both ends contains some lead shot. It is turned end for end 300 times, each time allowing the shot to fall the length of the tube and come to rest before inverting it again. If the shot loses a negligible amount of heat to the tube, what is the increase in temperature? Try it. 22°

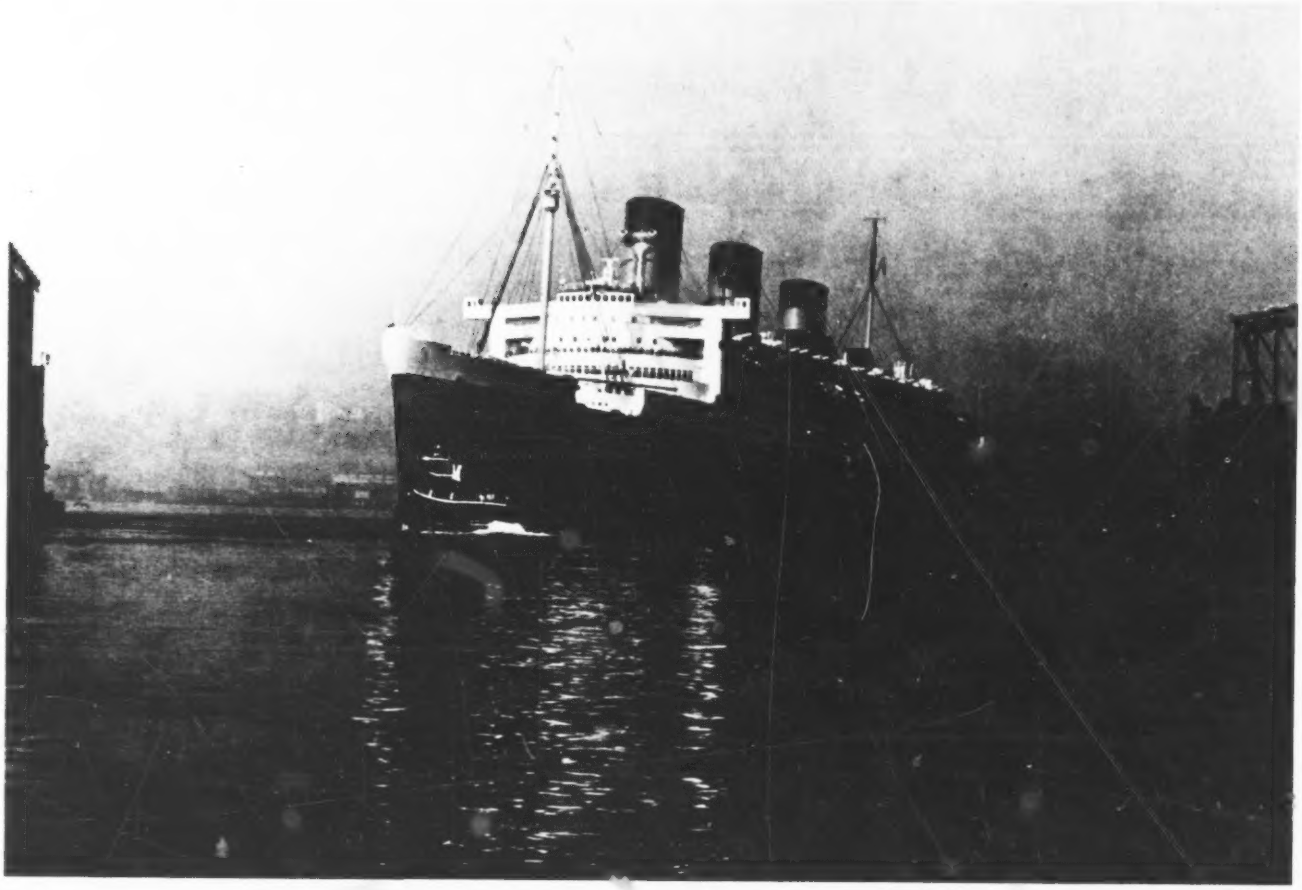
10.9 An electric coffee pot holds a kilogram (about a quart) of water and is rated at 600 watts. Starting from room temperature (20°C), estimate how long it will take the water to reach boiling temperature. What assumptions have you made?

9.3 min

10.10 Show that if a constant propelling force F keeps a vehicle moving at a constant speed v, the power required is equal to Fv. Derivation

10.11 The Queen Mary, one of Britain's largest steamships, has been retired to a marine museum on our west coast after completing 1,000 crossings of the Atlantic. Her mass is 81,000 tons (75 million kilograms) and her maximum engine power of 234,000 horsepower (174 million watts) gives her a maximum speed of 30.63 knots (16 meters per second).

a) What is her kinetic energy at full speed? 9.6×10^9 J
b) What constant force would be required to stop her from full speed within 10 nautical miles (20,000 meters)? 4.8×10^5 N
c) What power would be required to keep her going at full speed against this force? 7.68×10^6 watts
d) Assume that at maximum speed all the power output of her engines goes into overcoming water drag. If the engines are suddenly stopped, how far will the ship coast before stopping? (Assume water drag is constant.) 880 m
e) The assumptions made in (d) are not valid for the following reasons:
    1) Only about 60% of the power delivered to the propellor shafts results in a forward thrust to the ship; the rest results in turbulence, eventually warming the water.
    2) Water drag is less for lower speed.
    3) If the propellors are not free-wheeling, they add an increased drag. Which of the above reasons tend to increase, which to decrease the coasting distance? 1 & 2 increase, 3 decreases
f) Explain why tugboats are important for docking big ships. Change direction



10.12 Consider the following hypothetical values for a paddle-wheel experiment like Joule's: a 1 kilogram weight descends through a distance of 1 meter, turning a paddle-wheel immersed in 5 kilograms of water.

- a) About how many times must the weight be allowed to fall in order that the temperature of the water will be increased by 1/2 Celsius degree? 104.6 times
- b) List ways you could modify the experiment so that the same temperature rise would be produced with fewer falls of the weight? (There are at least four possible ways.) Increase weight, decrease amount of water, increase height of fall, lower specific heat

10.13 On his honeymoon in Switzerland, Joule attempted to measure the difference in temperature between the top and the bottom of a waterfall. Assuming that the amount of heat produced at the bottom is equal to the decrease in gravitational potential energy, calculate roughly the temperature difference you would expect to observe between the top and bottom of a waterfall about 50 meters high, such as Niagara Falls. 0.12°c

10.14 Devise an experiment to measure the power output of

- a) a man riding a bicycle.
- b) a motorcycle.
- c) an electric motor. Discussion

10.15 When a person's food intake supplies less energy than he uses, he starts "burning" his own stored fat for energy. The oxidation of a pound of animal fat provides about 4,300 kilocalories of energy. Suppose that on your present diet of 4,000 kilocalories a day you neither gain nor lose weight. If you cut your diet to 3,000 kilocalories and maintain your present physical activity, about how many weeks would it take to reduce your mass by 5 pounds? 3 weeks

10.16 About how many kilograms of boiled potatoes would you have to eat to supply the energy for a half-hour of swimming? Assume that your body is 20% efficient. 1.5 kg

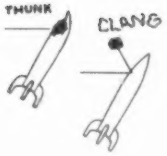
10.17 In order to engage in normal light work, an average native of India needs about 1,950 kilocalories of food energy a day, whereas an average West European needs about 3,000 kilocalories a day. Explain how each of the following statements makes the difference in energy need understandable.

- a) The average adult Indian weighs about 110 pounds; the average adult West European weighs about 150 pounds.
- b) India has a warm climate.
- c) The age distribution is different in India. Discussion

10.18 Show how the conservation laws for energy and momentum apply to a rocket lifting off. Discussion

10.19 In each of the following, trace the chain of energy transformations from the sun to the final form of energy:

- a) A pot of water is boiled on an electric stove.
- b) An automobile accelerates from rest on a level road, climbs a hill at constant speed, and comes to stop at a traffic light.
- c) A windmill in Holland pumps water out of a flooded field. Discussion



10.20 The two identical space vehicles shown here were drifting through interstellar space. Each was struck by a 10-kilogram meteor traveling at 100 m/sec. Which rocket was knocked further off course? Explain. Undamaged rocket

10.21 A 2-gram bullet is shot into a tree stump. It enters at a speed of 300 m/sec and comes to rest after having penetrated 5 cm in a straight line. Compute the average force on the bullet during the impact, and the work done. 1800 N 90 J

10.22 In the Prologue to Unit 1 of this course, it was stated that Fermi used materials containing hydrogen to slow down neutrons. Explain why collisions with light atoms would be more effective in slowing down neutrons than collisions with heavy atoms. Discussion

10.23 The actual cost of moving furniture and individuals various distances is shown in the following tables. Using these tables, discuss the statement: "the cost of moving is approximately proportional to the amount of work that has to be done on it, using the physicist's definition of work." Discussion

Truck Transportation (1965)

Weight	Moving rates (including pickup & delivery) from Boston to:		
	Chicago (967 miles)	Denver (1969 miles)	Los Angeles (2994 miles)
100 lbs	$ 18.40	$ 24.00	$ 27.25
500	92.00	120.00	136.25
1000	128.50	185.50	220.50
2000	225.00	336.00	406.00
4000	384.00	606.00	748.00
6000	576.00	909.00	1122.00

Air Cargo Transportation (1965)

Weight	Moving rates (including pickup & delivery) from Boston to:		
	Chicago	Denver	Los Angeles
100 lbs	$ 13.95	$ 25.57	$ 29.85
500	70.00	127.50	149.25
1000	129.00	216.50	283.50
2000	248.00	413.00	527.00
4000	480.00	796.00	1024.00
6000	708.00	1164.00	1255.00

Personal Transportation (1965)

	One way fare from Boston to:		
	Chicago	Denver	Los Angeles
Bus	$33.75	$58.40	$88.75
Train	$47.85	$79.31	$115.24
Airplane (jet coach)	$53.39	$107.36	$167.58

Chapter 11 The Kinetic Theory of Gases

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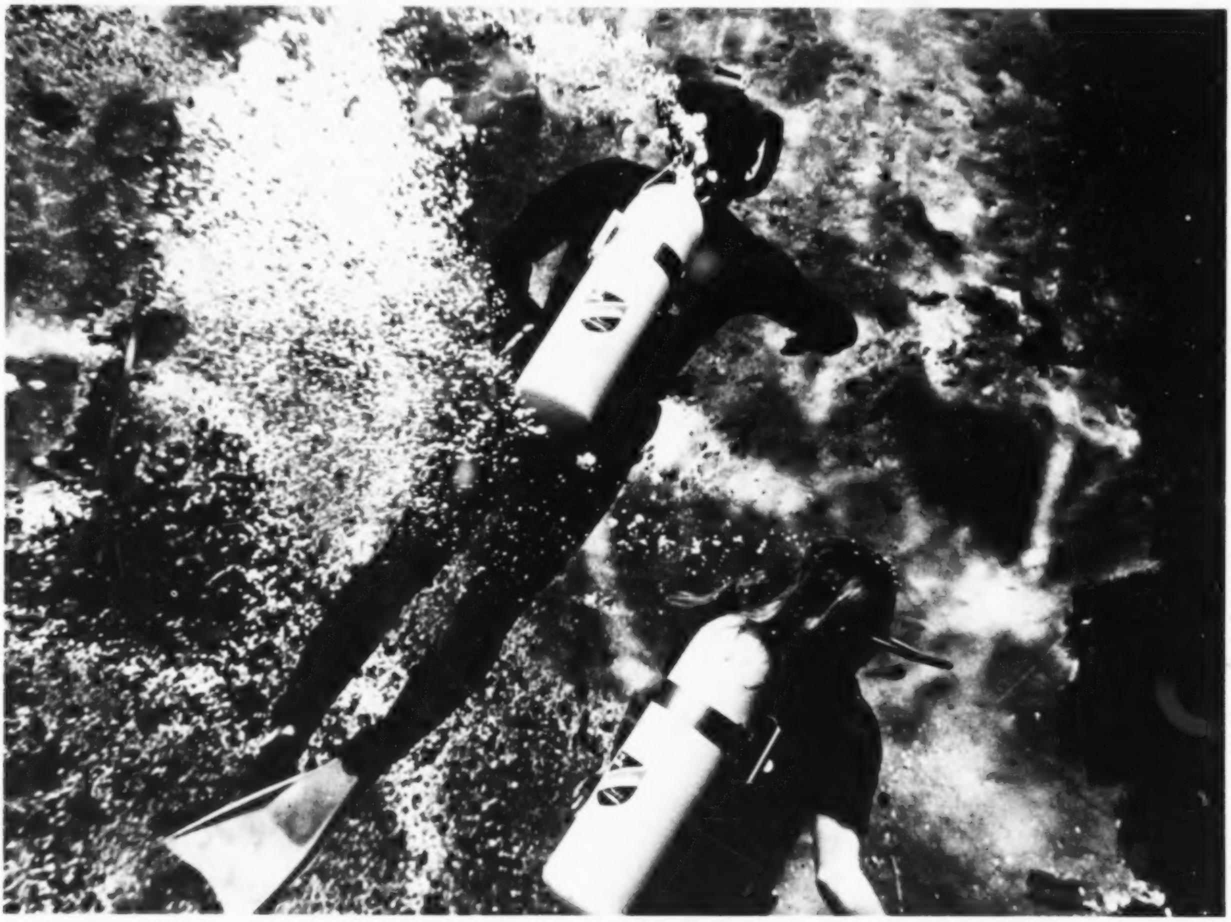
Bubbles of gas from high-pressure tanks expand as the pressure decreases on the way to the surface.



11.1 An overview. During the 1840's many scientists came to the conclusion that heat is not a substance but a form of energy which can be converted into other forms (see Chapter 10). Two of these scientists, James Prescott Joule and Rudolf Clausius, then took what seemed to be the next logical step: they assumed that since heat can be changed into mechanical energy and mechanical energy can be changed into heat, then heat itself might be a form of mechanical energy. They proposed, as a first approximation, that "heat energy" is simply the kinetic energy of atoms and molecules.

In general, when the substance is heated, the kinetic energy of its molecules increases and the molecules move farther apart against attractive intermolecular forces.

Since nineteenth-century scientists could not observe the motions of individual molecules, they could not check directly the hypothesis that heat is molecular kinetic energy. Instead, they first had to work out some mathematical predictions from the hypothesis, and then try to test these predictions by experiment. For theoretical reasons which we will explain below, it is easiest to test such hypotheses by working with the properties of gases, and therefore this chapter will deal primarily with what is called the kinetic theory of gases.

The development of the kinetic theory of gases in the nineteenth-century led to the last major triumph of Newtonian mechanics. By using a simple theoretical model of a gas, and applying Newton's laws of motion, scientists could deduce equations which related observable properties of gases—such as pressure, density and temperature—to the sizes and speeds of molecules. With these equations, kinetic theorists could

(1) explain the known relations between observable properties of gases, such as "Boyle's law";

(2) predict new relations, such as the fact that the viscosity of a gas increases with temperature but is independent of density;

(3) infer the sizes and speeds of the molecules.

Thus the success of kinetic theory indicated that Newtonian mechanics could be used to investigate the properties of matter as observed in the laboratory, and for the first time provided a reliable way of estimating the properties of individual molecules.

By applying Newtonian mechanics to a mechanical model of gases, however, the kinetic theorists made another discovery: they found physical situations in which Newtonian mechanics is not valid. According to kinetic theory, the energy of a molecule should be equally shared by the motions of all the atoms inside the molecule. But the properties of gases

Summary 11.1

1. Following the discovery of conservation of energy in the middle of the nineteenth century, scientists started to explore what seemed to be a natural extension of this discovery: the hypothesis that heat is just molecular kinetic energy. (We now know that heat involves other forms of internal energy as well.)

2. The kinetic theory of gases was successful insofar as it led to useful explanations of known properties of gases, prediction of new properties, and estimates of molecular sizes and speeds.

3. Research in kinetic theory also raised doubts about the complete validity of Newtonian mechanics as applied to molecules: the problem of energy-sharing ("equipartition theorem") and the problem of irreversibility.

Recall from chemistry that molecules may contain several atoms.

deduced from this "equal sharing" principle were definitely in disagreement with experiment. Newtonian mechanics could be applied successfully to the motions and collisions of molecules among each other in a gas, but not to the motions of atoms inside molecules. It was not until the twentieth century that an adequate theory of the behavior of atoms — quantum mechanics — was developed (see Unit 5).

Kinetic theory also revealed another apparent contradiction between Newtonian mechanics and observable properties of matter: this is the problem of "irreversibility." An inelastic collision is an example of an irreversible process; other examples are the mixing of two gases, or scrambling an egg. Can irreversible processes be described by a theory based on Newtonian mechanics, or do they involve some new fundamental law of nature? In discussing this problem from the viewpoint of kinetic theory, we will see how the concept of "randomness" was introduced into physics.

Modern physicists do not take seriously the "billiard ball" model of gas molecules—nor did nineteenth-century physicists, for that matter. The simple postulates of the model have to be modified in many respects in order to get a theory that agrees well with experimental data. Nevertheless, physicists are still very fond of the kinetic theory, and often present it as an example of how a physical theory should be developed. Perhaps this is only nostalgia for the old mechanistic style of explanation which has had to be abandoned in other areas of physics. But as an example of an ideal type of theory, kinetic theory has exerted a powerful influence on physics research and teaching. In Sec. 11.5, therefore, you will find one of the mathematical derivations from a mechanical model which is used in kinetic theory. This derivation is given, not to be studied in detail, but as an illustration of the kind of mathematical reasoning based on mechanical models which physicists have found to be useful in understanding nature.

Q1 Nineteenth-century kinetic theorists assumed that heat is

a) a fluid.
b) molecular.
c) the kinetic energy of molecules.
d) made of molecules.

Q2 In the kinetic theory of gases, it is assumed that Newton's laws of motion (do, do not) apply to the motion and collisions of molecules.

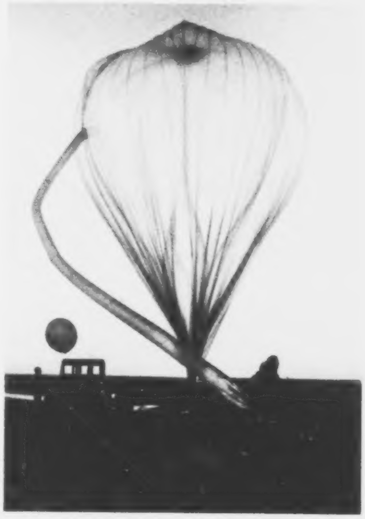
Q3 True or false: In the twentieth century Newtonian mechanics was found to be applicable not only to molecules but also to the atoms inside molecules.

Summary 11.2
1. According to kinetic theory, intermolecular forces should be relatively unimportant in gases as compared to liquids and solids; hence the theory of gases could be developed before much was known about these forces. 11.2

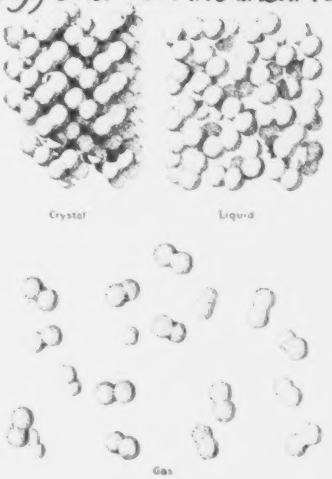
11.2 A model for the gaseous state. What is the difference between a gas and a liquid or solid? We know by observation that a liquid or solid has a definite volume. A gas, on the other hand, will expand to fill any container, or, if unconfined, will go off in all directions. We can also find out easily that gases have low densities compared to liquids and solids. According to the kinetic theory, the molecules in a gas are moving freely through empty space most of the time, occasionally colliding with each other or with the walls of their container. Furthermore, we assume that the forces between molecules act only at very short distances, whereas the molecules are usually far apart from each other. Therefore gas molecules, in this model, are considered "free" except during collisions. In liquids, on the other hand, the molecules are so close together that these forces keep them from flying apart. In solids, the molecules are usually even closer together, and the forces between them keep them in a definite orderly arrangement.

In the nineteenth century, little was known about the forces between molecules, so it was natural to apply the kinetic theory first to gases, where these forces should have little effect if the theory is right. Physicists therefore adopted a simple model of gases, in which the molecules are considered to behave like miniature billiard balls—that is, tiny spheres which exert no forces at all on each other except when they collide. All the collisions of these spheres are assumed to be elastic, so that the total kinetic energy of two spheres is the same before and after they collide. The total volume occupied by all these spheres is assumed to be very small compared to the total volume of the container.

Note that the word "model" is used in two different senses in science. In Chapter 10, we mentioned the model of Newcomen's engine which James Watt was given to repair. That was a working model which actually could do work, although it was much smaller than the original engine, and some of its parts were made of different materials. We are now discussing a theoretical model of a gas; this model exists only in our imagination. Like the points, lines, triangles and spheres which are studied in geometry, this theoretical model can be discussed mathematically, and the results may or may not be useful to us when we try to understand the real world of experience. In order to emphasize the fact that the model is only a theoretical one, we will use the word "particle" instead of atom or molecule. Atoms and molecules exist and


Weather balloon of U.S. Air Force.

Molecules of paired atoms, as appear in the bottom drawing, were a later refinement of the model. (The relative motion of the parts of a molecule involved forms of internal energy other than the linear kinetic energy.)



Sketch of the relative spacing of atoms in three states of matter.

from General Chemistry, 2nd edition, by Linus Pauling, W.H. Freeman and Company. © 1953.

2. It is important to distinguish between a theoretical model in science (such as is used in kinetic theory) and a working model. 3. The kinetic theory model of a gas consists of a large number of very small particles in rapid disordered motion; the particles move freely most of the time, exerting forces on each other only when they collide.

have definite properties that can be determined in the laboratory. Particles are imaginary objects, such as perfectly elastic spheres, whose properties as defined in our theoretical model may or may not be similar to those of atoms and molecules. We will develop the model in such a way that it can be used to explain as well as possible the particular properties of gases in which we are interested.

The word "gas" was originally derived from the Greek work "chaos"; it was first used by the Belgian chemist Joan Baptista van Helmont (1580-1644).

Our model of a gas consists of a large number of very small particles in rapid, disordered motion. We now know enough about gases to say more precisely how we should interpret this phrase if we want to have a useful model. "A large number" here means something like a billion billion (10^{18}) or more. "Very small" means about a hundred-millionth of a centimeter (10^{-8} cm, or 10^{-10} meters) in diameter. "Rapid motion" means a speed of a few hundred meters per second. In the following sections we will see how the model was used by kinetic theorists to arrive at these estimates.

What do we mean by "disordered"? In the nineteenth century, kinetic theorists assumed that if they could follow the motion of an individual molecule they would see it move in a definite way determined by Newton's laws of motion. However, it would be very difficult to calculate the properties of the kinetic theory gas model if we had to apply Newton's laws to a billion billion particles. We don't know in what direction any given particle will be moving. Furthermore, even though we can estimate the average speed of the particles, different particles will have different speeds and each particle will frequently change its speed. Therefore from our viewpoint the motions of the individual particles are disordered and unpredictable, and it is useful to assume that these motions are random even though we believe that they are really determined by the laws of motion.

See "The Law" and "Randomness and the Twentieth Century" in Project Physics Reader 3

On page 86 you will find a more detailed discussion of the idea of random fluctuations.

The idea of randomness originally entered physics through the kinetic theory of gases. The disadvantage of having to consider the motions of a large number of particles actually becomes an advantage as soon as one assumes that the particles move randomly. You can see why this is so by thinking about the results of flipping a coin. It would be very hard to predict how a single coin would behave, but you can confidently predict that flipping a million coins will give about 50 percent heads and 50 percent tails. Similarly, it is a safe bet that out of a billion billion molecules about as many will be moving in one direction as in another. It is also reasonable to expect that the particles are equally likely to be found anywhere inside the container, even though we don't

A good way to visualize the model is to look at a demonstration of Brownian motion. Particles which are small, but large enough to be seen in a microscope, move around randomly when suspended in a gas or liquid. Their motion is a result of their continual bombardment of molecules from all directions. 11.3

know where a given particle is at any particular time.

In summary: we are going to discuss the properties of a mechanical model for gases. The model consists of a large number of very small particles in rapid disordered motion. The particles move freely most of the time, exerting forces on each other only when they collide. The model is designed to represent the structure of real gases in some ways, but it is simplified in order to make calculations possible. By comparing the results of these calculations with the properties of gases it is possible to estimate the speeds and sizes of molecules, assuming that the model itself is a reasonably good description of gases.

See "On the kinetic theory of gases" and "The Law of Disorder" in Project Physics Reader 3

Q4 Molecules exert forces on one another

a) only when the molecules are far apart.
b) only when the molecules are close together.
c) all the time.
d) never.

Q5 Why was the kinetic theory first applied to gases rather than to liquids or solids?

11.3 The speeds of molecules. The basic idea of the kinetic theory—that heat is related to the kinetic energy of molecular motion—had been frequently suggested in the seventeenth century. In 1738, the Swiss mathematician Daniel Bernoulli showed how the kinetic theory could be used to explain a well-known property of gases: pressure is proportional to density (Boyle's law). Bernoulli assumed that the pressure of a gas is simply a result of the impacts of individual molecules striking the wall of the container. If the density were doubled, there would be twice as many molecules striking the wall per second, and hence twice the pressure.

Pressure is defined as the perpendicular force on a surface divided by the area of the surface.

Bernoulli also proved mathematically that the pressure would be proportional to the square of the speed of the molcules. Later scientists used this proof to establish relations between pressure, molecular speed and temperature (see Sec. 11.5). However, in Bernoulli's time, the modern temperature scales had not been established, and Bernoulli did not have enough experimental data to develop this part of his theory.

DANIEL BERNOVLLIVS

Daniel Bernoulli

Although Bernoulli's theory seems to have been the first step leading to the modern kinetic theory of gases, it was generally ignored by other scientists in the eighteenth century. One reason for this was that Newton had proposed a different theory in his Principia (1687). Newton showed that Boyle's law could be explained by assuming that a gas is composed of particles at rest that exert repulsive forces

Summary 11.3

1. Bernoulli (1738) proved that gas pressure is proportional to the square of the (average) speeds of the molecules. 2. Bernoulli's theory was ignored for a long time because Newton had proposed an alternative theory, in which pressure is due to repulsive forces between motionless molecules. 3. John Herapath (1820) rediscovered Bernoulli's result and used it to compute molecular speeds. In modern symbols, $P = \frac{1}{3} Dv^2$, where D = density of gas; if P and D are measured, v can be calculated.

on neighboring particles, if these forces are inversely proportional to the distance between particles. Although Newton did not claim that he had proved that gases really are composed of such repelling particles, other scientists were so impressed by Newton's discoveries in other areas of physics that they assumed his theory of gas pressure must also be right.

John Herapath

The kinetic theory of gases was proposed again in 1820 by an English physicist, John Herapath. Herapath rediscovered Bernoulli's results on the relation between pressure and density of a gas and the speeds of the particles. In modern symbols, we can express these results by the simple equation

$$P = \frac{1}{3} Dv^2 \qquad (11.1)$$

where P = pressure which the gas exerts on the container, D = density (mass/volume) and v = average speed. (This equation is important still, and will be derived in Sec. 11.5) Since we can determine the pressure and density of a gas by experiment, we can use this result to calculate the average speed of the molecules. Herapath did this, and found that the result was fairly close to the speed of sound in the gas, about 330 meters per second for air.

Herapath's calculation of the speed of an air molecule (first published in 1836) was an important event in the history of science, but it was ignored by most other scientists. Herapath's earlier work on the kinetic theory had been rejected for publication by the Royal Society of London, and despite a long and bitter battle (including letters to the Editor of the Times of London) Herapath had not succeeded in getting any recognition for his theory.

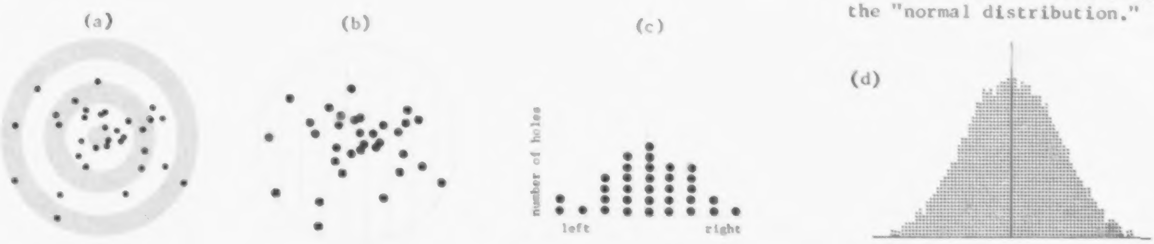
James Prescott Joule did see the value of Herapath's work, and in 1848 he read a paper to the Manchester Literary and Philosophical Society in which he tried to revive the kinetic theory. Joule showed how the speed of a hydrogen molecule could be computed (using Herapath's theory), and obtained a value of 2000 meters per second at 0°C, the freezing temperature of water.

Joule's paper also was ignored by other scientists, because physicists do not generally look in the publications of a "literary and philosophical society" for important papers. But once the equivalence of heat and mechanical energy was generally accepted, it was not long before other physicists independently started to work out the consequences of the assumption that heat is molecular energy. Rudolf Clausius in Germany published a paper in 1856 on "The Nature of the Motion we call Heat," which established the basic principles of kinetic theory in almost the form we accept them today. Soon afterwards, James Clerk Maxwell in Britain, and Ludwig Boltzmann in Austria, started to work out the full mathematical details of the theory.

The Maxwell velocity distribution. Not all molecules in a gas have the same speed. In 1859 Maxwell suggested that the speeds of molecules in a gas could be described by a statistical law: most molecules have speeds not very far from the average speed. but a few have much lower speeds and a few have much higher speeds.

Maxwell's law of distribution of molecular speeds can be illustrated by the following analogy: if a marksman shoots a gun at a target many times, some of the bullets will probably hit the bullseye, but others will miss by smaller or larger amounts. Suppose we count the number of bullets that hit the target at various distances to the left and right of the bullseye, and make a graph showing the number of bullets at these distances.

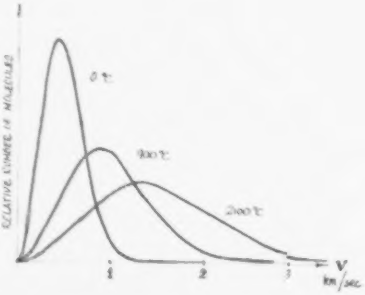
Students may be familiar with the bell shaped curve because it often applies to the distribution of scores on tests. Teachers sometimes use this curve in deciding which scores should correspond to various letter grades - hence the expression "grading on the curve", meaning that your grade depends not only on your own score but on how it is related to the distribution of scores in the class.



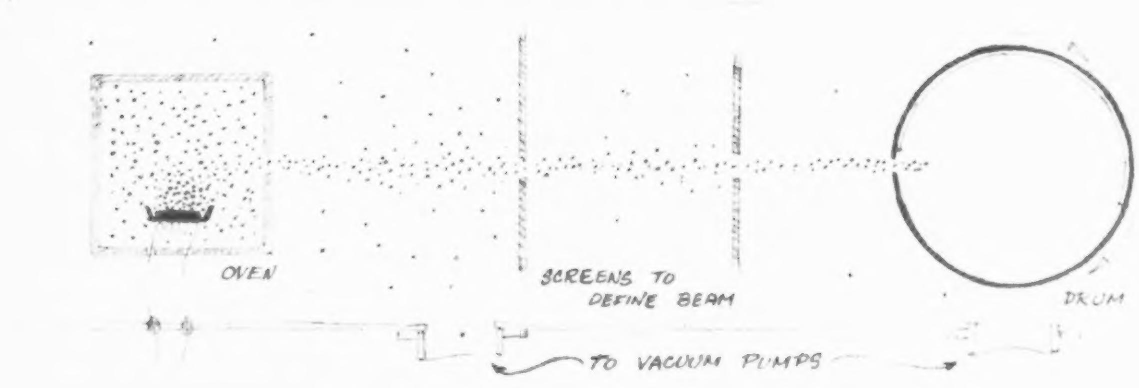
TARGET PRACTICE EXPERIMENT
(a) Scatter of holes in target; (b) target marked off in distance intervals left and right of center; (c) frequency distribution of holes left and right of center. If, as in (d), there is a very large number of cases, such distribution often closely approximate the mathematical curve called the "normal distribution."

The graph showing the distribution of misses illustrates a general principle of statistics, the law of normal distribution: if any quantity varies randomly about an average value, then the graph showing the frequencies of deviations from the average will look something like the one shown in the margin. There will be a peak at about the average value and a sharp decline on either side. A similar "bell shaped curve," as it is frequently called, describes the distribution of many kinds of physical measurements. During the nineteenth century, for example, statisticians found that the normal distribution law applies even to large collections of people, as in the distribution of human heights.

Maxwell's distribution law for speeds is shown in the margin. Note that the peak shifts to higher speeds as the temperature increases. The tail of the curve is much longer on the right (high speeds) than on the left (low speeds), since no molecules can have less than zero speed.

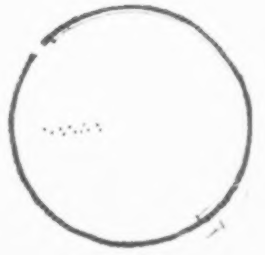


Maxwell's distribution of molecular speeds for 3 different temperatures.

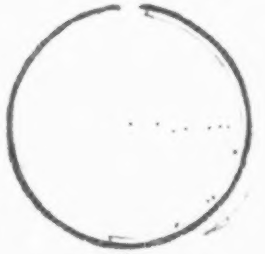


Direct Measurement of Molecular Speeds

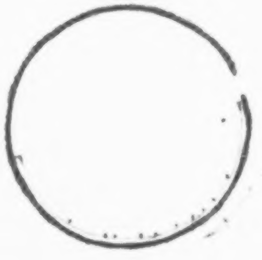
A narrow beam of molecules is formed by letting a hot gas pass through a series of slits. In order to keep the beam from spreading out, collisions with randomly moving molecules must be avoided; therefore, the source of gas and the slits are housed in a highly evacuated chamber. The molecules are then allowed to pass through a slit in the side of a cylindrical drum which can be spun very rapidly. The general scheme is shown above.



As the drum rotates, the slit moves out of the beam of molecules so that no more molecules can enter until the drum has rotated through a whole revolution. Meanwhile the molecules in the drum continue moving to the right, some moving fast and some moving slowly.

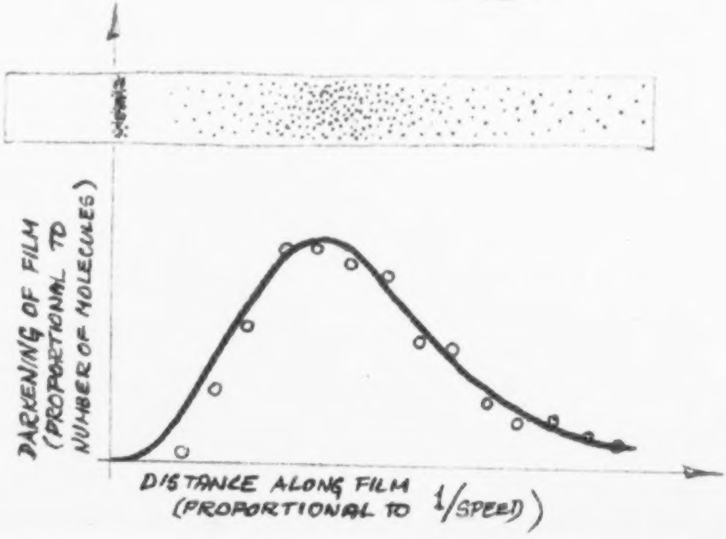


Fastened to the inside of the drum is a sensitive film which acts as a detector; any molecule striking the film leaves a mark. The faster molecules strike the film first, before the drum has rotated very far.



The slower molecules hit the film later, after the drum has rotated farther. In other words, molecules of different speeds strike different parts of the film. The darkness of the film at any point is proportional to the number of molecules which hit it there. Measurement of the darkening of the film shows the distribution of molecular speeds.

The speckled strip at the right represents the unrolled film, showing the impact position of molecules over many revolutions of the drum. The heavy band indicates where the beam struck the film before the drum started rotating.



A comparison of the experimental results with those predicted from theory is shown in the graph. The dots show the experimental results and the solid line represents the predictions from the kinetic theory.

11.4

What reason do we have for thinking that Maxwell's distribution law really applies to molecular speeds? Several successful predictions based on this law gave indirect support to it, but it was not until the 1920's that a direct experimental check was possible. Otto Stern in Germany, and later Zartmann in the United States, devised a method for measuring the speeds in a beam of molecules (see the illustration of Zartmann's method on the preceding page). Stern, Zartmann and others found that molecular speeds are indeed distributed in accordance with Maxwell's law.

Q6 In the kinetic theory of gases it is assumed that the pressure of a gas is due to

a) gas molecules colliding with one another.
b) gas molecules colliding with the walls of the container.
c) repulsive forces exerted by molecules on each other.

Q7 The average speed of molecules in a gas can be calculated if we know

a) the pressure of the gas and the volume of the container.
b) the mass of the gas and the volume of the container.
c) the pressure of the gas and its density.

11.4 The sizes of molecules. Is it reasonable to suppose that gases are composed of molecules that move at speeds of several hundred meters per second? In 1857, a Dutch meteorologist, Christian Buys-Ballot, pointed out that if this were really true, one would expect gases to diffuse and mix with each other very rapidly. But anyone who has studied chemistry knows that if hydrogen sulfide or chlorine is generated at one end of a large room, it may be several minutes before it is noticed at the other end. Yet, according to the kinetic-theory calculations we mentioned in the last section, each of the gas molecules should have crossed the room hundreds of times by then. Something must be wrong with our model.

Rudolf Clausius realized that this discrepancy was a valid objection to his own version of the kinetic theory. In his paper published in 1856, he had assumed that the size of the particles is so small that they can be treated like mathematical points. If this were true, particles would almost never collide with each other. However, in order to account for the gas properties pointed out by Buys-Ballot—that is, the slowness of diffusion and mixing—Clausius decided to change the model. He thought it was likely that in real gases, the molecules are not mathematical points but have a finite size. He realized that if the model were made more realistic by assuming that the particles have finite size and can therefore collide with each other, it would then be possible to explain why gases do not diffuse rapidly.

The larger molecules are, the more likely they are to collide with each other.

Averages and Fluctuations

Molecules are too small, too numerous, and too fast for us to measure the speed of any one molecule, or its kinetic energy, or how far it moves before colliding with another molecule. For this reason the kinetic theory of gases concerns itself with making predictions about average values. The theory enables us to predict quite precisely the average speed of the molecules in a sample of gas, or the average kinetic energy, or the average distance the molecules move between collisions.

The average of a small number of cases cannot be predicted very well. Although the average height of adult American men is 5'9½", it would be very unlikely that the average height of any particular group of 10 men would be that value. However, statistical predictions can be very precise for very large sets of values. The average height of all adult men in Ohio would be very close to 5'9½". The precision in predicting average values for very large samples is what makes the kinetic theory of gases so successful, for molecules are very numerous indeed.

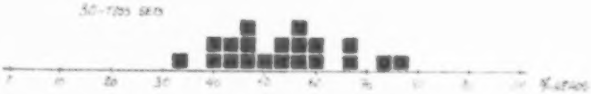
A simple example of a statistical prediction is the statement that if a coin is tossed many times, it will land "heads" 50 percent of the time and "tails" 50 percent of the time. For small sets of tosses, there will be many "fluctuations" away from the predicted average of 50% heads. The first chart at the right shows the percentage of heads in twenty sets of 30 tosses each. Because there are more ways that 30 tosses can split 15-15 than split any other way, 50% is the most probable value for every set. But often the split is not 15-15.

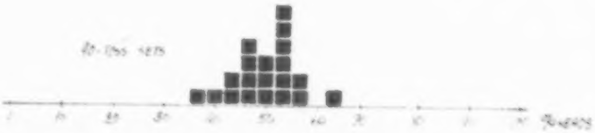
The next chart shows the percentage of heads in each of twenty 90-toss sets. There are still fluctuations from the most probable 45-45 split, but the fluctuations are generally smaller compared to the total number of tosses. Large fluctuations from 50% are less common than for the smaller 30-toss sets.

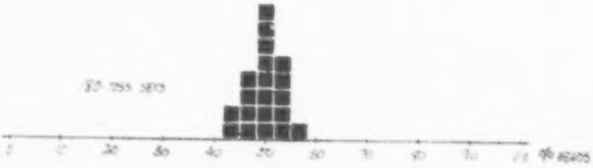
The last chart shows the percentage of heads in each of twenty 180-toss sets. There are certainly fluctuations, as there always will be, but they are generally smaller compared to the total number of tosses in a set. Large fluctuations from 50% are still less common.

Statistical theory shows that the average fluctuation from 50% for sets of tosses shrinks in proportion to the square root of the number of tosses. We can use this rule to compare the average fluctuation for sets of, say, 100,000,000 tosses with the average fluctuation for sets of 100 tosses: since the 100,000,000-toss sets have 1,000,000 times as many tosses as the 100-toss sets, their average fluctuation in percentage of heads should be 1,000 times smaller!

These same principles hold for fluctuations from average values of any randomly-distributed quantities, such as molecular speed, or distance between collisions. Since even a thimble-full of air contains about a quintillion (10^{18}) molecules, large fluctuations in observable gas behavior are extremely unlikely.







Summary 11.4

1. Buys-Ballot (1857) noted that if it were true (as claimed by early kinetic theorists) that gas molecules move at speeds of several hundred meters per second, diffusion and mixing should occur much more rapidly than they actually do. 11.4

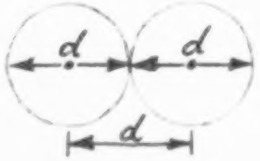
Even though an individual molecule has an instantaneous speed of several hundred meters per second, it changes its direction of motion every time it collides with another molecule. Consequently it doesn't get very far away from its starting point.

2. To meet this objection, Clausius changed the model to give the particles a finite size. Hence they can only go a relatively short distance (the mean free path) before colliding with other particles and changing their direction of motion.

Clausius now was faced with the dilemma that plagues every theoretical physicist. If a simple model is modified to explain more observable properties, it will then be more complicated, and one usually will not be able to deduce the predictions of the model from its basic assumptions without making some approximations. If the theoretical predictions don't agree with the experimental data, one doesn't know whether this is because one of the assumptions of the model is wrong, or because some error was introduced by the approximations made in doing the calculation from the model. (This situation has been somewhat improved in the twentieth century by the availability of fast electronic computers, but the problem is still a serious one.) The development of a theory often involved a compromise between two criteria: adequate explanation of the data and mathematical convenience.

Notice that this model takes some account of short-range repulsive forces between molecules (whatever force acts when they collide), but still leaves out any attractive forces.

Clausius found a temporary solution by making only a small change in the model: he assumed that the particles are not points but spheres of diameter d. Two particles will collide with each other if their centers come within a distance d; all collisions are still assumed to be perfectly elastic.



Using his new model, Clausius proved mathematically that the "mean free path" of a particle (defined as the average distance it travels between collisions) is inversely proportional to the square of the diameter of the particles. The probability that a particle will collide with another one is proportional to the cross-sectional area of a particle, and this area is proportional to the square of its diameter. The bigger the particle, the more likely it is to collide with others, and the shorter its mean free path will be.

The relation between mean free path, collision probabilty and molecular size will be investigated in the experiment which goes along with this chapter. Mathematical derivations will also be found in the discussion of this experiment in the Student Handbook.

Within a few years it became clear that the new model was a great improvement over the old one. It turned out that other properties of gases also depend on the size of the molecules, and by combining data on several such properties it was possible to work backwards and determine fairly accurate values for molecular sizes.

It was first necessary to find a precise theoretical relation between molecular size and a measurable property of gases. This was done by Maxwell soon after Clausius' paper on the

3. Maxwell proved that, according to the kinetic theory as modified by Clausius, the viscosity of a gas should increase with temperature and should be independent of density, unlike the behavior of liquids. 4. Experiments by Maxwell and others confirmed his predictions about viscosity, thus providing new arguments for the validity of kinetic theory.

See "James Clerk Maxwell" in Project Physics Reader 3

mean free path was published. Fortunately, a record of some of Maxwell's earliest thoughts on kinetic theory has been preserved in his correspondence. On May 30, 1859, Maxwell wrote a letter to Sir George Gabriel Stokes, a prominent mathematical physicist and expert on the properties of fluids:

> I saw in the Philosophical Magazine of February '59, a paper by Clausius on the 'mean length of path of a particle of air or gas between consecutive collisions,'.... I thought that it might be worth while examining the hypothesis of free particles acting by impact and comparing it with phenomena which seem to depend on this 'mean path.' I have therefore begun at the beginning and drawn up the theory of the motions and collisions of free particles acting only by impact, applying it to internal friction [viscosity] of gases, diffusion of gases, and conduction of heat through a gas...
>
> ...I do not know how far such speculations may be found to agree with facts, even if they do not it is well to know that Clausius' (or rather Herapath's) theory is wrong and at any rate as I found myself able and willing to deduce the laws of motion of systems of particles acting on each other only by impact, I have done so as an exercise in mechanics. One curious result is that μ [the viscosity coefficient] is independent of density... This is certainly very unexpected, that the friction should be as great in a rare as in a dense gas. The reason is, that in the rare gas the mean path is greater, so that the frictional action extends to greater distances.
>
> Have you the means of refuting this result of the hypothesis?

Notice that Maxwell seems intrigued by the mathematical properties of the model. Yet he expects that the only contribution that his calculations will make to science may be to refute the theory by showing that it leads to predictions that disagree with experiment.

Maxwell's calculations showed that the viscosity of a gas should be proportional to the mass and average speed of the individual molecules, and inversely proportional to the cross-section area of each molecule:

$$\text{viscosity} \propto \frac{mv}{A}.$$

According to kinetic theory, the absolute temperature of the gas is proportional to the square of the average speed of the molecules (Sec. 11.5). Maxwell's formula therefore indicates that the viscosity of a gas should increase with the temperature; it should be proportional to the square root of the absolute temperature. This would be markedly different from the familiar behavior of liquids, whose viscosity decreases with temperature.

Cold honey.

Stokes' reply to Maxwell's letter has been lost, but we can guess that he told Maxwell that the existing experimental

5. Loschmidt (1865) showed that molecular size could be estimated by combining the mean free path (from viscosity measurements) and the volume occupied by the substance when condensed to a liquid.

data (on the viscosity of gases at different temperatures and densities) were inadequate to test the kinetic-theory prediction. At the time, no one had any reason to think that gases would behave differently from liquids in this respect. The need to make careful measurements of gas viscosity did not arise until Maxwell showed that such measurements would be of great theoretical significance in testing the kinetic theory of gases.

Since the predicted increase of viscosity with Temperature was rather small, it was necessary to make measurements over a large range of temperatures in order to test the theory.

Although his fame rests mainly on his theoretical discoveries, Maxwell was quite competent in doing experiments, and he decided to make his own measurements of the viscosity of air. He used a laboratory in his own house in London; one can imagine the puzzlement of his neighbors who could look in the window:

> For some days a large fire was kept in the room, though it was in the midst of very hot weather. Kettles were kept on the fire, and large quantities of steam allowed to flow into the room. Mrs. Maxwell acted as stoker, which was very exhausting work when maintained for several consecutive hours. After this the room was kept cool, for subsequent experiments, by the employment of a considerable amount of ice.

Maxwell had already published his prediction about gas viscosity, along with several other mathematical theorems deduced from the assumptions of the model, before he did these experiments. It was therefore a pleasant surprise when he found that the prediction was right after all. The viscosity of a gas does increase with temperature, and does not change at all over a very large range of densities. This success not only made Maxwell himself a firm believer in the correctness of the kinetic theory, but also helped to convert many other scientists who had been somewhat skeptical.

Now that a definite relation had been established between an observable property of gases—viscosity—and the size of molecules, it was possible to use experimental data to obtain information about molecules. However, Maxwell's theory related the viscosity not only to the cross-section area of a single molecule, but also to its mass. The mass of a molecule was still an unknown quantity, although one could measure the total mass of all N molecules in the gas. It was still necessary to estimate the value of N, the number of molecules in the gas.

In 1865, the Austrian physicist Josef Loschmidt made the first quantitative estimate of molecular size from kinetic theory by combining viscosity measurements with data on the comparative volumes of liquids and gases. Loschmidt reasoned

From kinetic theory, viscosity = (const.) $\frac{mv}{A}$
v is known from pressure and density (see Eq. 11.1)
$A = \frac{\pi}{4} d^2$ where d = diameter of a molecule,
m = mass of a molecule, and
N_m = total mass of gas.
If we know viscosity, v, N, and total mass of gas, we can find m and d. But N and d are both unknown, and so far there is only one equation to relate them to one observable quantity (viscosity).

Loschmidt's Solution: liquid volume is approximately equal to Nd^3, if each molecule occupies a cube of side d. Liquid volume can be measured and set equal to Nd^3; from viscosity, we can get a value for Nd^2. Thus we can determine d:

$$d = \frac{Nd^3}{Nd^2}$$

Having found d, we can now go back to the previous equations, for example: liquid volume = Nd^3, and substitute to find N.

11.4

that molecules are probably closely packed together in the liquid state, so that the volume of a given mass of liquid is approximately the same as the total volume of all the molecules in the liquid. The volume of the liquid can therefore be used to calculate the product Nd^3, and measurements of the same substance in the gaseous state can be used to calculate Nd^2. It was then possible to compute N and d separately.

To illustrate the huge size of Loschmidt's number, note that if a tiny hole were made in a completely evacuated 100-watt light bulb, of such a size that 100,000 air molecules were admitted each second, it would take one billion years for enough molecules to enter the bulb to make the pressure inside equal to atmospheric pressure.

Loschmidt used this method to estimate the diameter of an air molecule. He obtained a value of about a millionth of a millimeter (or 10^{-9} meters). This is about 4 times as large as modern values, but it is amazingly accurate considering the fact that before 1865 no one knew whether a molecule was bigger than a thousandth of a millimeter or smaller than a trillionth of a millimeter. In fact, as Lord Kelvin remarked five years later,

> The idea of an atom has been so constantly associated with incredible assumptions of infinite strength, absolute rigidity, mystical actions at a distance and indivisibility, that chemists and many other reasonable naturalists of modern times, losing all patience with it, have dismissed it to the realms of metaphysics, and made it smaller than 'anything we can conceive.'

Kelvin showed that other methods could also be used to estimate the size of atoms. None of these methods gave results as reliable as did the kinetic theory, but it was encouraging that they all led to the same order of magnitude (within 50 percent) for the size of a molecule.

A closely related quantity, "Avogadro's number," is defined as the number of molecules in a gram-molecular weight of gas. Although Avogadro postulated that this number should be the same for all gases, he had no way of actually calculating it.

Loschmidt's method of calculating d could also be used to calculate N, the number of molecules in a given volume of gas. The number of molecules in a cubic centimeter of gas (at 1 atmosphere pressure and 0°C) is now known as Loschmidt's number; its presently accepted value is 2.687×10^{19}.

Q8 In his kinetic-theory model Clausius assumed that the particles have a finite size, instead of being mathematical points, because

a) obviously everything must have some size.
b) it was necessary to assume a finite size in order to calculate the speed of molecules.
c) the size of a molecule was already well known before Clausius' time.
d) by assuming finite-size molecules the theory could account for the slowness of diffusion.

Q9 Maxwell originally thought that he could refute the kinetic theory by

a) proving that not all molecules have the same speed.
b) proving that molecules have a finite size.
c) proving that the theoretical prediction that gas viscosity is independent of density disagrees with experiment.
d) proving that gas viscosity does not decrease with increase in temperature.

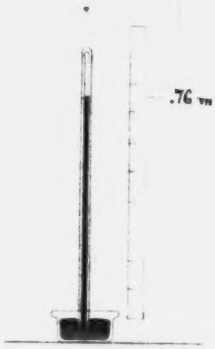
Omitting Section 11.5 will in no way alter the story-line of Chapter 11.

11.5 Predicting the behavior of gases from the kinetic theory.

Galileo, in his Dialogues Concerning Two New Sciences (1638), noted that a vacuum pump cannot lift water more than about 34 feet (10½ meters). This fact was well known; pumps were being used to obtain drinking water from wells and to remove water from flooded mines. One important consequence of the limited ability of pumps to lift water was that some other method was needed to pump water out of deep mines. This need provided the initial stimulus for the development of steam engines (Sec. 10.5). Another consequence was that physicists in the seventeenth century became curious about why the vacuum pump worked at all, as well as about why there should be a limit to its ability to raise water.

This section treats some optional special topics.

Air Pressure. As a result of experiments and reasoning by Torricelli (a student of Galileo), Guericke, Pascal and Robert Boyle, it was fairly well established by 1660 that the vacuum pump works because of air pressure. This pressure is sufficient to balance a column of water high enough to exert an equal pressure in the opposite direction. If mercury, which is 14 times as dense as water, is used instead, the air pressure can raise it only $\frac{1}{14}$ as far, that is, about 0.76 meter. This is a more convenient height for doing laboratory experiments and therefore much of the seventeenth-century research on air pressure was done with the mercury "barometer" designed by Torricelli.



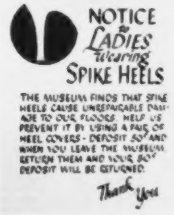
Torricelli's barometer was a glass tube standing in a pool of mercury. The air pressure on the pool supported the column of mercury in the tube.

See "The Barometer Story" in Project Physics Reader 3

It seems curious at first glance that the height of the mercury column which can be supported by air pressure does not depend on the diameter of the tube—that is, it doesn't depend on the total amount of mercury, but only on its height. To understand the reason for this we must distinguish between pressure and force. Pressure is defined as the magnitude of the force on a surface divided by the area of the surface:

$$P = F/A. \qquad (11.2)$$

A large force may produce only a small pressure if it is spread over a large enough area; for example you can walk on snow without sinking in if you wear large snowshoes. On the other hand, a small force may produce a very large pressure if it is concentrated in a small area. Women wearing spike heels can ruin a wooden floor or carpet; the pressure at the place where their heels touch the floor is greater than that under an elephant's foot.


NOTICE
to
LADIES
Wearing
SPIKE HEELS

THE MUSEUM FINDS THAT SPIKE HEELS CAUSE UNREPAIRABLE DAMAGE TO OUR FLOORS. HELP US PREVENT IT BY USING A PAIR OF HEEL COVERS - DEPOSIT 50¢ AND WHEN YOU LEAVE THE MUSEUM RETURN THEM AND YOUR 50¢ DEPOSIT WILL BE RETURNED.

Thank you

Force is a vector, but we only use the component of force in a direction perpendicular to the surface. Pressure is not a vector.

In 1661 two English scientists, Richard Towneley and Henry Power, discovered a basic relation between the pressure exerted by a gas and its density: the pressure is proportional to the density. Thus if the density of a given mass of air is doubled, by compressing it, its pressure will be twice as great. Since the density is defined as mass divided by volume, this relation can also be stated: the product pressure × volume is constant. Robert Boyle confirmed this relation by extensive experiments, and it is generally known as Boyle's law.

Boyle's law: pressure is proportional to density, if the temperature is held constant when the density changes.

Kinetic explanation of gas pressure. According to the kinetic theory of gases, the pressure of a gas is the average result of the continual impacts of many small particles against the wall of the container. It is therefore reasonable that the pressure should be proportional to the density; the greater the density, the greater the number of particles colliding with the wall. Moreover the pressure must depend on the speed of the individual particles, which determines the force exerted on the wall during each impact and the frequency of the impacts.

See "The Great Molecular Theory of Gases" in Project Physics Reader 3

We are now going to study the model of a gas described in Sec. 11.2: "a large number of very small particles in rapid disordered motion." Rather than trying to analyze the motions of particles moving in all directions with many different velocities, we fix our attention on the particles that are simply bouncing back and forth between two opposite walls of a box. Hardly any molecules in a real gas would actually move in just this way, but the basic physical factors involved in the theory can be understood fairly well with this simplified model. We will assume, then, that all the particles in our model are moving with the same speed, either right or left. In addition, we will assume that particles never hit each other, but when they hit the sides of the box they bounce off elastically, simply reversing their direction.

We are assuming here that the particles are points with zero size, so that collisions between particles can be ignored. If the particles did have finite size the results of the calculation would be slightly different, but the approximation used here is accurate enough for most purposes.

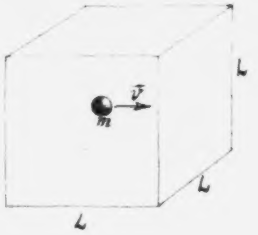
For a later part of the argument we will want to be able to move one of the walls, so we will make that wall a piston which snugly fits into the box. It can be shown from the laws of conservation of momentum and energy that when a very light particle bounces elastically off of a much more massive stationary object, very little of its kinetic energy is transferred. That is, the particle reverses direction with very little loss in speed. Bombardment by a tremendously large number of molecules would move the wall, however, so we will provide an outside force just great enough to keep the wall in place.

work = force x distance; if distance = 0, then work = 0

How large a force will these particles exert on the piston when they hit it? According to Newton's second law, the force on the particle is equal to the product of its mass times its acceleration ($\vec{F} = m\vec{a}$). As was shown in Sec. 9.4, the force can also be written as

$$\vec{F} = \frac{\Delta(m\vec{v})}{\Delta t}$$

where $\Delta(m\vec{v})$ is the change in momentum. To find the average force acting on the wall we need to find the change in momentum per second due to molecule-wall collisions. By Newton's third law the average force acting on the wall is equal and opposite to the average force acting on the molecules.



Let a single molecule of mass m move in a cubical container of volume L^3 as shown in the figure. The molecule, moving with a speed v_x, is about to collide with the right-hand wall. The momentum of the molecule just before collision is mv_x. The molecule collides elastically with the wall and rebounds with the same speed. Therefore the momentum after the collision is $m(-v_x) = -mv_x$. The change in the momentum of the molecule as a result of this collision is

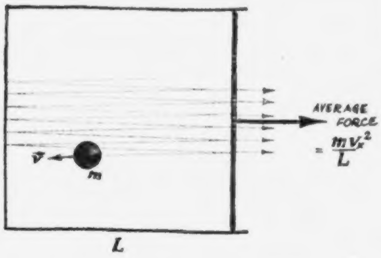
$$-mv_x - mv_x = -2mv_x.$$

The time between collisions of one molecule with the right-hand wall is the time required to cover a distance 2L at a speed of v_x; that is, $2L/v_x$. If $2L/v_x$ = the time between collisions, then $v_x/2L$ = the number of collisions per second. Thus, the change in momentum per second is given by

Note that all the vectors considered in this derivation have only two possible directions: to the right or to the left. We can therefore indicate direction by using + and - signs respectively.

$$\begin{pmatrix}\text{change in momentum}\\ \text{per second}\end{pmatrix} = \begin{pmatrix}\text{change in momentum}\\ \text{in one collision}\end{pmatrix} \times \begin{pmatrix}\text{number of collisions}\\ \text{per second}\end{pmatrix}$$

$$-mv_x^2/L \quad = \quad (-2mv_x) \quad \times \quad (v_x/2L)$$



But by Newton's second law the change in momentum per second equals the average force. Therefore, $-mv_x^2/L$ = average force acting on molecule due to the wall; and by Newton's third law, $+mv_x^2/L$ = average force acting on the wall due to the molecule. Thus the pressure due to one molecule moving with a speed v_x is

$$P = F/A = F/L^2 = mv_x^2/L^3 = mv_x^2/V.$$

There are N molecules in the container. They will not all move with the same speed, but we only need to know the average speed in order to find the pressure. More precisely, we need the average of the square of their speeds in the x-direction, and we call this quantity, $\overline{v_x^2}$. The pressure on the wall due to N molecules will be N times the pressure due to one molecule, or $P = Nm\overline{v_x^2}/V$. We can express the square of the average speed in terms of the velocity components as follows: $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$. Also, if the motion is random, then there is no preferred direction of motion and $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$. These last two expressions can be combined to give

$$\overline{v^2} = 3\overline{v_x^2} \quad \text{or} \quad \overline{v_x^2} = \frac{1}{3}\,\overline{v^2}.$$

By substituting this expression into our pressure formula, we get

$$P = \frac{1}{3}\,N\,\overline{mv^2}/\,V\,.$$

Recall that density is
$D = \dfrac{\text{mass}}{\text{volume}} = \dfrac{Nm}{V}$

Our final expression for the pressure in terms of molecular speed v and density D is therefore

$$P = \frac{1}{3}\,D\overline{v^2}. \qquad (11.3)$$

Note that in equation (11.1) we wrote $P = \frac{1}{3}Dv^2$ and called v the average speed. Equation (11.3) is more precise: $P = \frac{1}{3}D\overline{v^2}$, where $\overline{v^2}$ is av. squared speed. (Av. speed)^2 = av.(speed^2) only for symmetric distributions. This can be demonstrated with numbers; e.g.

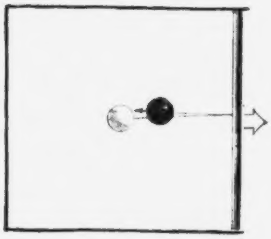
$3 + 4 + 2 + 3 = 12$
Average $= \frac{12}{4} = 3$
Average$^2 = 3^2 = 9$

$3^2 + 4^2 + 2^2 + 3^2$
$= 9 + 16 + 4 + 9 = 38$
Average $= \frac{38}{4} = 9\frac{1}{2}$

This formula agrees with Boyle's law—pressure is proportional to density—if we can assume that the other factor on the right-hand side of Eq. (11.3), namely $\overline{v^2}$, is constant. Since the mass of the particles is constant anyway, this amounts to the same thing as assuming that the total kinetic energy of all the particles ($\frac{1}{2}N\overline{mv^2}$) is constant.

Why should the total kinetic energy of a gas of particles remain constant when its pressure and density change? If we can answer that question, we will be able to say that the kinetic-theory model explains one of the properties of real gases, Boyle's law.

At first sight it would seem that the kinetic energy of the system would not remain constant when we change the pressure or density. Suppose we reduce the outside force that holds the piston in place. What will happen? The force on the piston resulting from the collisions of the particles will now be greater than the outside force, and the piston will start to move to the right.



As long as the piston was stationary, the particles did not do any work on it, and the piston did not do any work on the particles. But if the piston moves in the same direction as the force exerted on it by the particles, then the particles must be doing work on the piston. The energy needed to do this work must come from somewhere. But the only source of energy in our model is the kinetic energy of the particles. Therefore the kinetic energy of the particles must decrease.

Another way to look at this problem is to apply the laws of conservation of momentum and mechanical energy to the collisions between the particles and the piston. According to the principles stated in Chapter 10, if a light particle collides with a heavy piston which is moving in the same direction (that is, away from the particle), then the speed of the particle will be smaller when it bounces off, even if the collision is perfectly elastic. Therefore its kinetic energy is smaller.

If we had increased the outside force of the piston instead of decreasing it, just the opposite would happen. The piston would move to the left, doing work on the particles and increasing their kinetic energy. This result could also be predicted from the principles of conservation of momentum and mechanical energy.

The kinetic energy of the particles in our model is not constant when pressure and density change. On the contrary, it will increase when the pressure increases, and decrease when the pressure decreases. Therefore the model does not explain Boyle's law—unless we can find some reason why the kinetic energy of the particles should remain constant when the pressure changes.

In order to keep the kinetic energy of the particles constant, even though changes in pressure would tend to change the kinetic energy, we must provide some supply of energy. This energy supply must have the following two properties:

(1) it must add kinetic energy to the gas particles whenever they lose energy because the pressure decreases;

(2) it must take away kinetic energy whenever the particles increase their kinetic energy because the pressure increases.

In other words, the energy-supply must always act in such a way as to maintain a constant amount of kinetic energy in a gas.

If such an energy supply were included in the kinetic-theory model, this model would then provide a satisfactory explanation for the pressure of air and other gases.

The effect of temperature on gas pressure. Robert Boyle, writing in 1660 about the pressure of air, recognized that heating a gas would increase its volume. Many experiments were done throughout the eighteenth century on the expansion of gases by heat, but the results were not consistent enough to establish a quantitative relation between volume and temperature.

It was not until about 1800 that the law of thermal expansion of gases was definitely established by the French chemist Joseph-Louis Gay-Lussac (1778-1850). Gay-Lussac found that all the gases he studied—air, oxygen, hydrogen, nitro-

Maxwell suggested the following analogy: "When air is compressed the sides of the vessel are moving to meet the molecules like a cricket bat swung forward to meet the ball, and the molecules like the cricket ball rebound with a greater velocity than before. When air is allowed to expand the sides of the vessel are retreating like the cricketer's hands when he is stopping the ball and the molecules rebound with diminished velocity. Hence air becomes warmer when compressed and cooler when allowed to expand." (from notes for an unpublished lecture, in the Maxwell collection at Cambridge University)

See Study Guide 11.14

Summary 11.5

1. The now familiar concept of air pressure was established in the seventeenth century; pressure is defined as force/area.

2. Power and Towneley discovered the relation now known as Boyle's law (1661): pressure is proportional to density.

3. Boyle's law can be derived from the kinetic theory model, by considering collisions of particles with the walls of the container, provided we can assume that the mean square speed of the particles is constant when the pressure changes.

4. In order to justify this assumption, we must introduce the concept of a constant-temperature system in which there is some mechanism to maintain constancy of v^2 — otherwise, according to the principles of conservation of momentum and energy, v^2 must increase when pressure increases, and decrease when pressure decreases.

5. The effect of temperature on gas pressure and volume was established experimentally by Gay-Lussac; the data can be summarized by the ideal gas law, $P = KD(t + 273)$ with t = Celsius temperature.

6. It is impossible to cool any gas below $t = -273°$; even if the ideal gas law does not apply it is still impossible to reach this temperature. This limit is related to the third law of thermodynamics.

7. The "absolute" (Kelvin) temperature is defined as $T = t + 273$; the average molecular kinetic energy of an ideal gas is then simply proportional to its absolute temperature.

On the Celsius scale, water freezes at 0^{o} and boils at 100^{o}, when the pressure is equal to normal atmospheric pressure. On the Fahrenheit scale, water freezes at 32^{o} and boils at 212^{o}.

gen, nitrous oxide, ammonia, hydrochloric acid, sulfur dioxide and carbon dioxide—changed their volume by the same amount when the temperature changed by the same amount. The amount of volume change was always proportional to the change in temperature, as long as the pressure remained constant. On the other hand, if the volume (or density) was held constant, the change in pressure would always be proportional to the change in temperature.

The experimental data obtained by Power, Towneley, Boyle, Gay-Lussac and many other scientists can be summarized by a single equation, known as the ideal gas law

$$P = kD(t + 273) \qquad (11.4)$$

where t is the temperature on the Celsius scale and k is a number which is constant when the pressure, volume and temperature of the same sample of gas are changed.

This equation is called the ideal gas law because it does not apply accurately to real gases except at very low pressures. It is not a law of physics in the same sense as the law of conservation of momentum, but rather a first approximation to the properties of real gases. It is not valid when the pressure is so high, or the temperature is so low, that the gas may change to a liquid.

Why does the number 273 appear in the ideal gas law? Simply because we are measuring temperature on the Celsius scale. If we had chosen to use the Fahrenheit scale, the equation for the ideal gas law would be

$$P = kD(t + 460)$$

where t is the temperature on the Fahrenheit scale. In other words, the fact that the number is 273 or 460 has no real significance, but depends on our choice of a particular scale for measuring temperature. However, it is significant that the pressure and volume of a gas depend on temperature in such a way that the product of pressure times volume would be zero when the temperature decreases to a certain value.

The value of this lowest possible temperature is -273.16° Celsius (459.69° Fahrenheit). Both experiment and thermodynamic theory have shown that it is impossible actually to cool anything all the way down to this temperature. However, temperature can be lowered in a series of cooling operations to within a small fraction of a degree above the limit.

Lord Kelvin proposed to define a new temperature scale, called the absolute temperature scale. Sometimes it is called the Kelvin scale. The absolute temperature T is equal to 273 degrees more than the Celsius temperature, t:

$$T = t + 273 \qquad (11.5)$$

The temperature $t = -273^{\circ}C$, which is unattainable, is now $T = 0$ on the absolute scale, and is called the absolute zero of temperature.

The ideal gas law may now be written in the simpler form:

$$P = kDT \qquad (11.6)$$

Note that the ideal gas law includes Boyle's law as a special case: when the temperature is held constant, the pressure is proportional to the density.

Heat and molecular kinetic energy. Now that we have redefined Boyle's law by adding the condition that the temperature must be kept constant when pressure and density change, we can go back to the kinetic-theory model and see what is needed to put it into agreement with the properties of gases. To do this, we compare the two equations

$$P = \frac{1}{3} D\overline{v^2} \text{ (theory)} \qquad (11.3)$$

and

$$P = kDT \text{ (experiment)}. \qquad (11.6)$$

These two equations are consistent only if we assume that

$$\frac{1}{3} D\overline{v^2} = kDT:$$

that is, that

$$\overline{v^2} = 3kT.$$

If we multiply both sides of the last equation by $\frac{1}{2}$ m, we get the interesting result that

$$\frac{1}{2} m\overline{v^2} = \frac{3}{2} mkT.$$

Thus the theory implies that the average kinetic energy per particle is proportional to the absolute temperature!

We pointed out earlier that the kinetic-theory model of a gas would not be able to provide an explanation of Boyle's law unless there were some kind of energy supply which could keep the kinetic energy of the particles constant when the pressure changes. Now we know how this energy supply should be controlled: by a thermostat. If we simply keep the surroundings of the gas at a fixed temperature, then the average kinetic energy of the molecules will also remain fixed. Whenever the kinetic energy momentarily decreases (for example, during expansion) the temperature of the gas will drop below that of its surroundings. Heat will then flow into the gas until its temperature comes back up to the temperature of the surroundings. Whenever the kinetic energy momentarily increases (for example, during compression) the temperature of the gas will rise above that of its surroundings. Heat will then flow out of the gas until its temperature comes back down to the temperature of the surroundings. The natural tendency of heat to flow from hot bodies to cold bodies is just what we need to keep the average kinetic energy of the particles constant.

Q10 The pressure of a gas depends upon the square of the speed of the molecules because

- a) the momentum changes upon impact with the wall.
- b) the time between impacts is shorter for high speed molecules.
- c) a combination of (a) and (b).
- d) the area of the wall is measured in squared units of length.

Q11 If a piston is pushed into a container of gas, what will happen to the total kinetic energy of the molecules of gas?

Q12 Which of the following results when the ideal gas law is combined with the kinetic theory?

- a) P is proportional to T.
- b) P is proportional to v^2.
- c) $\frac{1}{2}mv^2$ is proportional to T.

11.6 The second law of thermodynamics and the dissipation of energy. At the beginning of this unit we mentioned a basic philosophical theme of the Newtonian cosmology: the idea that the world is like a machine whose parts never wear out, and which never runs down. This idea inspired the search for conservation laws for matter and motion, and up to now it might seem that the search has been successful. We can measure "matter" by mass, and "motion" by momentum or kinetic energy. By 1850 the law of conservation of mass had been firmly established in chemistry, and the laws of conservation of momentum and energy had been firmly established in physics.

Yet these conservation laws could not banish the suspicion that somehow the world is running down, and the parts of the machine are wearing out. Energy may be conserved in burning fuel, but it loses its usefulness as the heat diffuses into the atmosphere. Mass may be conserved in scrambling an egg, but structure is lost. In these transformations, something is conserved, but also something is lost. Some processes are irreversible—they will not run backwards.

The first attempts to formulate quantitative laws for irreversible processes in physics were stimulated by the development of steam engines. During the eighteenth and nineteenth centuries, the efficiency of steam engines—the amount of mechanical work that could be obtained from a given

Summary 11.6

1. Despite the acceptance of the Newtonian world machine doctrine, it could not be denied that certain natural processes appear to be irreversible.

2. Carnot's analysis of the efficiency of steam engines (1824) led to the conclusion that there is a fixed upper limit on the amount of mechanical energy that can be obtained from a given amount of heat; furthermore, all real engines fail to attain the theoretical limit because of irreversible flow of heat from hot to cold.

3. Kelvin (1852) generalized Carnot's conclusions (now known as the second law of thermodynamics) by asserting that there is a universal tendency in nature toward the dissipation of energy.

4. Clausius reformulated Kelvin's dissipation principle in terms of a new concept, entropy (1865), which can be regarded as a measure of disorder and always increases.

Our life runs down in sending
  up the clock.
The brook runs down in sending
  up our life.
The sun runs down in sending up
  the brook.
And there is something sending
  up the sun.
It is this backward motion to-
  ward the source,
Against the stream, that most we
  see ourselves in,
The tribute of the current to
  the source.
It is from this in nature we
  are from.
It is most us.
  (Robert Frost, *West-Running Brook* p. 37)

In Newtonian mechanics, interactions of particles are reversible: they could happen backward as well as forward. But most interactions we observe are not reversible.

amount of fuel energy—was steadily increased (see Sec. 10.6). In 1824, a young French engineer, Sadi Carnot, published a short book entitled Reflections on the Motive Power of Fire. Carnot raised the question: what is the maximum efficiency of an engine? By careful analysis of the flow of heat in the engine, Carnot proved that there is a maximum efficiency, always less than 100%. There is a fixed upper limit on the amount of mechanical energy that can be obtained from a given amount of heat by using an engine, and this limit can never be exceeded regardless of what substance—steam, air, or anything else—is used in the engine.

Even more ominous than the existence of this limit on efficiency was Carnot's conclusion that all real engines fail to attain the theoretical limit in practice. The reason is that whenever a difference of temperature exists between two bodies, or two parts of the same body, there is a possibility of doing work by allowing heat to expand a gas as the heat flows from one body to the other. But if heat flows by itself from a hot body to a cold body, and we do not design our engine properly, we will lose the chance of doing work that might have been done.

Carnot's analysis of steam engines shows that the process of equalization of temperature by the flow of heat from hot bodies to cold bodies represents a waste of mechanical energy. This is what we mean when we say that energy is "degraded" or "dissipated"—the total amount of energy is always the same, but energy tends to transform itself into less useful forms.

After the discovery of the law of conservation of energy, Carnot's conclusions about steam engines were incorporated into the new theory of heat (thermodynamics) and became known as the second law of thermodynamics. This law has been stated in various ways, all of which are roughly equivalent, and express the idea that the tendency of heat to flow from hot to cold makes it impossible to obtain the maximum amount● of mechanical energy from a given amount of heat.

Carnot's analysis of steam engines implies more than this purely negative statement, however. In 1852, Lord Kelvin generalized the second law of thermodynamics by asserting that there is a universal tendency in nature toward the dissipation of energy. Another way of stating this principle was suggested by Rudolf Clausius, in 1865. Clausius introduced a new concept, entropy, which he defined in terms of the heat transferred from one body to another. We will not discuss the technical meaning of entropy in thermodynamics

5. If Kelvin's principle were applicable to the universe, one would expect eventually a state in which all bodies have reached the same temperature, so that it is no longer possible to obtain useful work from heat, and all life must cease; this is the so-called "heat death."

Entropy: in thermodynamics, the change in entropy of a system, resulting from a transfer of heat H from a body at temperature T_1, is defined as

$$\Delta S_1 = \frac{H}{T_1}$$

Here T is the temperature on the absolute scale. When the same heat is received by another body at temperature T_2, the change in entropy is

$$\Delta S_2 = \frac{H}{T_2}$$

If T_2 is less than T_1, the total change in entropy,

$$\Delta S = \Delta S_1 + \Delta S_2,$$

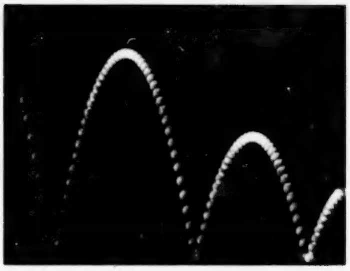
will be positive. The change in entropy would be zero only if $T_1 = T_2$.

The total amount of entropy in a body cannot be measured directly but is usually defined in such a way that $S = 0$ when $T = 0$.

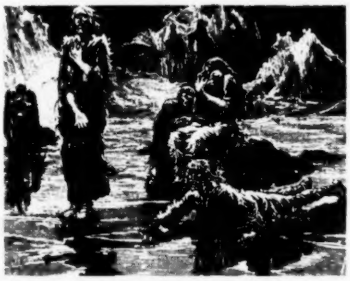
The second law of thermodynamics can be written as $\Sigma \Delta S > 0$, where Σ denotes summation over all systems affected by a reaction.

● That is, the mechanical equivalent of the heat—4.18 joules per calorie.

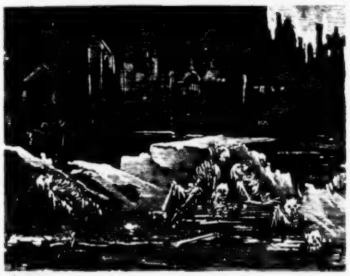
but simply state that whenever heat flows from a hot body to a cold body, entropy increases. It also increases whenever mechanical energy is changed into internal energy, as in inelastic collisions and frictional processes. All these changes can be identified with increasing disorder of the system. And indeed entropy can be defined as a measure of the disorder of a system. So the generalized version of the second law of thermodynamics, as stated by Clausius, is simply: the entropy of a system always tends to increase.



Irreversible processes are processes for which entropy increases; hence they cannot be run backwards without violating the second law of thermodynamics. For example, heat will not flow by itself from cold bodies to hot bodies; a ball dropped on the floor will not bounce back higher than its original position; and an egg will not unscramble itself. All these (and many other) events, which could take place without violating any of the principles of Newtonian mechanics, are forbidden by the second law of thermodynamics.



"La misérable race humaine périra par le froid."



"Ce sera la fin."

Lord Kelvin predicted, on the basis of his principle of dissipation of energy, that all bodies in the universe would eventually reach the same temperature by exchanging heat with each other. When this happens, it will be impossible to produce any useful work from heat, since work can only be done when heat flows from a hot body to a cold body. The sun and other stars would eventually grow cold, all life on earth would cease and the universe would be dead. This "heat death," which seemed to be an inevitable consequence of thermodynamics, aroused some popular interest at the end of the nineteenth century, and was described in several books written at that time, such as H. G. Wells' The Time Machine. The American historian Henry Adams, who learned about thermodynamics through the works of one of America's greatest scientists, J. Willard Gibbs,• argued that the second law could be applied to human history. He wrote a series of essays which were published under the title The Degradation of the Democratic Dogma. The French astronomer Camille Flammarion wrote a book describing all the possible ways in which the world could end; we have reproduced from his book two illustrations showing an artist's conception of the heat death.

Q13 The "heat death of the universe" refers to a state

a) in which all mechanical energy has been transformed into heat energy.
b) in which all heat energy has been transformed into other forms of energy.
c) in which the temperature of the universe decreases to absolute zero.
d) in which the supply of coal and oil has been used up.

•Gibbs, a professor at Yale, made the first extensive applications of thermodynamics to chemistry in the 1870's. Later he developed a generalized version of kinetic theory known as statistical mechanics, which is the basis of most modern calculations of the properties of materials.

Q14 Which of the following statements are consistent with the second law of thermodynamics?

a) Heat does not naturally flow from cold bodies to hot bodies.
b) Energy tends to transform itself into less useful forms.
c) No engine can transform all its heat input into mechanical energy.
d) Most processes in nature are reversible.

11.7 Maxwell's demon and the statistical view of the second law of thermodynamics. Is there any way of avoiding the heat death? Is irreversibility a basic law of physics, or only an approximation based on our limited experience of natural processes?

See "Maxwell's Demon" in Project Physics Reader 3

The Austrian physicist Ludwig Boltzmann used the kinetic theory of gases to investigate the nature of irreversibility, and concluded that the tendency toward dissipation of energy is not an absolute law of physics but only a statistical one. Boltzmann argued that if one were to list all the possible arrangements of molecules in a gas, nearly all of them would have to be considered "disordered." Only a few of them, for example, would have all the molecules in one corner of an otherwise-empty container. It is to be expected that if we start from an ordered arrangement of molecules, the arrangement will inevitably become less ordered, simply because most possible arrangements are random. Similarly, if we put a hot body (whose molecules are moving rapidly) in contact with a cold body (whose molecules are moving slowly), it is almost certain that after a short time both bodies will have nearly the same temperature, simply because there are many more possible arrangements of molecular speeds in which fast and slow molecules are mixed together, than arrangements in which most of the fast molecules are in one place and most of the slow molecules are in another place.

To illustrate Boltzmann's argument we might consider shuffling a pack of cards. Most possible arrangements of the cards are more or less disordered. If we start with an ordered arrangement—for example, the cards sorted by rank and suit—then shuffling would almost certainly lead to a more disordered arrangement. Nevertheless it does occasionally happen that a player is dealt 13 spades even if no one has stacked the deck.

According to Boltzmann's view, it is almost certain that disorder will increase in any natural process that we can actually observe. The second law is therefore a statistical law that applies to collections of large numbers of molecules, but may have no meaning when applied to individual molecules. Since it is a statistical law, there is a remote possibility that a noticeably large fluctuation may occur in which energy is concentrated rather than dissipated.

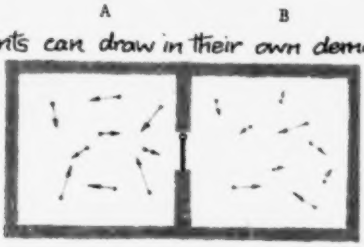
Summary 11.7
1. Boltzmann showed that the irreversibility of natural processes such as the mixing of gases could be explained simply as a reasonable consequence of the fact that most possible arrangements are disordered; hence if there is initially any order at all, it is very likely to disappear.

2. Boltzmann's theory suggests that the second law is only statistically valid and might occasionally be violated by large fluctuations. 3. Maxwell suggested that an imaginary being who could detect individual molecules to sort them out would be able to make heat flow from cold to hot; hence in agreement with Boltzmann's arguments, irreversibility is not a fundamental law of physics, but merely a result of our inability to deal with individual molecules.

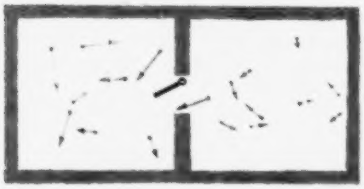
For example, the molecules in a glass of water are usually moving randomly in all directions but they might all just happen to move in the same direction at the same time. The water would then jump out of the glass. (The glass would have to move downward at the same time since momentum must still be conserved.) In this case a disordered motion has suddenly turned into an ordered motion; entropy has decreased instead of increased, and the second law of thermodynamics (regarded as an absolute law of physics) has been violated. Such large fluctuations seem extremely unlikely, yet if they can occur at all we must recognize that the second law is not a fundamental law of physics.

How Maxwell's "demon" could use a small, massless door to increase the order of a system and make heat flow from a cold gas to a hot gas.

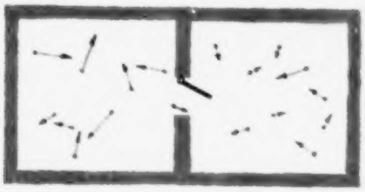
Students can draw in their own demons.



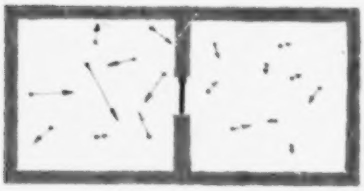
(a) Initially the average KE of molecules is greater in A.



(b) Only fast molecules are allowed to go from B to A.



(c) Only slow molecules are allowed to go from A to B.



(d) As this continues, the average KE in A increases and the average KE in B decreases.

Maxwell proposed a "thought experiment" to show how the second law of thermodynamics could be violated by an imaginary person who could observe individual molecules and sort them out, thereby causing heat to flow from cold to hot. Suppose a container of gas is divided into two parts, A and B, by a diaphragm. Initially the gas in A is hotter than the gas in B. This means that the molecules in A have greater average speeds than those in B. However, since the speeds are distributed according to Maxwell's distribution law (Sec. 11.3), a few molecules in A will have speeds less than the average in B, and a few molecules in B will have speeds greater than the average in A.

Maxwell saw that there would be a possibility of making heat flow from a cold gas to a hot gas because of this overlapping of the distributions for gases at different temperature. "Now conceive a finite being," Maxwell suggested, "who knows the paths and velocities of all the molecules by simple inspection but who can do no work except open and close a hole in the diaphragm by means of a slide without mass." (If the slide has no mass, no work will be needed to move it.) Let this "finite being" observe the molecules in A, and when he sees one coming whose speed is less than the average speed of the molecules in B, let him open the hole and let it go into B. Now the average speed of the molecules of B will be even lower than it was before. Next, let him watch for a molecule of B, whose speed is greater than the average speed in A, and when it comes to the hole let him draw the slide and let it go into A. Now the average speed in A will be even higher than it was before. Maxwell concludes:

> Then the number of molecules in A and B are the same as at first, but the energy in A is increased and that in B diminished, that is, the hot system has got hotter and the cold colder and yet no work has been done, only the intelligence of a very observant and neat-fingered being has been employed.

The imaginary "being who knows the paths and velocities of all the molecules" has come to be known as "Maxwell's Demon." Maxwell's thought experiment shows that if there were any way to sort out individual molecules, the principle of dissipation of energy could be violated. Some biologists have suggested that certain large molecules, such as enzymes, may actually be able to guide the motions of smaller molecules, building up ordered molecular systems in living beings in just this way.

Q15 In each of the following, which situation is more ordered?

a) an unbroken egg; a scrambled egg
b) a glass of ice and warm water; a glass of water at uniform temperature

Q16 True or false?

a) Maxwell's demon was able to circumvent the second law of thermodynamics.
b) Modern physics has made a Maxwell's demon.
c) Maxwell believed in the existence of his demon.

11.8 Time's arrow and the recurrence paradox.

Does time have a direction? Is there any real difference between going forward and going backward in time? These questions lie at the root of the problem of irreversibility.

Toward the end of the nineteenth century, a small but influential group of scientists began to question the basic philosophical assumptions of Newtonian mechanics and even the very idea of atoms. The Austrian physicist Ernst Mach criticized Newton's concepts of force, mass and absolute space, and argued that scientific theories should not depend on assuming the existence of things (such as atoms) which could not be directly observed. Typical of the attacks on atomic theory is the argument which the mathematician Ernst Zermelo and others advanced against kinetic theory: the second law of thermodynamics is an absolutely valid law of physics because it agrees with all the experimental data, but kinetic theory allows the possibility of exceptions to this law; hence kinetic theory must be wrong.

The critics of kinetic theory could point to two apparent contradictions between the kinetic theory (or in fact any molecular theory based on Newton's laws of mechanics) and the principle of dissipation of energy: the reversibility paradox and the recurrence paradox. Both paradoxes are based on possible exceptions to the second law. In considering their relevance to the validity of kinetic theory, we have to decide whether it is good enough to show that these exceptions would occur extremely rarely, or whether we must exclude them entirely.

Summary 11.8

1. Critics of kinetic theory, late in the nineteenth century, argued that the theory cannot be consistent with Newtonian mechanics and with the observed irreversibility of natural processes, at the same time.

2. The reversibility paradox (Kelvin and Loschmidt) is based on the fact that Newton's laws are reversible in time; hence any sequence of molecular events can be run backwards as well as forwards; hence a principle of increasing entropy cannot be inferred from kinetic theory.

See "The Arrow of Time" in Project Physics Reader 3

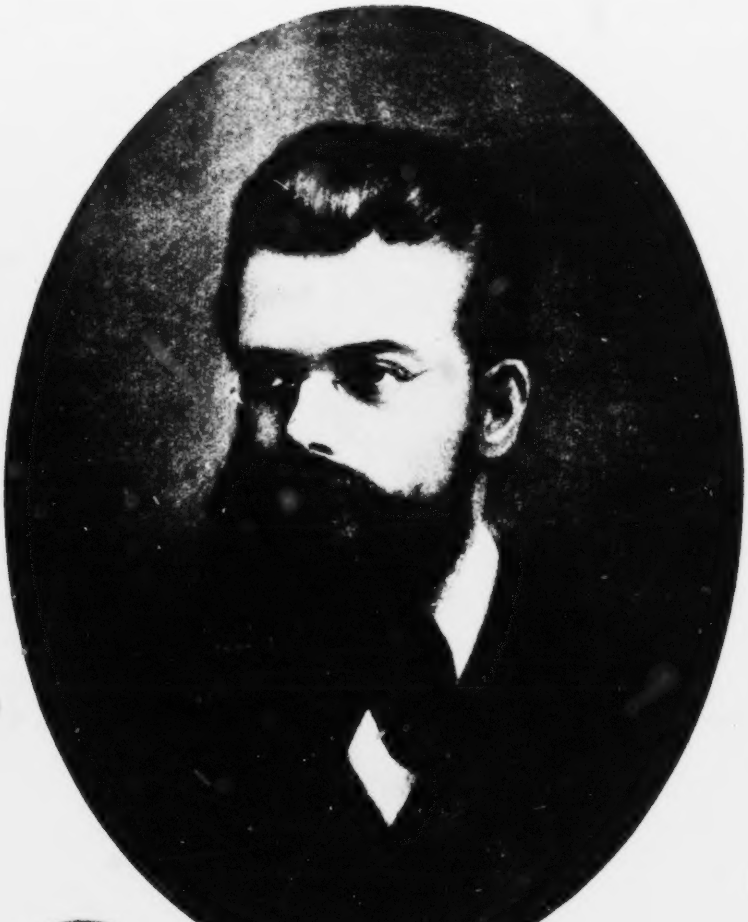
3. The recurrence paradox (Poincaré, Nietzsche, Zermelo) is based on the idea that any bounded mechanical system governed by Newton's laws must eventually come back to its original state; hence there could be no "heat death" in a universe governed strictly by Newton's laws on the molecular level.

4. Einstein's theory of Brownian motion (1906) showed that the fluctuations predicted by kinetic theory could actually be observed and measured quantitatively under certain circumstances - thereby providing new evidence for the validity of kinetic theory and the atomic viewpoint.

The term "Mach number" for speeds of airplanes honors Mach's contributions to the physics of fluids.

A few of the scientists who made significant contributions to the development of the kinetic theory of gases and thermodynamics

1



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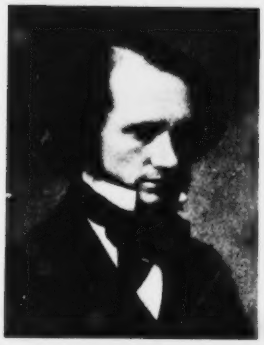
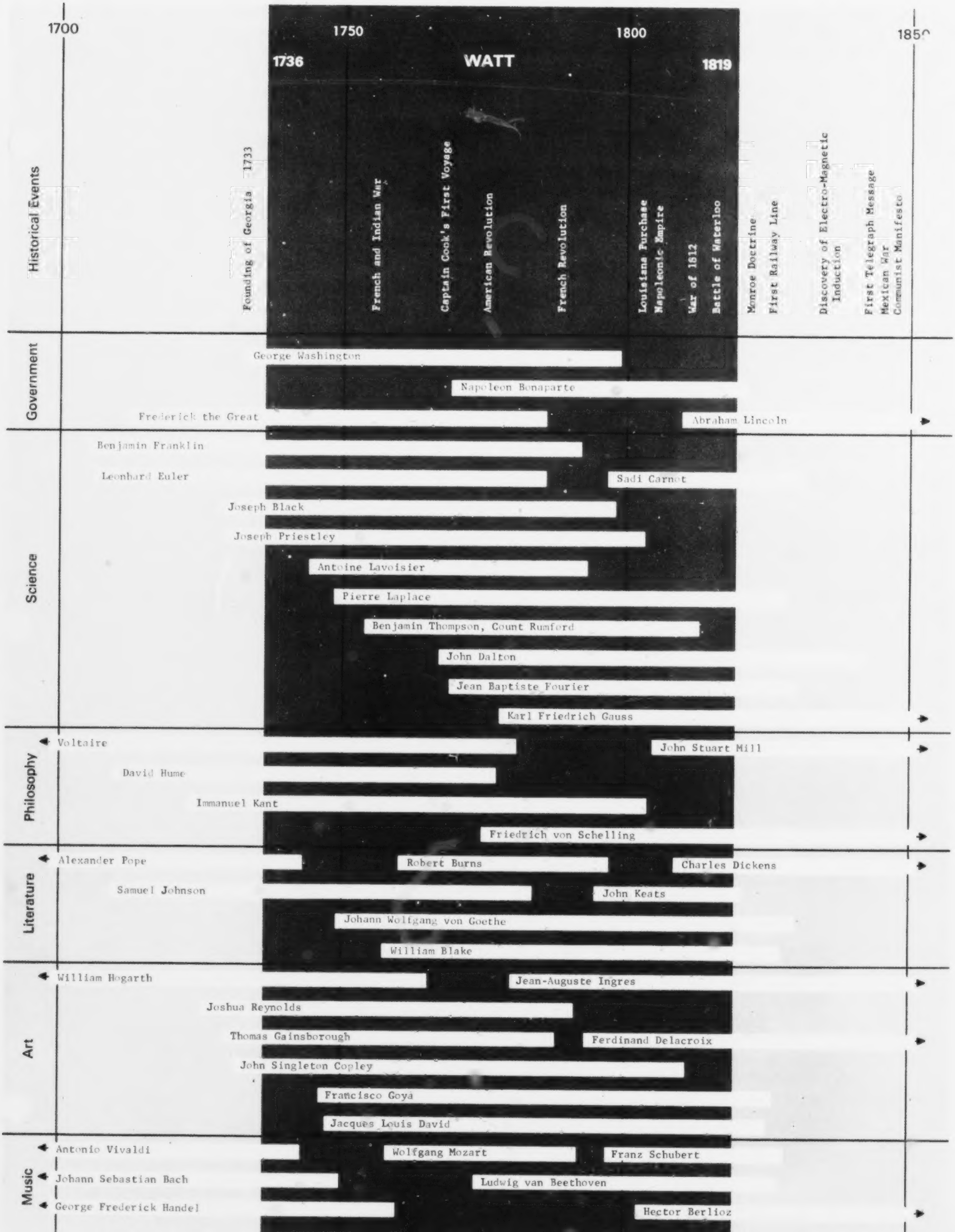


3

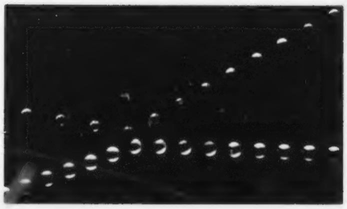
4

5

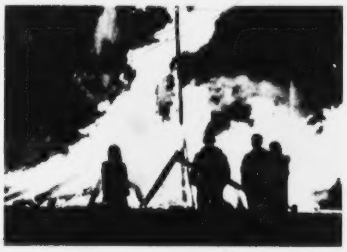
1 James Clerk Maxwell
2 Ludwig Boltzmann
3 William Thomson (Lord Kelvin)
4 Sadi Carnot
5 Rudolf Clausius


1700
1750
1800
1850
1736
WATT
1819
Historical Events
Founding of Georgia 1733
French and Indian War
Captain Cook's First Voyage
American Revolution
French Revolution
Louisiana Purchase
Napoleonic Empire
War of 1812
Battle of Waterloo
Monroe Doctrine
First Railway Line
Discovery of Electro-Magnetic Induction
First Telegraph Message
Mexican War
Communist Manifesto
Government
George Washington
Napoleon Bonaparte
Frederick the Great
Abraham Lincoln
Science
Benjamin Franklin
Leonhard Euler
Sadi Carnot
Joseph Black
Joseph Priestley
Antoine Lavoisier
Pierre Laplace
Benjamin Thompson, Count Rumford
John Dalton
Jean Baptiste Fourier
Karl Friedrich Gauss
Philosophy
Voltaire
John Stuart Mill
David Hume
Immanuel Kant
Friedrich von Schelling
Literature
Alexander Pope
Robert Burns
Charles Dickens
Samuel Johnson
John Keats
Johann Wolfgang von Goethe
William Blake
Art
William Hogarth
Jean-Auguste Ingres
Joshua Reynolds
Thomas Gainsborough
Ferdinand Delacroix
John Singleton Copley
Francisco Goya
Jacques Louis David
Music
Antonio Vivaldi
Wolfgang Mozart
Franz Schubert
Johann Sebastian Bach
Ludwig van Beethoven
George Frederick Handel
Hector Berlioz

11.8



The reversibility paradox: Can a model based on reversible effects explain a world in which so many events are irreversible?



The reversibility paradox was discovered in the 1870's by Lord Kelvin and Josef Loschmidt, both of whom were supporters of atomic theory; it was not regarded as a serious objection to the kinetic theory until the 1890's. The paradox is based on the simple fact that Newton's laws of motion are reversible in time. Imagine that we could take a motion picture of the molecules of a gas, colliding elastically according to the assumptions of kinetic theory. When we showed the motion picture, there would be no way to tell whether it was being run forward or backward—either way would show valid sequences of collisions. But motion pictures of interactions involving large objects (containing many molecules) do have obvious differences between forward and backward time directions.

There is nothing in Newton's laws of motion which distinguishes going backward from going forward in time. How can the kinetic theory explain irreversible processes if it is based on laws of motion which are reversible? The existence of irreversible processes seems to indicate that time flows in a definite direction—from past to future—in contradiction to Newton's laws of motion.

As Lord Kelvin expressed the paradox,

> If...the motion of every particle of matter in the universe were precisely reversed at any instant, the course of nature would be simply reversed for ever after. The bursting bubble of foam at the foot of a waterfall would reunite and descend into the water; the thermal motions would reconcentrate their energy, and throw the mass up the fall in drops reforming into a close column of ascending water. Heat which had been generated by the friction of solids and dissipated by conduction, and radiation with absorption, would come again to the place of contact, and throw the moving body back against the force to which it had previously yielded. Boulders would recover from the mud the materials required to rebuild them into their previous jagged forms, and would become reunited to the mountain peak from which they had formerly broken away. And if also the materialistic hypothesis of life were true, living creatures would grow backwards, with conscious knowledge of the future, but no memory of the past, and would become again unborn. But the real phenomena of life infinitely transcend human science; and speculation regarding consequences of their imagined reversal is utterly unprofitable.

Kelvin himself, and later Boltzmann, used statistical ideas to explain why we do not observe such reversals. There are a large number of possible disordered arrangements of water molecules at the bottom of a waterfall; only an extremely small number of these arrangements would lead to the process described in the above quotation if we could reverse the velocity of every molecule. Reversals of this kind are possible in principle, but very unlikely.

The recurrence paradox revived an idea that had been frequently used in ancient philosophies: the myth of the "eternal return." According to this myth, the history of the world is cyclic; all historical events are repeated over and over again, and the people who are now dead will someday be born again and go through the same life. The German philosopher Friedrich Nietzsche was convinced of the truth of this idea, and even tried to prove it using the principle of conservation of energy. He wrote:

> If the universe may be conceived as a definite quantity of energy, as a definite number of centres of energy—and every other concept remains indefinite and therefore useless—it follows therefrom that the universe must go through a calculable number of combinations in the great game of chance which constitutes its existence. In infinity, at some moment or other, every possible combination must once have been realized; not only this, but it must have been realized an infinite number of times.

The World's great age begins
anew,
The golden years return,
The earth doth like a snake
renew
His winter weeds outworn...
Another Athens shall arise
And to remoter time
Bequeath, like sunset to the
skies,
The splendour of its prime...

Percy Bysshe Shelley, "Hellas"
(1822)

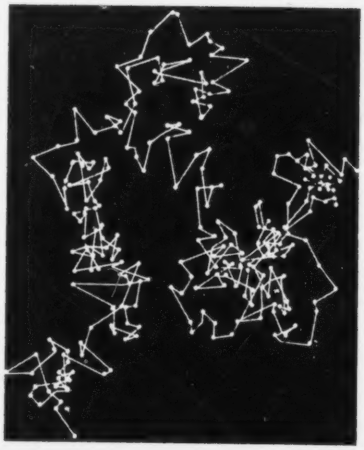
Nietzsche claimed that his proof of the eternal return refuted the theory of the heat death. At about the same time, in 1889, the French mathematician Henri Poincaré published a similar theorem on the recurrence of mechanical systems. According to Poincaré, his recurrence theorem implied that while the universe might undergo a heat death, it would ultimately come alive again:

> A bounded world, governed only by the laws of mechanics, will always pass through a state very close to its initial state. On the other hand, according to accepted experimental laws (if one attributes absolute validity to them, and if one is willing to press their consequences to the extreme), the universe tends toward a certain final state, from which it will never depart. In this final state, which will be a kind of death, all bodies will be at rest at the same temperature.
>
> ...the kinetic theories can extricate themselves from this contradiction. The world, according to them, tends at first toward a state where it remains for a long time without apparent change; and this is consistent with experience; but it does not remain that way forever; ...it merely stays there for an enormously long time, a time which is longer the more numerous are the molecules. This state will not be the final death of the universe, but a sort of slumber, from which it will awake after millions of centuries.
>
> According to this theory, to see heat pass from a cold body to a warm one, it will not be necessary to have the acute vision, the intelligence, and the dexterity of Maxwell's demon; it will suffice to have a little patience.

Though Poincaré was willing to accept the possibility of a violation of the second law after a very long time, others were less tolerant. In 1896, Ernst Zermelo (at that time a student of Max Planck) published a paper attacking not only the kinetic theory but the mechanistic conception of the world in general, on the grounds that it contradicted the second law. Boltzmann replied, repeating his earlier explanations of the statistical nature of irreversibility. When these did not satisfy Zermelo, Boltzmann (half-seriously) proposed the following hypothesis: the history of the universe is really cyclic, so that the energy of all its molecules must eventually be reconcentrated in order to start the next cycle. During this process of reconcentration, all natural processes will go backward, as described by Kelvin (above). However, the human sense of time depends on natural processes going on in our own brains. If the processes are reversed, our sense of time will also be reversed. Therefore we could never actually observe "time going backward" since we would be going backward along with time.

The final outcome of the dispute between Boltzmann and his critics was that both sides were partly right and partly wrong. Mach and Zermelo were correct in their belief that molecular and atomic processes cannot be adequately described by Newton's laws of mechanics; we will come back to this subject in Unit 5. Gases are not collections of little billiard balls. But Boltzmann was right in his belief in the usefulness of the molecular model; the kinetic theory is very nearly correct except for those properties involving the detailed structure of molecules.

In 1905, Albert Einstein pointed out that the fluctuations predicted by kinetic theory should produce an effect which could be observed and measured quantitatively: the motion of very small particles in liquids. Subsequent studies of this motion (called "Brownian motion" after Robert Brown, the English botanist who had observed it in 1828) confirmed Einstein's theoretical calculations. This new success of kinetic theory, along with the discoveries in radioactivity and atomic physics at the beginning of the twentieth century, persuaded almost all the skeptics that atoms really exist. But the problem of irreversibility and the question of whether the laws of physics must distinguish between past and future, are issues that still interest physicists today.



Record of a particle in Brownian motion. Successive positions, recorded every 20 seconds, are connected by straight lines. The actual paths between recorded positions would be as erratic as the overall path.

Q17 The kinetic energy of a falling stone is transformed into heat when the stone strikes the ground. Obviously this is an irreversible process; we never see the heat transform into kinetic energy of the stone, so that the stone rises off the ground. The reason that the process is irreversible is that

a) Newton's laws of motion prohibit the reversed process.
b) there is a very small probability that the disordered molecules will happen to arrange themselves in the way necessary for the reversed process to occur.
c) the reversed process would not conserve energy.

Q18 Which of the following is a reversible process?

- a) a pendulum swinging in air.
- b) water falling in a cataract.
- c) two molecules colliding perfectly elastically.
- d) an ice cube melting in a glass of warm water.

Q19 The recurrence theorem states that any given arrangement of molecules will be repeated after a long enough time. The theorem

- a) appears to contradict the principle of dissipation of energy.
- b) was discovered by ancient philosophers.
- c) was disproved by Poincaré.
- d) applies only to the molecules of living systems.

These ruins of a Greek temple at Delphi are meant to symbolize the encroachment of disorder.

Study Guide

11.1 List some of the directly observable properties of gases. Pressure, Density, temp, Viscosity

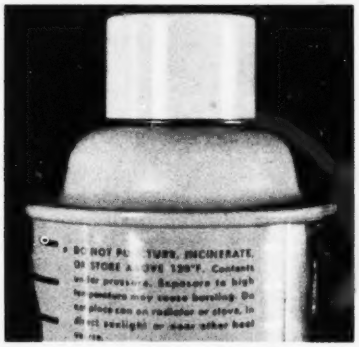
11.2 What could kinetic theorists explain about gases? Discussion

11.3 Where did Newtonian mechanics run into difficulties in explaining the behavior of molecules? Discussion

11.4 Distinguish between two uses of the word "model" in science. Working model, Theoretical model

11.5 Randomness can be used in predicting the results of flipping a large number of coins. Give some other examples where randomness is useful. Discussion

11.6 The speed of sound in a gas is about the same as the average speed of the gas molecules. Is this a coincidence? Discuss. No, discussion



11.7 Consider the curves showing Maxwell's distribution of molecular speeds.

a) All show a peak. Probable speed
b) The peaks move toward higher speed at higher temperatures. Average speed
c) They are not symmetrical like normal distribution curves. Explain these characteristics on the basis of the kinetic model. No negative speeds

11.8 Many products are now sold in spray cans. Explain in terms of the kinetic theory of gases why it is dangerous to expose the cans to high temperatures. $P \sim T$

11.9 Benjamin Franklin in 1765 observed that not more than a teaspoonful of oil covered half an acre of a pond. Suppose that one cubic centimeter of oil forms a continuous layer one molecule thick that just covers an area on water of 1000 square meters.

a) How thick is the layer? 10^{-9} m
b) What is the diameter of a single molecule of the oil? 10^{-9} m

11.10 How did Clausius modify the simple "model" for a gas? What was this new model able to explain? Discussion

11.11 How did Josef Loschmidt estimate the size of a molecule? Discussion

11.12 The atmospheric pressure of air is balanced by a column of mercury of height 0.76 meters or by 10.5 meters of water. Air is approximately a thousand times less dense than water. Why can you not say the atmosphere is only 10,000 meters deep? Density changes

11.13 The spike heel on a girl's shoe is a square, one centimeter on an edge. If her mass is 50 kilograms, how many atmospheres of pressure are exerted when she balances on one heel? 50 atmospheres

11.14 If a light particle rebounds from a massive, stationary piston with almost no loss of speed, then, according to the principle of Galilean relativity, it would still do so from a moving piston in the frame of reference of the moving piston. Show that the rebound speed as measured in the laboratory would be less from a retreating piston, as is claimed at the top of p. 95. (Hint: Express the speed of the particle relative to the piston in terms of their speeds in the laboratory frame.) Discussion

11.15 Clausius' statement of the second law of thermodynamics is: "Heat will not of its own accord pass from a cooler to a hotter body." Show in words how a refrigerator can operate. Discussion

11.16 When a gas is compressed by pushing in a piston, its temperature increases.

a) Explain this fact in two ways: first, by using the first law of thermodynamics and second, by using the kinetic theory of gases. Discussion

b) The compressed air eventually cools down to the same temperature as the surroundings. Explain this heat transfer in terms of molecular collisions. Discussion

11.17 Why, if there is a tendency for heat to flow from hot to cold, will not the universe eventually reach absolute zero? Discussion

11.18 How did Maxwell's demon hope to circumvent the second law of thermodynamics? Discussion

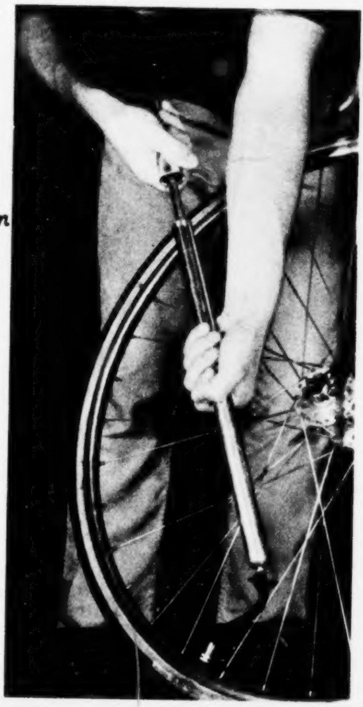
11.19

a) Explain what is meant by the statement that Newton's laws of motion are time-reversible. Discussion

b) Describe how a paradox arises when the time-reversibility of Newton's laws of motion is combined with the second law of thermodynamics. Discussion

11.20 Since molecular motions are random, one might expect that any given arrangement of molecules would recur if he waited long enough. Explain how a paradox arises when this prediction is combined with the second law of thermodynamics. Discussion

11.21 Many philosophical and religious systems of the Far East and the Middle East include the ideas of eternal return and resurrection. Read about some of these philosophies and discuss them in the light of your knowledge of the second law of thermodynamics. Discussion



Chapter 12 Waves

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The first three sections of this chapter can be handled as a reading assignment, with a short discussion in class about "propagating". It is essential, however, that lab work accompany the reading.

12.1

12.1 Introduction. Waves are all around us. Water waves, especially, whether giant rollers in the middle of the ocean or gently formed rain ripples on a still pond, are sources of wonder or amusement. If the earth's crust shifts, violent waves cause tremors thousands of miles away. A musician plucks a guitar string and sound waves pulse against our ears. Someone stumbles on a crowded dance floor and a wave of bumping or crowding spreads through the adjacent dancers. Wave disturbances may come in a concentrated bundle like the shock front from a single clap of the hands or from an airplane flying at supersonic speeds. Or the disturbances may come in succession like the train of waves sent out from a steadily vibrating source, such as an alarm clock.

Summary 12.1

1. Nature is full of waves, full of organized motion that transfers energy without transporting matter.

2. Science often uses two or more viewpoints to describe and study the same things or events. This chapter introduces waves from the viewpoint of cooperating particles transmitting disturbances throughout a medium. A different picture of waves will appear in the next unit.

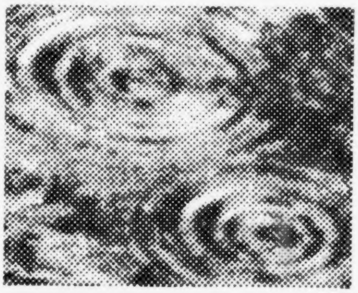
As physics has progressed over the last hundred years, vibrations and waves of a less obvious kind have been discovered. Electromagnetic waves in particular have been found to be fundamental to nearly everything we can sense about our universe. Most of our explanations of energy transfer involve waves.

So far we have been thinking of motion in terms of individual particles. As we begin to study the cooperative motion of collections of particles, we shall recognize how intimately related are the particle and wave models we make of events in nature.

If you look at a black and white photograph in a newspaper or magazine with a magnifying glass, you discover that the picture is made up of many little black dots printed on a white page (up to 20,000 dots per square inch). If you do not use the magnifier, you will not see the individual dots, but a pattern with all possible shadings between completely black and completely white. The two views emphasize different aspects of the same thing.

In much the same way, the physicist often has available several ways of viewing events. For the most part, a particle view has been emphasized in the first three units. In Unit 2, for example, we treated each planet as a particle experiencing the sun's gravitational attraction. The behavior of the solar system was described in terms of the positions, velocities and accelerations of point-like objects, but this viewpoint is far from a complete description of our planetary neighbors.

In the last chapter we saw two different descriptions of a gas. One was in terms of the behavior of the individual particles making up the gas. We used Newton's laws of motion to describe what each particle does, and then we used average



A small section from the lower right of the photograph on the opposite page.

A: "Least-Time" or "Least-Energy" Situations

The chief reason for our concern with waves is their subsequent importance in electromagnetic radiation and in quantum mechanics. We do stress the medium, however, although it is possible to sidestep the medium issue by relying on field ideas, because we want to lay the classical viewpoint which made the Michelson-Morley result such a shock to physics.

values to describe the behavior of the gas as a whole. But we also discussed a gas with the aid of concepts such as pressure, temperature, heat and entropy, which refer to a sample of gas as a whole. This is the viewpoint of thermodynamics, which does not depend on assuming Newton's laws or even the existence of particles. Each of these viewpoints served a useful purpose and helped us to understand what we cannot directly see.

See "Introduction to Waves" and "What is a Wave" in Project Physics Reader 3

Now we are about to study waves, and once again we find alternative points of view. Most of the waves discussed in this chapter can be described in terms of the behavior of particles, but we also want to understand waves as disturbances traveling in a continuous medium. We want, in other words, to see the picture as a whole, not only individual dots.

12.2 Properties of waves. What is a wave? We begin our study of waves with a simple example. Suppose that two people are holding opposite ends of a rope. One person snaps the rope up and down quickly. That puts a sort of hump in the rope, which travels along the rope toward the other person. The traveling hump is a wave.

Originally, the rope is held motionless. The height of each point on the rope depends only upon its position along the rope. When one person snaps the rope, he creates a rapid change in the height of one end, and the disturbance then moves away from its source. The height of each point on the rope depends upon time as well as position.

The disturbance is a pattern of displacement along the rope. The motion of the displacement pattern is an example of a wave. The snapping of the one end is the source of the wave, and the rope is the medium in which the wave moves. These four terms are common to all mechanical wave situations.

Consider another example. Drop a pebble into a pool of still water, and a series of circular crests and troughs spread over the water's surface. This moving displacement pattern of the water surface is a wave. The pebble is the source, the moving pattern of crests and troughs is the wave and the water surface is the wave's medium.

In rope waves and water waves, we can see the media, the source and the disturbance. However, in our analysis we want to concentrate on the wave—the moving pattern. Each of these waves consists of changing displacements from the equilibrium height of successive parts of the medium. Thus we can refer to these as waves of displacement.

E30: Introduction to waves

High-speed photograph of ripples produced by a milk drop.

Suspension by strings from overhead is better, especially for longitudinal waves.

If we can see the medium and recognize the displacements, then we can see waves. As we proceed, we should be prepared to find waves in media we cannot see (such as air) or which are disturbances in properties we cannot detect with our eyes (such as pressure, or electric fields).

A loose spring coil can be used to demonstrate three differend kinds of waves. If the end of the spring is moved from side to side, as in Fig. 12.1 (a), a wave of side-to-side displacement will travel along the spring. If the end of the spring is moved back and forth, as in Fig. 12.1 (b), a wave of back-and-forth displacement will travel along the spring. If the end of the spring is twisted, a wave of angular displacement will travel along the spring. Waves like Fig. 12.1 (a), in which the displacements of the spring are perpendicular to the direction the wave travels, are called transverse waves. Waves like Fig. 12.1 (b), in which the displacements are in the direction the wave travels, are called longitudinal waves. And waves like Fig. 12.1 (c), in which the displacements are twisting in the direction of travel of the wave, are called torsional waves.

All three can be set up in solids. In fluids, however, transverse and torsional waves die out very quickly if they can be produced at all. Thus sound waves in air and in water are longitudinal—the molecules of the medium are displaced back and forth along the direction that the sound travels.

It is often useful to make a graph of wave patterns. It is important to note that the graph always has a transverse appearance, even if it represents a longitudinal or torsional wave. Thus in Fig. 12.2 the pattern of compressions in a sound wave is represented by a graph. The graph line goes up and down to represent the increasing and decreasing density of the air, not to represent an up and down motion of the air.

A complete description of transverse waves involves a variable which descriptions of longitudinal or torsional waves do not: the direction of displacement. The displacements of a longitudinal wave can be in only one direction—the direction of travel of the wave. Similarly, the angular displacements of a torsional wave can be around only one axis—the direction of travel of the wave. But the displacements of a transverse wave can be in any and all of an infinite number of directions. This is easily seen on a rope by shaking one end around randomly instead of straight up and down or straight left and right. For simplicity, the diagrams of rope and spring waves in this chapter have shown

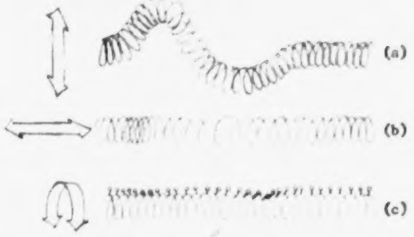


Fig. 12.1 Three types of waves on a coil spring. In (c), small markers have been put on the top of each coil.

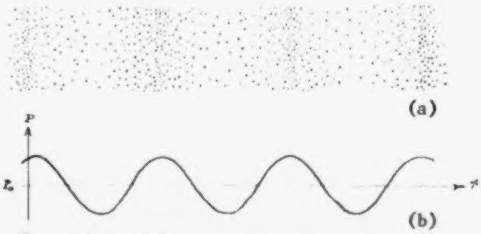


Fig. 12.2 (a) "Snapshot" representation of a sound wave progressing to the right. The dots represent the density of air molecules. (b) Graph of air pressure P vs. position x at the instant of the snapshot. P_o is the normal, undisturbed value of pressure.

Emphasize that this wiggle is a graph of some quantity and not a diagram of something moving up and down.

A caution in using "slinkies": stretching both ends increases the tension and decreases the linear density, so that experiments on the effect on velocity of either of these factors are confounded by the effect of the other.

A: Mechanical wave machines

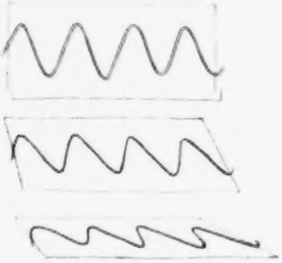


Fig. 12.3 Different polarization planes of a transverse wave.

transverse displacements consistently in a single plane.

When the displacement pattern of a transverse wave lies in a single plane, the wave is said to be polarized. For waves on ropes and springs we can observe the polarization directly. However, there is a general way of identifying a polarized wave, whether we can see the wave directly or not: find some effect of the wave which depends on angular position. An example of the principle is illustrated in the diagram below, where the interaction of a rope wave with a slotted board is shown to depend on the angle of the slotted board. Each of the three sketches begins with the same wave.

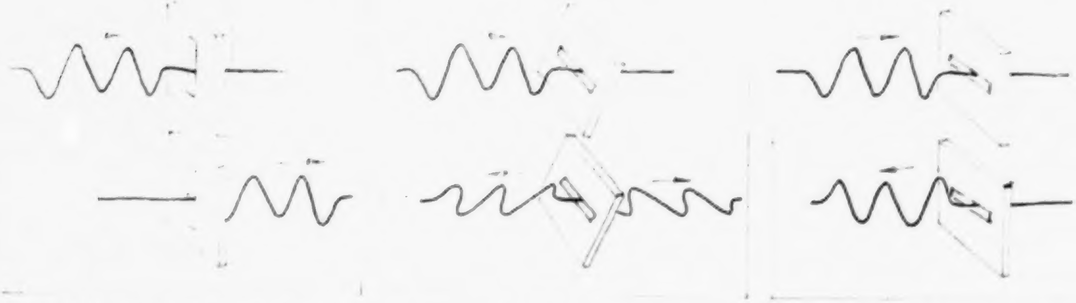


Fig. 12.4 The interaction of a polarized rope wave is sketched for three different orientations of a slotted board. The same short wave train approaches the slotted board in each of the three sketches. Depending on the orientation of the slot, the train of waves (a) goes entirely through the slot; (b) is partly reflected and partly transmitted; or (c) is completely reflected.

In general, if we can find some effect of a wave which depends similarly on angular orientation, we can conclude that the wave is polarized. Further, we can conclude that the wave must be transverse rather than longitudinal or torsional. Some interesting and important examples of this principle will be presented in Chapter 13.

All three kinds of wave have an important characteristic in common. The disturbances move through the media and away from their sources and continue on their own. We stress this particular characteristic by saying that these waves "propagate," which means we imply more than that they "travel" or "move."

An example will clarify the difference between waves which do propagate and those which do not. Almost every description of the great wheat plains of our middle west, in Canada, or in Central Europe contains a passage about the beautiful wind-formed waves that roll for miles across them. The disturbance is the swaying motion of the wheat. And the regions of that disturbance do indeed travel, but they do not propagate. That is, the disturbance does not originate at a source and then go on by itself but needs to be continually fanned by the wind. When the wind stops, the disturbance stops too. In this regard the traveling "waves" of swaying wheat are not at all the same as our rope and water waves,

Summary 12.2

1. All mechanical waves originate from a source and propagate a displacement pattern through a medium.

2. Longitudinal waves are moving patterns of compression and rarefaction along the direction of propagation. Transverse waves are moving patterns of displacement perpendicular to the direction of propagation. Torsional waves are moving patterns of angular displacement in planes perpendicular to the direction of propagation.

3. A transverse wave is polarized when its displacement pattern lies in a single plane.

and we shall therefore concentrate on waves that originate at sources and propagate. For the purposes of this chapter, waves are disturbances which propagate in a medium.

Q1 What kinds of mechanical waves can propagate in a solid?

Q2 What kinds of mechanical waves can propagate in a fluid?

Q3 What kinds of mechanical waves can be polarized?

Q4 Suppose that a mouse runs along under a rug, causing a bump in the rug that travels across the room. Is this moving disturbance a propagating wave?

D 37 : Wave propagation
D 38 : Energy transport

12.3 Wave propagation. Waves and their behavior may perhaps best be studied by beginning with large mechanical models and focussing our attention on pulses. Consider, for example, a freight train with many cars attached to a powerful locomotive standing still at a railroad crossing. If the locomotive starts abruptly, a disturbance or a displacement wave will be transmitted down the line of cars to the very last one. The shock of the starting displacement proceeds from locomotive to caboose, clacking through the couplings one by one. In this example, the locomotive was the source, the freight cars and their couplings were the medium and the "bump" traveling along the line of cars was the wave. The disturbance proceeds all the way from end to end, and with it goes energy in the form of displacement and motion. Yet no particles of matter are transferred that far; each car only jerks ahead. How much time does it take for the effect of a disturbance created at one point to reach a distant point? The time interval depends upon the speed with which the disturbance or wave propagates. That, in turn, depends upon the type of wave and the characteristics of the medium. In any case, the effect of a disturbance is never transmitted instantaneously. Time is needed for each part of the medium to transfer its energy to the next part.

Summary 12.3
1. Wave propagation involves the transfer of energy without the transfer of matter.

2. The speed of the propagation depends on the elasticity (or stiffness) and density of the medium.

A very important point: energy transfer can occur without matter transfer.

A commonly given example of a wave is a starting or stopping wave in a line of automobiles. This is not strictly a propagating wave because it depends on individual decisions of the drivers.

An engine starting abruptly can start a "bang" wave along a line of cars.

F26: Progressive waves, transverse and longitudinal

12.4

The series of sketches in Fig. 12.5 represent a wave on a rope as seen by a series of frames of a motion picture film, the frames being taken at equal time intervals. The pieces of rope do not travel along with the wave, but each bit of the rope goes through an up-and-down motion while the wave moves to the right. Except at the source of the disturbance on the left, each bit of the rope goes through exactly the same motion as the bit to its left, but its displacement is delayed a moment from the bits closer to the source.

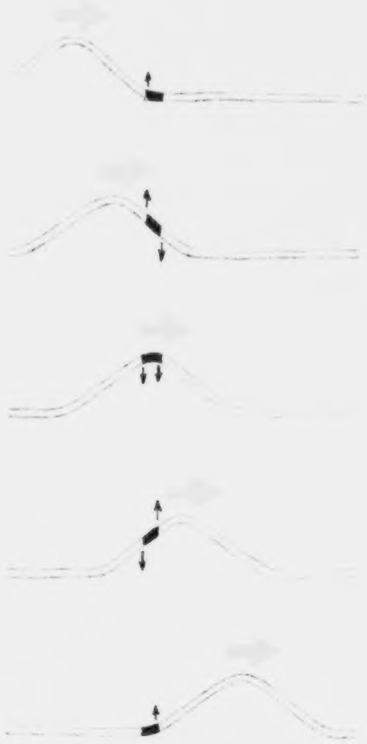


Fig. 12.5 A rough representation of the forces on a small section of rope as a transverse pulse moves past it.

Consider a small section of the rope, labeled X in the diagrams in the margin. When the pulse traveling on the rope first reaches X, the section of rope just ahead of X exerts an upward force on X. As X moves upward, a restoring force pulling X down arises from the next section on. The further up X moves, the greater the restoring forces becomes. So eventually X stops moving up and starts down again. The section of rope ahead of X is now exerting a restoring force and the section behind it is exerting an upward force, so the trip down is similar, but opposite, to the trip up. Finally X is returned to the equilibrium position and both forces vanish.

The time required for X to go up and down, that is, the time required for the pulse to pass, depends on two factors: the magnitude of the forces on X and the mass of X. To put it another way, and more generally: the speed with which a wave propagates depends on the stiffness and on the density of the medium. The stiffer the medium, the greater will be the force a section exerts on neighboring sections, and so the greater will be the propagation speed. The greater the density of the medium, the less it will respond to forces, and so the slower will be the propagation. It can be shown that the speed of propagation of a wave depends on the ratio of a stiffness factor and a density factor.

The exact meaning of stiffness and density factors is different for different kinds of waves and different media. For tight strings, for example, the stiffness factor is the tension T in the string and the density factor is the mass per unit length m/ℓ. The propagation speed v is

$$v = \sqrt{\frac{T}{m/\ell}} \, .$$

Q5 What is transferred along the direction of wave motion?

Q6 On what two properties of a medium does wave speed depend?

12.4 Periodic waves. Most of the disturbances we have considered up to now have been short-lived and sudden. The waves set up by single disturbances are called pulses, e.g., the snapping of one end of a rope or the dropping of a stone in a pond, or the sudden bumping of one end of a train. In each case we see a pattern running along the medium with a certain velocity. Continuous regular rhythmic disturbances in a medium result from periodic vibrations which cause periodic waves in that medium. A good example of such a vibration is

Point out similarity to circular motion (Chapter 4) where same terms were previously introduced. 12.4

a swinging pendulum. The swinging is periodic in time, and the pendulum bob executes simple harmonic motion. Another example is the up-and-down motion of a weight at the end of a good spring. The maximum displacement from the position of equilibrium is called the amplitude A and is shown in Fig. 12.6 The time taken to complete one oscillation is called the period and labeled T, while the frequency, or number of vibrations per second, is symbolized by f.

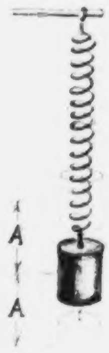


Fig. 12.6

What happens when such a vibration is applied to the end of a rope? Let us think of an experiment where one end of a rope is fastened to an oscillating weight. As the weight vibrates up and down, we observe a wave "traveling" along the length of the rope.

We observe moving crests and troughs along the length of a uniform rope. The source executes simple harmonic motion up and down, and ideally every point along the length of the rope executes simple harmonic motion in turn. The wave travels forward to the right as crests and troughs in turn replace one another, but the points along the rope simply oscillate up and down following the motion of the source. The distance between any two consecutive crests or any two consecutive troughs is always found to be the same along the length of the rope. This distance is called the wavelength of the periodic wave, and is denoted by the Greek letter λ (lambda). As in Fig. 12.7, the amplitude of the wave is represented by A.

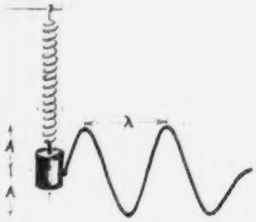


Fig. 12.7

When a single pulse moves from one part of the medium to another, it is fairly clear what is meant by the speed of the pulse. All we need in principle is a clock and a meter stick; watch the front edge of the pulse, and the speed of the pulse is quickly found. But what if we cannot observe the source or the beginning or ending of a wave train? We will show that the speed of a periodic wave can be easily found from its frequency and wavelength.

As a wave progresses, each point in the medium oscillates periodically with the frequency and period of the source. Figure 12.8 illustrates a periodic wave moving to the right, frozen every 1/4 period. Follow the progress of the crest that started out at the extreme left. The time it takes this crest to move a distance of one wavelength is the time it takes a point in the medium to go through one complete oscillation. That is, the crest moves one wavelength λ in one period of oscillation T. The speed of the crest is therefore

$$v = \frac{\text{distance travelled}}{\text{time taken}} = \frac{\lambda}{T}.$$

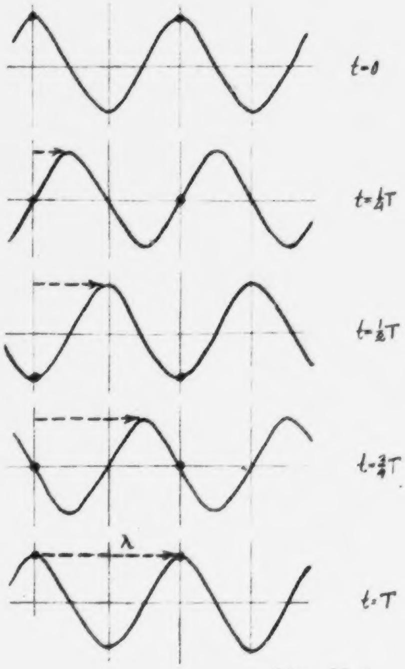


Fig. 12.8

A: Graphical addition of waves

Summary 12.4

1. Periodic waves are those with a single displacement pattern repeated over and over again.

The wave generated by a simple harmonic vibration is a sine wave. It has the same shape as a graph of the sine function familiar in trigonometry.

2. Four definitions pertain to all wave motion, whether mechanical or not:
   (a) Period (T): time for one complete oscillation.
   (b) Wavelength (λ): distance between two identical points on identical waves.
   (c) Frequency (f): number of crests passing a point per second.
   (d) Amplitude (A): maximum displacement from equilibrium condition.

3. Points on a wave which are λ, 2λ, 3λ, etc. apart are in phase with each other, and points which are $\lambda/2$, $3\lambda/2$, $5\lambda/2$, etc. are out of phase with each other.

4. The speed (v) of a wave equals the product of its wavelength (λ) and frequency (f): $v = \lambda f$.

The speed of this wave is the same thing as the speed of any one of its crests. We say, therefore, that the speed of the wave,

$$v = \frac{\text{wavelength}}{\text{period of oscillation}} = \frac{\lambda}{T}.$$

But $T = \frac{1}{f}$, where f = frequency (see Unit 1, Chapter 4, page 106). Therefore $v = f\lambda$, or wave speed = frequency × wavelength.

We can also write this relationship as $\lambda = \frac{v}{f}$ or $f = \frac{v}{\lambda}$. These expressions imply that, for waves of the same speed, the frequency and wavelength are inversely proportional—a wave of twice the frequency would have only half the wavelength, and vice versa. This inverse relation of frequency and wavelength will be useful in other units in this course.

In Fig. 12.9, sets of points are marked which are in step. The crest points C and C′ have reached maximum displacement positions in their vibrations in the upward direction. The trough points D and D′ have reached maximum displacement positions in the downward direction. The points C and C′ have been chosen such that they have identical displacements and velocities at any instant of time. Their vibrations are identical and in unison. The same is true for the points D and D′. Indeed there are indefinitely many such points along the length of the wave which are vibrating identically. Note that C and C′ by definition are a distance λ apart, and so are D and D′. Points such as C and C′ are said to be in phase with one another as are also points D and D′. Points separated from one another through distances of λ, 2λ, 3λ, ... $n\lambda$, are all in phase with one another. These can be anywhere along the length of the wave and need not correspond with only the high or low points. For example, points such as P, P′, P′′, are all in phase with one another. They are each separated from the other by the distance λ.

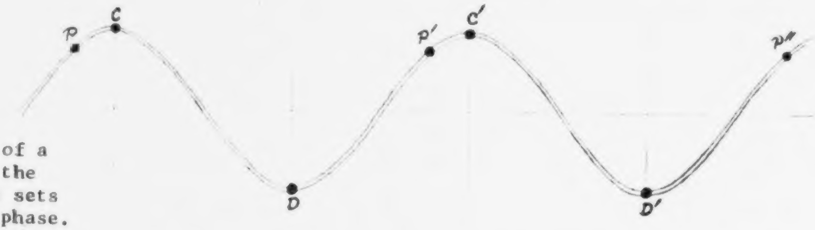


Fig. 12.9 A "snapshot" of a periodic wave moving to the right. Letters indicate sets of points with the same phase.

Some of the points are exactly out of step. Point C reaches its maximum upward displacement at the same time that D reaches its maximum downward displacement. At the instant that C begins to go down, D begins to go up. Points

D46: Beats and harmonic synthesis

such as these are called one-half wave out of phase with respect to one another; C and D′ also are one-half wave out of phase. Points separated from one another through distances of $\frac{\lambda}{2}$, $\frac{3\lambda}{2}$, $\frac{5\lambda}{2}$,... are one-half wave out of phase.

Q7 Of the wave variables—frequency, wavelength, period, amplitude and polarization—which ones describe
1) space properties of waves?
2) time properties of waves?

Q8 How can "wavelength" be defined for a wave that isn't a regular sine wave?

Q9 A vibration of 100 cycles per second produces a wave.
1) What is the wave frequency?
2) What is the period of the wave?
3) If the wave speed is 10 meters per second, what is the wavelength? (You can look back to find the relationship you need to answer this.)

Q10 If points X and Y on a periodic wave are one-half wave "out of phase" with each other, which of the following must be true?
a) X oscillates at half the frequency at which Y oscillates.
b) X and Y always move in opposite directions.
c) X is a distance of one-half wavelength from Y.

12.5 When waves meet: the superposition principle. So far we have considered single waves in isolation from other waves. What happens when two waves encounter each other in the same medium? Suppose that two waves approach each other on a rope, one traveling to the right and one traveling to the left. The series of sketches in Fig. 12.10 show what happens. The waves pass through each other without either being modified. After the encounter, each wave shape looks just as it did before and is traveling along just as it was before. This phenomenon of passing through each other unchanged is common to all types of waves. You can easily see that it is true for surface ripples on water. (Look back, for example, to the opening photograph for the chapter.) You could infer that it must be true for sound waves by recalling that two conversations can take place across a table without either distorting the other.

But what is going on during the time when the two waves overlap? They add up. At each instant the rope's displacement at each point in the overlap region is just the sum of the displacements that would be caused by each of the two waves alone. If two waves travel toward each other on a rope, one having a maximum displacement of 1 cm upward and the other a maximum displacement of 2 cm upward, the total maximum upward displacement of the rope while these two waves pass each other is 3 cm.

What a wonderfully simple behavior, and how easy it makes everything! Each wave proceeds along the rope making its

This should definitely be preceded by a demonstration. Start with two equal pulses meeting so that some students might claim they bounce off each other then go to a large and a small pulse to show they indeed pass through each other unchanged. Superposition as they pass is seen as the small pulse rides over the large one.

D39: Superposition

Photograph of this happening on a slinky in Unit 3 Reader.

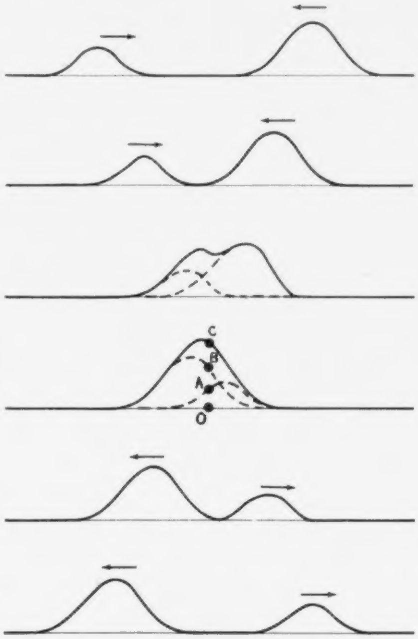
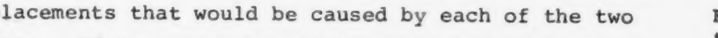


Fig. 12.10 The superposition of two rope waves. The dashed curves are the contributions of the individual waves. As an example, in the fourth sketch the displacement OA plus the displacement OB gives the resulting displacement of the rope OC.

T15: Superposition
T26: Square wave analysis

A train of short, small pulses started at one end and longer, larger pulses from the other will further suggest the superposition principle. Simple addition of displacement is the most reasonable model to try first.



L38 : Superposition of waves

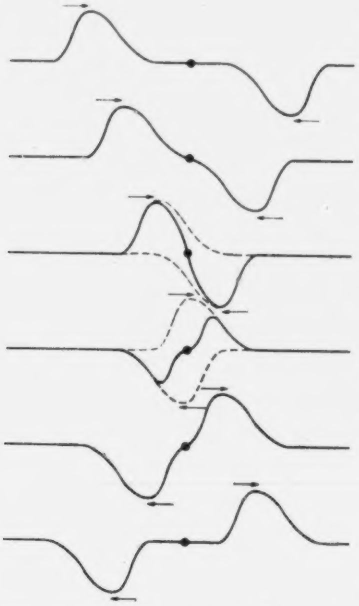


Fig. 12.11 These are, of course, only drawings. No evidence for simple additivity is presented here.

own particular contribution to the rope's displacement no matter what any other wave may be doing. If we want to know what the rope looks like at any instant, all we need do is add up the displacements due to each wave at each point along the rope. We say that waves obey a superposition principle. Figure 12.11 shows another case of wave superposition. Notice that when the displacements are in opposite directions, they tend to cancel each other. This still fits the addition rule, since one direction of displacement is considered negative.

The superposition principle applies no matter how many separate waves or disturbances are present in the medium. The examples shown in Figs. 12.10 and 12.11 illustrate the principle applied when only two waves are present, but we can discover by experiment that the superposition is just as valid when there are three, ten, or any number of waves. Each wave makes its own contribution and the net result is simply the sum of all the individual contributions.

This simple additive property of waves permits us to add waves graphically. You should check the diagrams with a ruler to see that the net displacement (full line) is just the sum of the individual displacements (dashed lines) in these two cases.

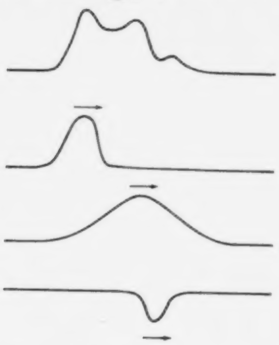


Fig. 12.12

We can turn the superposition principle around. If it is true that waves add as we have described, then we can think of a complex wave as the sum of a set of simpler waves. In Fig. 12.12 at the left, a complex wave has been analyzed into a set of three simpler waves.

The French mathematician Jean-Baptiste Fourier announced a theorem in 1807 that any periodic oscillation, regardless of its complexity, can be analyzed as the sum of a series of simpler regular wave motions. Fourier was interested in the theory of heat, sound, light and electricity, and his theorem became a basic tool for harmonic analysis in all these areas. Fourier showed the general validity of the superposition principle.

Q11 Two periodic displacement waves of amplitudes A_1 and A_2 are passing through a point P. What will be the greatest displacement of point P?

Q12 The superposition principle states that wave amplitudes add. How then can waves cancel each other out?

Jean-Baptiste Fourier (1768-1830), a tailor's son who became a brilliant mathematician in Napoleonic France.

Little is made of the powerful idea hinted at here. If you have time you might want to take a brief excursion into harmonic analysis with such topics as timbre, electric organs, speech synthesis, harmonic distortion in hi fi, etc. Refer to "Ears, Architects of Harmony" in the Reader.

Summary 12.5

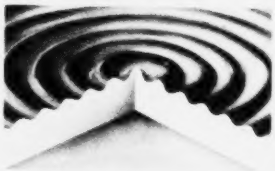
1. The simple additivity of waves is called the superposition principle – it is one of the basic tools of wave theory.
2. Complex waves can often be analyzed as the sum of a number of simple sine waves.

D43: Interference patterns

12.6 A two source interference pattern. We can see the superposition principle at work in the important wave phenomenon known as interference. Figure 12.13 is a photograph of ripples spreading away from a small sphere, vibrating up and down into the water surface. What we see here is the spatial pattern of the water level at an instant. Fig. 12.14 is a photograph of the water surface when it is agitated by two vibrating spheres. The two small sources go through their up and down motions together, that is, the sources are in phase. The photograph catches the pattern of the overlapping waves at one instant, called an interference pattern.

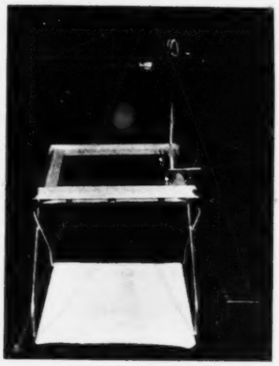
Can we interpret what we see in this photograph in terms of what we already know about waves? And can we describe how the pattern will change with time? If you tilt the page so that you are viewing Fig. 12.14 from a glancing direction, you will see more clearly some spokes or strips of intermediate shade—neither as bright as the crest, nor as dark as the trough of waves. This feature can be easily explained by the superposition principle.

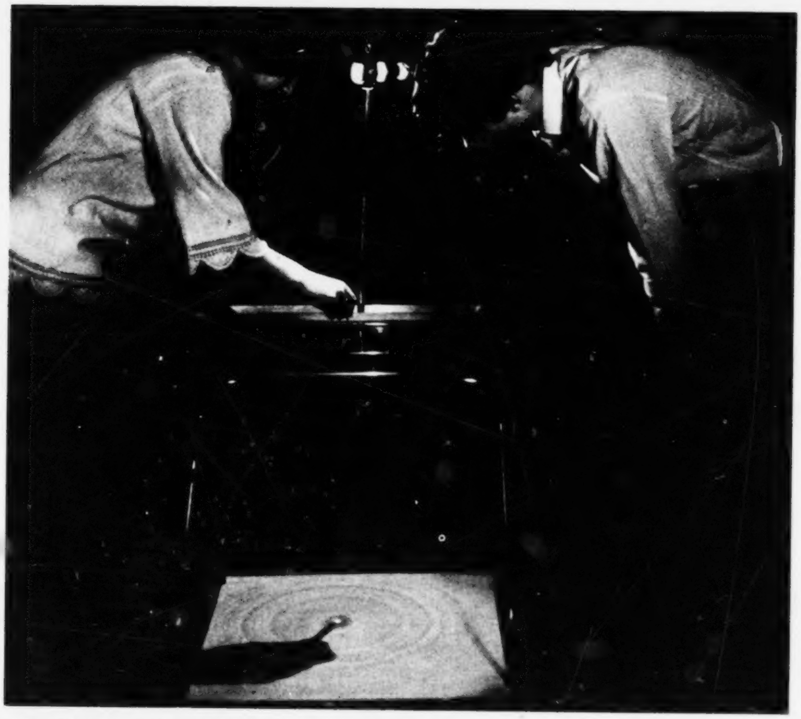
12.6



Drawing of surface wave on water.

A "ripple tank" set-up for viewing the behavior of waves.





The ripple tank, being used here by students to observe a circular pulse, can be fitted with vibrator to produce periodic wavetrains. Fig. 12.13 is an instantaneous photograph of the shadows of ripples produced by a vibrating point source. For Fig. 12.14 there were two point sources vibrating in phase.

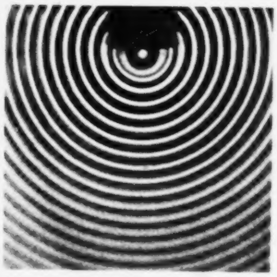


Fig. 12.13

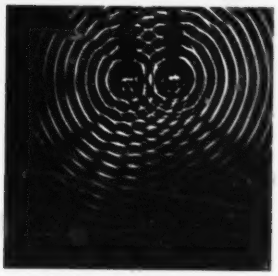
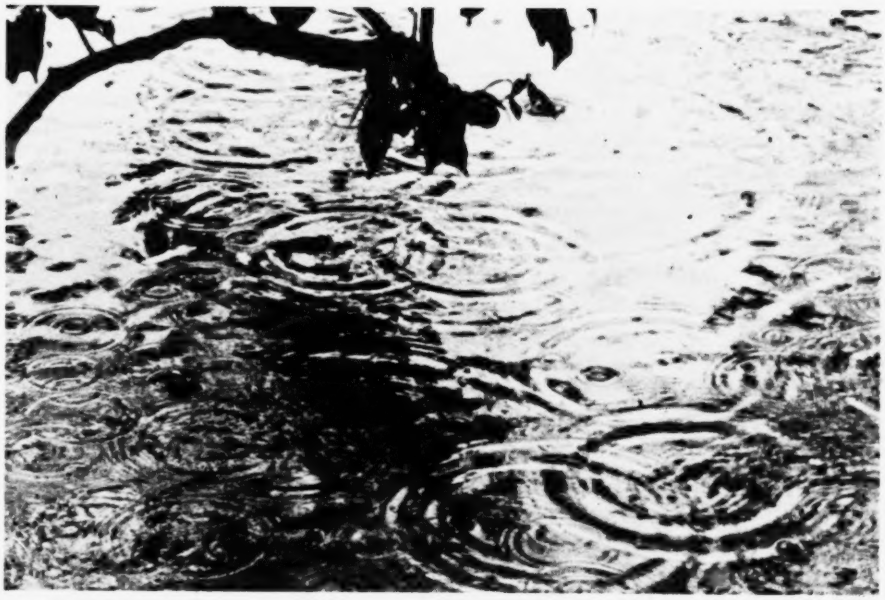


Fig. 12.14

T28 : Two-slit interference



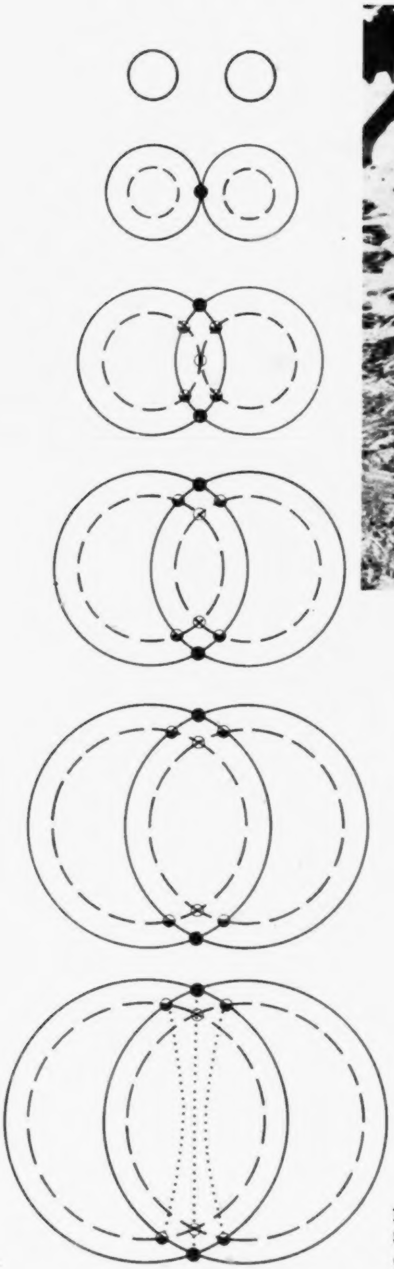


Fig. 12.15
Superposition of
circular pulses.

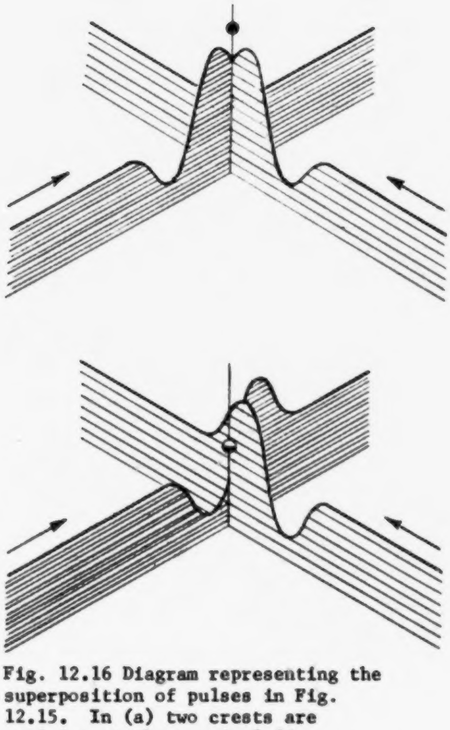
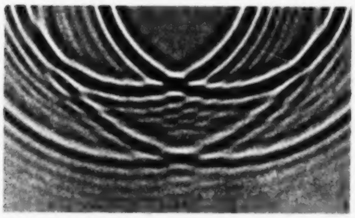


Fig. 12.16 Diagram representing the superposition of pulses in Fig. 12.15. In (a) two crests are arriving at the vertical line. In (b) a crest is arriving together with a trough. The dark and half-dark balls show the net displacement.



Amplitude diminishes as the pulses spread, so that the amplitudes of the two pulses
are not equal except along the median lines. Consequently, cancellation is never complete. The
124per cent discrepancy of r's becomes less at greater distances and so cancellation improves with
distance, as can be seen in Fig. 12.14.

Suppose that two sources produce identical pulses at the same instant, each pulse containing one crest and one trough as shown in Fig. 12.15. The height of each crest above the undisturbed level is equal to the depth of each trough below. The sketches show the patterns of the water surface after equal time intervals. As the pulses spread out, the points at which they overlap move too. In the figure we have placed a completely darkened circle wherever a crest overlaps another crest, a blank circle wherever a trough overlaps another trough, and a half-darkened circle wherever a crest overlaps a trough. Applying the superposition principle to this situation, we conclude that the water level is highest at the completely darkened circles, lowest at the blank circles, and at the equilibrium height at the half-darkened circles. Each of the sketches in Fig. 12.15 represents the spatial pattern of the water level at an instant. The dotted curves in the last sketch in Fig. 12.15 are the paths followed by the overlap regions during the time covered by the earlier sketches.

At points in Fig. 12.15 which are marked with darkened and blank circles the two pulses arrive in phase, as shown in Fig. 12.16(a). The waves reinforce each other causing a greater amplitude and are thus said to interfere constructively. All such points are at the same distance from each source.

At the points in Fig. 12.15 marked with half-darkened circles, the two pulses arrive completely out of phase, as shown in Fig. 12.16(b). Here the waves are said to interfere destructively, leaving the water surface undisturbed. All such points are one crest-trough distance further from one source than from the other.

Now we can interpret the photograph of Fig. 12.14. The centers of what we called "strips of alternating character" are areas where waves cancel or reinforce each other, called nodal or antinodal lines, respectively. Look closely at Fig. 12.17 and notice its symmetry. The central spoke or strip labeled A_0 is an antinode where reinforcement is complete. As the waves spread out, points on these lines are displaced up and down much more than they would due to either wave alone. The outside nodes labeled N_4 represent lines where destructive interference is at a maximum. As the waves spread out, points on these lines move up and down much less than they would due to either wave alone. You should compare the drawing in Fig. 12.17 with the photograph in Fig. 12.14 to make sure you know which are the antinodal lines and which are the nodal lines.

Summary 12.6

1. Interference patterns result from two or more periodic waves superposing.

2. Destructive interference occurs along nodal lines, where waves from two sources are exactly out of phase. Constructive interference occurs along antinodal lines, where waves from two sources are exactly in phase.

3. If we know the direction of a particular nodal line and the separation of the sources, we can calculate the wavelength (λ).

A: Moiré patterns

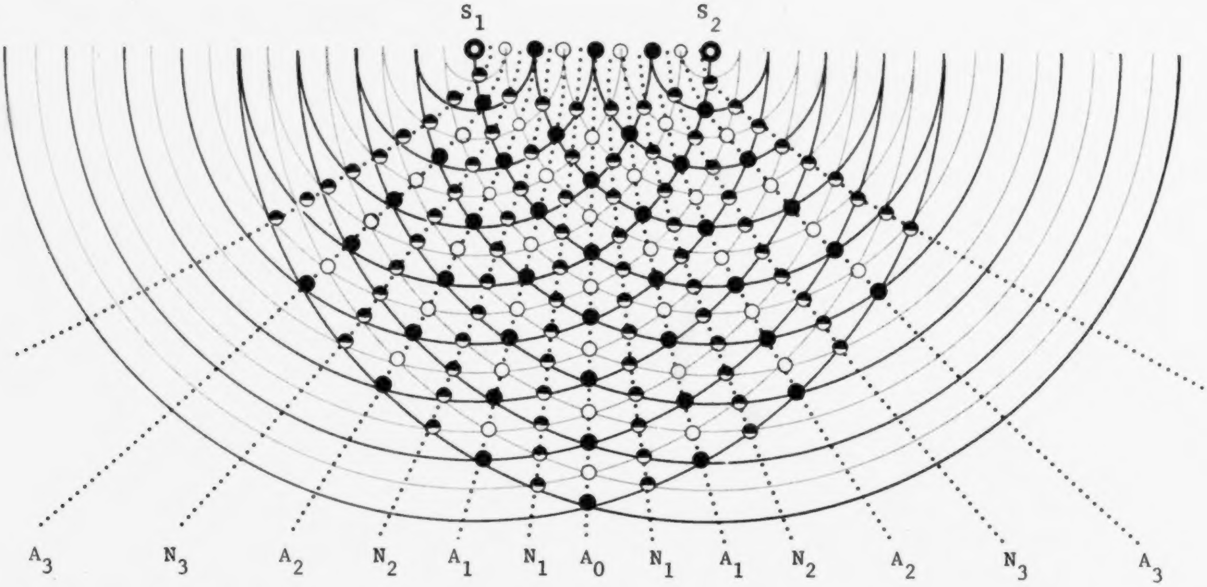


Fig. 12.17 A "snapshot" diagram of the interference pattern due to two in-phase periodic sources separated by four wavelengths. The letters A and N designate antinodal and nodal lines. The dark circles indicate where crest is meeting crest, the blank circles where trough is meeting trough, and the half-dark circles where crest is meeting trough.

If we know the wavelength λ and the source separation, then we can calculate the angles that these lines make with the central line A_0. Or, if we know these angles and the source separation, then we can calculate the wavelength. The observation of a two-source interference pattern allows us to make a wavelength calculation, even if we are unable to see the waves directly! For example, an interference pattern in sound can be created with two loudspeakers being driven in phase, and nodal and antinodal points can be located by ear. But if you do the experiment indoors, you may discover unexpected nodal points or "dead spots" caused by reflections from walls or other obstacles.

What determines the positions of the nodal and antinodal lines? Figure 12.18 shows part of the pattern of Figure 12.17. At any point P on an antinodal line, the waves from the two sources arrive in phase. This occurs where P is some whole number of wavelengths further from one source than from the other; that is, where the difference in distance $\overline{S_2P} - \overline{S_1P} = n\lambda$, λ being the wavelength and n being any whole number (including zero). At any point Q on a nodal line, the waves from the two sources arrive exactly out of phase. This occurs where Q is some odd number of half-wavelengths ($\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, etc.) further from one source than from the other; that is, where $\overline{S_1Q} - \overline{S_2Q} = (n + \frac{1}{2})\lambda$. If the distance ℓ from midway between the sources to a detection point is much larger than the source separation d, so that the point lies on the relatively straight part of the nodal or antinodal line, then there is a simple relationship between the node position, the wavelength λ and source separation d. The details of the relationship and the calculation of wavelength are described on the next page.

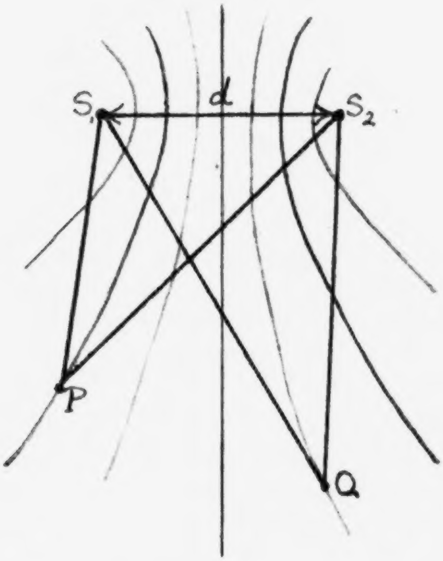


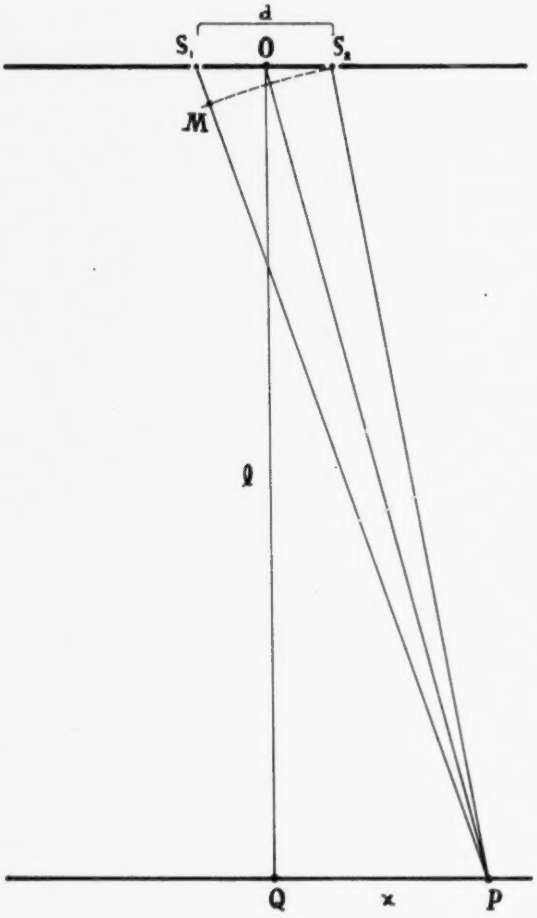
Fig. 12.18

Calculating Wave Lengths from an Interference Pattern

d = separation between sources S_1 and S_2 (they may be two sources that are truly in phase, or two slits through which a previously prepared wave front passes)

ℓ = distance from sources to detection line parallel to the two sources

x = distance from center line to point P along the detection line



Waves reaching P from S_1 travel farther than waves reaching P from S_2. If the extra distance is λ (or 2λ, 3λ, etc.), the waves will arrive in phase and P is a point of strong wave disturbance. If the extra distance is $\frac{1}{2}\lambda$ (or $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, etc.), the waves will arrive out of phase and P is a point of weak wave disturbance.

With P as center we draw an arc of a circle of radius $\overline{PS_2}$; it is indicated on the figure by the dotted line $\overline{S_2M}$. Then line $\overline{PS_2}$ = line $\overline{PM}$ and therefore the extra distance that the wave from S_1 travels is the length of the segment $\overline{S_1M}$.

Now if d is very small compared to ℓ, the circular arc $\overline{S_2M}$ is a very small piece of a large circle and is practically the same as a straight line. Also the angle S_1MS_2 is very nearly 90°, so that triangle S_1S_2M can be regarded as a right triangle. Furthermore angle S_1S_2M is equal to angle POQ. Then right triangle S_1S_2M is similar to right triangle POQ.

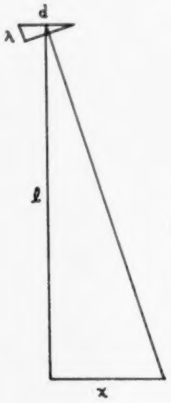
So
$$\frac{\overline{S_1M}}{\overline{S_1S_2}} = \frac{x}{\overline{OP}}. \qquad (12.1)$$

But $\overline{S_1M}$ is the extra distance traveled by the wave from source S_1. For P to be a maximum of disturbance, S_1M must equal λ (or 2λ, 3λ, etc.), $\overline{S_1S_2}$ is the source separation d. OP is very nearly equal to ℓ. Then Eq. (12.1) becomes

$$\frac{\lambda}{d} = \frac{x}{\ell} \qquad (12.2)$$

and

$$\lambda = \frac{d}{\ell}x \qquad (12.3)$$



If we measure the source separation d, the distance ℓ, and the distance x to the first disturbance maximum beside the center, we can calculate the wavelength from Eq. (12.3).

This analysis allows us to calculate the wavelength of any wave phenomenon, whether it is water ripples, sound, light, etc.; it will therefore be found very useful in later Units.

Slinkies and heavier coil springs are ideal for these demonstrations.

Q13 Are nodal lines in interference patterns regions of cancellation or regions of reinforcement?

Q14 What are antinodal lines?

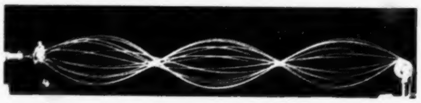
Q15 Nodal points in an interference pattern occur where
a) the waves arrive "out of phase."
b) the waves arrive "in phase."
c) the point is equidistant from the wave sources.
d) the point is one-half wavelength from both sources.

Q16 Under what circumstances do waves from two in-phase sources arrive at a point out of phase?

Summary 12.7
1. Periodic waves travelling in opposite directions can produce "standing waves"— oscillation patterns with fixed nodes.

2. Standing waves set up by multiple reflections in a bounded medium can have only a certain set of frequencies.

12.7 Standing waves. If both ends of a rope or spring are shaken very carefully, with the same frequency and same amplitude, a very interesting phenomenon can be produced. The interference of the identical waves coming from both ends will result in certain points on the rope not moving at all! In between these nodal points, the rope oscillates back and forth with no apparent propagation of wave patterns. This phenomenon is called a "standing wave" or "stationary wave" and is pretty to watch. (Using the superposition principle, you can show that this is just what would be expected from the addition of the two oppositely traveling waves.)



A vibrator at the left produces a wave train that runs along the rope and reflects from the fixed end at the right. The sum of the oncoming and the reflected waves is a standing wave pattern. This stroboscopic photograph shows 12 instantaneous positions of the rope.

T27: Standing waves
T29: Interference pattern analysis

The same effect can be produced by the interference of a continuous wave with its reflection. To make standing waves, there do not have to be two people shaking the opposite ends of the rope; one end can be tied to a hook on a wall. The train of waves sent down the rope by shaking one end will reflect back from the fixed hook and interfere with the new, oncoming wave and can produce the standing pattern of nodes and oscillation. In fact, you can go further and tie both ends of the rope to hooks and pluck or hit the rope. From the plucked point a pair of waves go out in opposite directions and then reflect back and forth from the fixed hooks. The interference of these reflected waves traveling in opposite directions can produce a standing pattern just as before. Standing waves on guitars, violins, pianos and all other stringed instruments are produced in just this fashion. Because the vibrations of strings are in standing waves, the vibration frequencies depend on the speed of wave propagation along the string and the length of the string.

Painting on a Greek vase from 5th century B.C.

Museum of Fine Arts, Boston

The connection between length of string and musical tone, recognized over two thousand years ago, was of the greatest importance in contributing to the idea that nature is built on mathematical principles. When strings of equal tautness and diameter are plucked, pleasing harmonies result if the lengths of the strings are in the ratios of small whole numbers. Thus the ratio 2:1 gives the octave, 3:2 the musical fifth and 4:3 the musical fourth. This striking connection between music

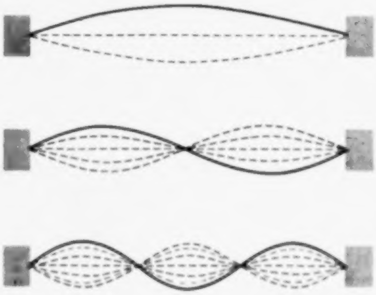
L39: Standing waves on a string

D45: Standing waves

L42 : Vibrations of a rubber hose
L43 : Vibrations of a wire

and numbers encouraged the Pythagoreans to search for other numerical harmonies in the universe. The Pythagorean ideal strongly affected Greek science and became an inspiration for much of Kepler's work. In a generalized form, the ideal flourishes to this day in many beautiful applications of mathematics to physical experience.

Using nothing more than the superposition principle, we can now state the harmonic relationship much more precisely. Standing patterns can be produced by reflections from the boundaries of a medium only for certain wavelengths (or frequencies). In the example of a string fixed at both ends, the two ends are fixed and so must be nodal points. Thus the longest traveling waves that can set up standing waves on a rope will be those for which one-half wavelength just fits on the rope. Shorter waves can produce standing patterns with more nodes, but only when some number of one-half wavelengths just fit on the rope. The shorter wavelengths correspond to higher frequencies, so the principle can be stated that on any bounded medium, only certain frequencies of standing waves can be set up. On an idealized string, there are in principle an unlimited number of frequencies, all simple multiples of the lowest frequency. That is, if f_0 is the lowest possible frequency of standing wave, the other possible standing waves would have frequencies $2f_0$, $3f_0$,.... These higher frequencies are called "overtones" or "harmonics" of the "fundamental" frequency f_0.



In real media, there are practical upper limits to the possible frequencies and the overtones are not exactly simple multiples of the fundamental frequency (that is, the overtones are not strictly harmonic). In more complicated systems than stretched strings, like the enclosure of a saxophone, the overtones may not be even approximately harmonic. As you might guess from the superposition principle, standing waves of different frequencies can exist in a medium at the same time. A plucked guitar string, for example, will oscillate in a pattern which is the superposition of the standing waves of many overtones. The relative oscillation energies of the different overtones of string instruments or in the enclosure of horns and organ pipes determine the "quality" of the sound they produce. The difference in the balance of overtones is what makes the sound of a violin distinct from the sound of a trumpet, and both distinct from a soprano voice, even if all these are sounding with the same fundamental frequency.

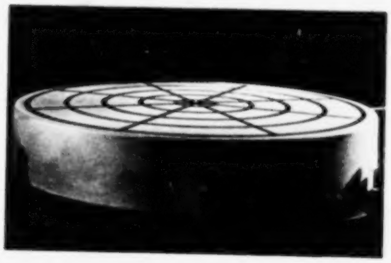
The production of multiple vibration modes is discussed in the excellent Science Study Series paperback Horns, Strings and Harmony. Part of the treatment is in the Reader.

Bathroom singers and pop-bottle blowers will be familiar with characteristic frequencies of cavities.

Q17 When two identical waves of frequency f, traveling in opposite directions, interfere to produce a standing wave, what is the motion of the medium at
1) the nodes of the standing wave?
2) the antinodes (loops) of the standing wave?

F27: Stationary longitudinal waves
F28: Stationary transverse waves

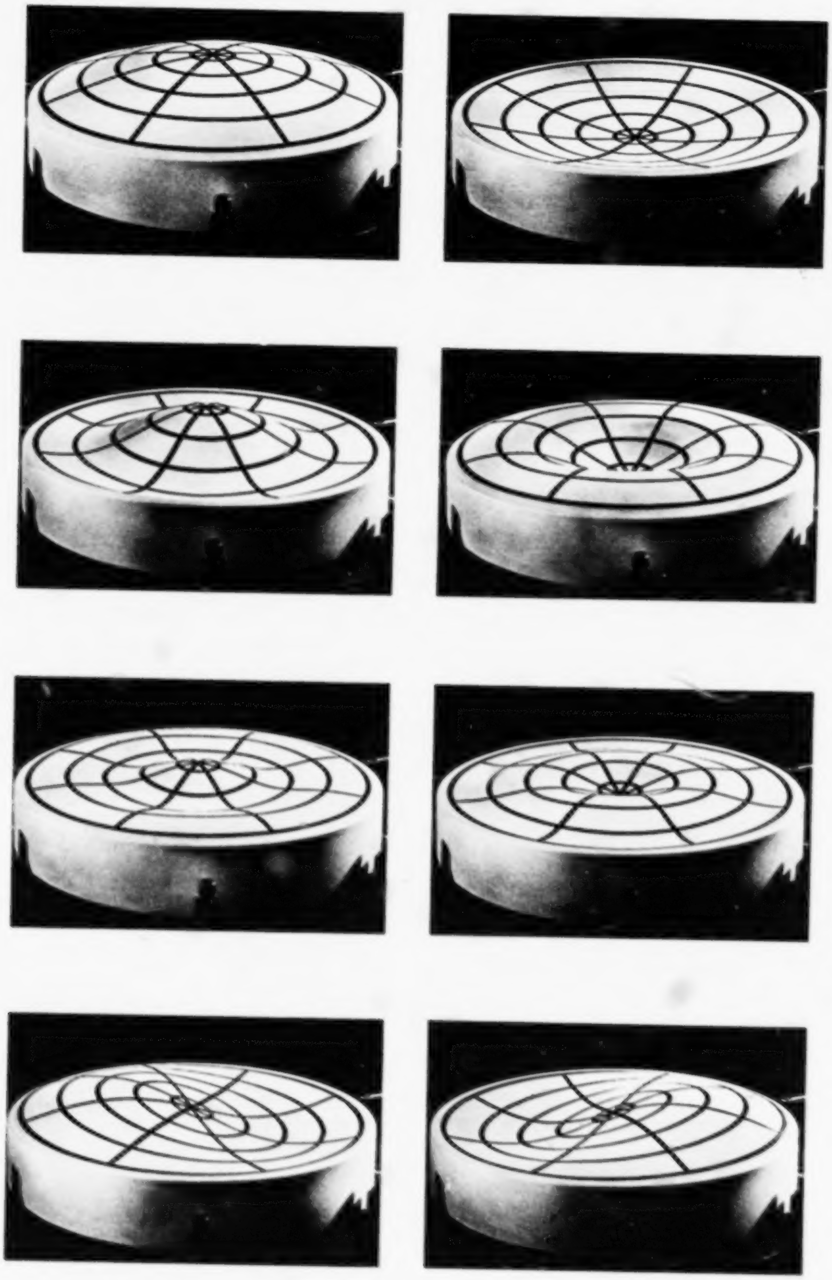
A: Standing waves on drum and violin



These phenomena appear in the film loop "Vibrations of a Drum", L44.

A rubber "drumhead" first at rest, then made to vibrate in each of four of its many possible modes.

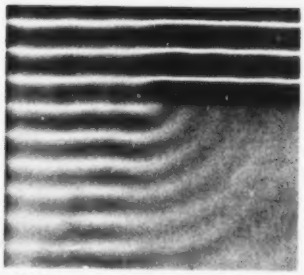
A number of characteristic frequencies of the surface of a bowl of milk can be excited by tapping the bowl. Try Jello!



Q18 If the two interfering waves have wavelength λ, what is the distance between the nodal points of the standing wave?

Q19 What is the wavelength of the longest traveling waves which can produce a standing wave on a string of length L?

Q20 Can standing waves of any frequency, as long as it is higher than the fundamental, be set up in a bounded medium?



Diffraction of ripples around the edge of a barrier.

12.8 Wave fronts and diffraction. Waves go around corners. We are so used to sound waves doing this that we scarcely notice it. This phenomenon of the energy of waves spreading into what we would expect to be "shadow" regions is called diffraction.

Once again we turn to water waves to see this point best. From among all the arrangements of barriers that can result in diffraction, we will concentrate on two. The first of these is shown in Fig. 12.20, which is a photograph of straight water waves being diffracted through a narrow slit in a straight barrier. The barrier is parallel to the wave's crest lines, and the slit is less than one wavelength wide. Figure 12.21 is a photograph of the second barrier arrangement we want to investigate. There are two narrow slits in the barrier, and the pattern of the diffracted wave is the same as that given by two point sources which are in phase. We get the same kind of result no matter how many narrow slits we put in the barrier placed parallel to the incoming wave. That is, the pattern of the diffracted wave is just that which would be produced if a point source were put at the center of each slit position, with such sources in phase.

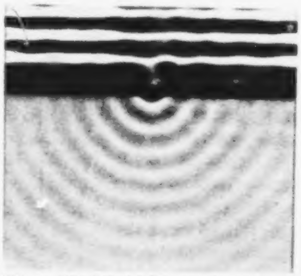


Fig. 12.20 Diffraction of ripples through narrow opening.

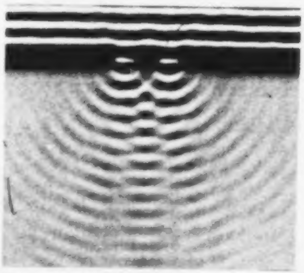


Fig. 12.21 Diffraction of ripples by two narrow openings. The diffracted waves spread out and overlap, producing the same interference effect as we studied for two actual sources in Section 12.6.

We can describe these and all other diffraction patterns if we understand a characteristic of waves, first enunciated by Christiaan Huygens in 1678 and now known as Huygens' principle. Before giving a statement of the principle, we need the definition of a wave front.

What we have called crest lines and trough lines of water waves are special cases of wave fronts. For a water wave, a wave front is an imaginary line along which every point on the water's surface is in exactly the same stage of its vibrational motion, that is, in the same phase. Crest lines are wave fronts, since every point on the water's surface along a crest line has just reached its maximum displacement upward, is momentarily at rest, and will start downward an instant later. For "straight-line" water waves, all wave fronts are straight lines parallel to each other. For circular waves, the wave fronts are all circles.

D44: Diffraction

For sound waves created by a handclap, the wave fronts become at a distance very nearly spherical surfaces. At large distances from such a source of sound, i.e., where the radii of the spherical wave fronts are large, a small section of the wave front is nearly flat. Thus circular or spherical wave fronts become virtually straight-line or flat-plane wave fronts at great distances from their sources.

Now here is Huygens' principle as it is generally stated today: every point on a wave front may be considered to behave as a point source for waves generated in the direction of the wave's propagation. As Huygens said:

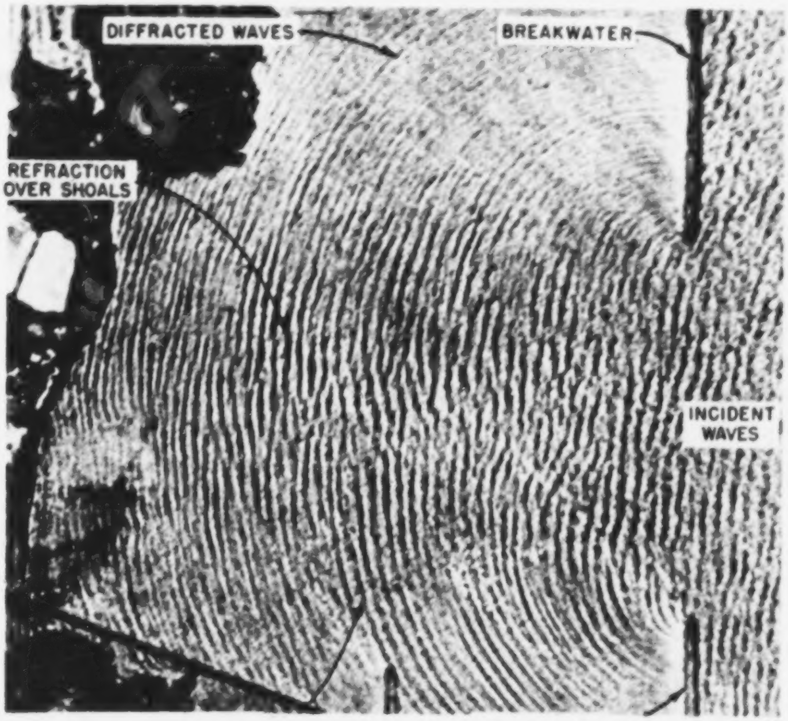
> There is the further consideration in the emanation of these waves, that each particle of matter in which a wave spreads, ought not to communicate its motion only to the next particle which is in the straight line drawn from the luminous [source] point, but that it also imparts some of it necessarily to all others which touch it and which oppose themselves to its movement. So it arises that around each particle there is made a wave of which that particle is the center.

The one-slit and two-slit diffraction patterns are certainly consistent with Huygens' principle. In the two-slit case (Fig. 12.21), each wave front arrives at the two slits at the same time, so the oscillations in the slit are in phase. That is why the interference pattern produced by the waves diffracted through these slits matches that produced by two point sources which oscillate in phase.

Summary 12.8
1. A wave front is a line or surface on which at every point the wave has the same phase.

2. Huygen's principle holds that every point on a wave front may be considered to behave as a point source for more waves.

3. The diffraction of waves through holes can be explained by Huygen's principle. The narrower the slit, in comparison to the wave length, the more the emerging wave becomes circular or spherical, as if it came from a point source.



Diffraction of ocean waves at a breakwater.

Compare with top photograph on page 131.

We can understand all diffraction patterns if we keep both Huygens' principle and the superposition principle in mind. For example, consider a slit, wider than one wavelength, in which case the diffraction pattern contains nodal lines. Figure 12.22 shows why nodal lines appear. There surely exist points like P that are just λ farther from one side of the slit A than from the other side B. In that case, AP and OP differ by one-half wavelength λ/2. In keeping with Huygens' principle, we imagine the points A and O to be in-phase point sources of circular waves. But since AP and OP differ by λ/2, the two waves will arrive at P completely out of phase. So, according to the superposition principle, the waves from A and O will cancel at point P.

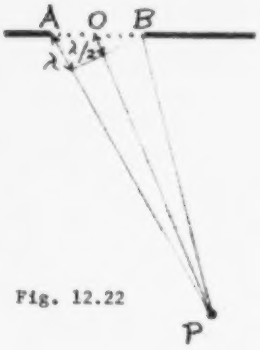


Fig. 12.22

But the argument made for the pair of "sources" at A and O can also be made for the pair consisting of the first point to the right of A and the first to the right of O. In fact, the same is true for each such matched pair of points all the way across the slit. Since the waves originating at each pair cancel, the point P is a nodal point on a nodal line. Then we should see nodal lines in the diffraction pattern of a slit wider than λ, and we do, as Fig. 12.23 shows. If the slit width is less than λ, then there can be no nodal point, since no point can be a distance λ farther from one side of the slit than from the other. Slits of widths less than λ behave nearly as point sources. The narrower they are, the more nearly their behavior is that of point sources.

We can measure the wavelength of a wave by investigating the interference pattern where the diffracted waves overlap. For example, we can analyze the two-slit pattern (Fig. 12.21) in just the same way we analyzed the two-source interference pattern in Sec. 12.6.

The larger the wavelength compared to the distance between the slits, for two-slit interference, the more the interference pattern spreads out. That is, as λ increases or d decreases, the nodal and antinodal lines make increasingly large angles with the straight-ahead direction. Similarly, for single-slit diffraction, the pattern spreads when the ratio of wavelength to the slit width increases. For a given slit width, the longer wavelength diffraction is the more pronounced. That is why, when you hear a band playing around a corner, you hear the bass drums and tubas better than the piccolos and cornets even if they have equal energy output.

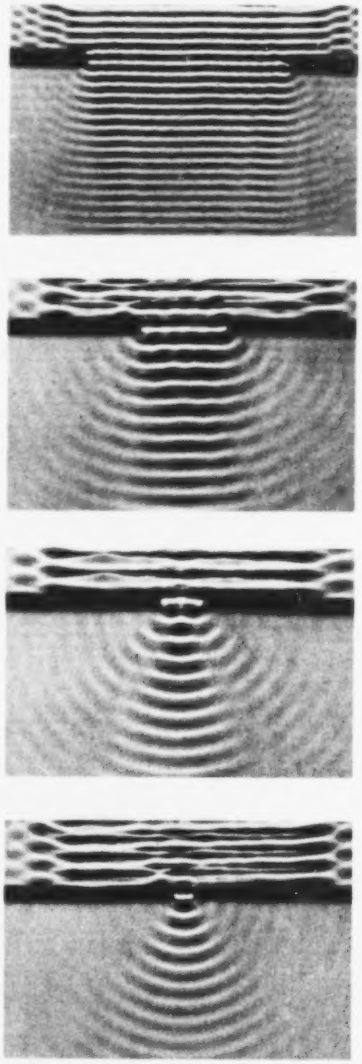


Fig. 12.23

Sound waves have been understood to behave according to these rules of interference and diffraction for a long time, but light, as we shall see in Chapter 13, was only proven to exhibit interference and diffraction after 1800. The reason

Perception of the phenomena rather than verbalization is of the essence here. Testing should reflect this by allowing students to sketch what will happen rather than tell about it.

Summary 12.9

1. Waves usually reflect from any boundary to their media.

2. The angle of reflection equals the angle of incidence.

3. A transverse wave is flipped upside down when reflected from a fixed end.

4. The behavior of a wave can be conveniently described in terms of rays.

5. Waves can be focused by curved reflecting surfaces.

• This section can be treated briskly; it is essential that students be familiar with the phenomena before they come to the text.

12.9

for this delayed discovery is primarily the fact that diffraction effects are much less for very short wavelengths. Measurements of light had to be extraordinarily accurate to show that light can bend around corners and can be made to interfere. These properties of light seem contrary to everyday experience, in contrast to the next two properties of wave motion we shall consider, namely reflection and refraction.

Q21 What characteristic do all points on a wave front have in common?

Q22 State Huygens principle.

Q23 Why can't there be nodal lines in a diffraction pattern from an opening less than one-half wavelength wide?

Q24 What happens to the diffraction pattern from an opening as the wavelength increases?

Q25 Diffraction of light went unnoticed for centuries because
- a) light travels so fast.
- b) light has such a short wavelength.
- c) light was seldom sent through holes.
- d) light waves have a very small amplitude.

D46: Reflection

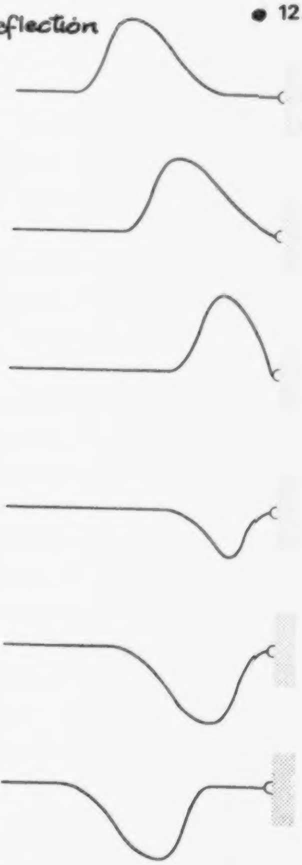


Fig. 12.24

• 12.9 Reflection. In addition to passing through one another and spreading around obstacles in their paths, waves also bounce away wherever they reach any boundary to their media. Echoes are familiar examples of the reflection of sound waves. The property of reflection is shared by all waves, and again the superposition principle, as well as Huygens' principle, will help us understand what is happening when reflection occurs.

We send a wave down the rope toward an end which is tied to a massive solid object, such as a wall in a building. The force that the wave exerts at the rope's end cannot do any work when the end does not move. If the rope is tied tightly to a hook securely fastened to a massive wall, the energy carried in the wave will not be absorbed at the rope's end. Then the wave will bounce back or be reflected, and ideally it will continue to carry the same energy.

What will the wave look like after it is reflected? The striking result is that the wave is flipped upside down on reflection. As the wave comes in from left to right and encounters the fixed hook, the hook must exert a force on the rope while the reflection is taking place. The description of the detailed way in which that force varies in time is complicated. The net effect is that an inverted wave of the same form is sent back down the rope.

So far we have been dealing with reflections of one-dimensional waves. If we now turn our attention to the two-dimensional water surface waves, we can have variously shaped crest lines, variously shaped barriers from which to get re-

The hook and building will not be completely unmoved, but the energy transferred will be negligible.

Even if the students are using ripple tanks, they should be encouraged to use sinks and tubs at home – perhaps to show the family. Wave physics isn't restricted to specialized devices in the lab. 12.9

flections, and various directions in which the waves can approach the barrier.

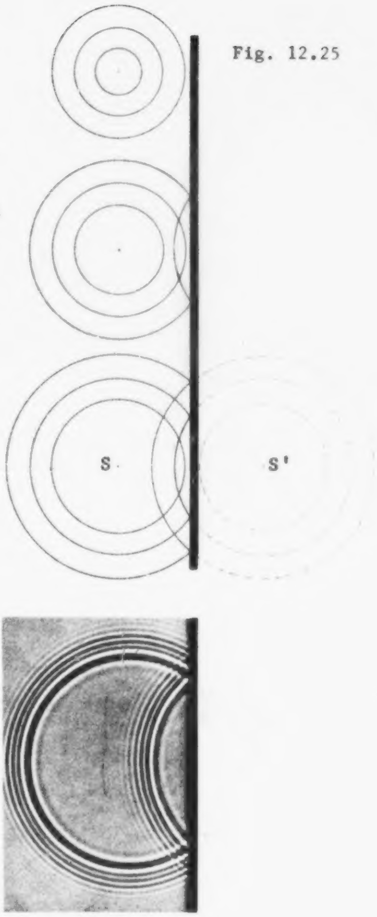


Fig. 12.25

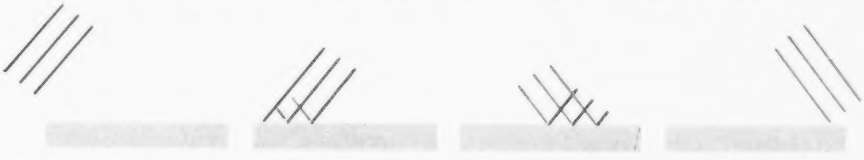
If you have never watched closely as water waves are reflected from a fixed barrier, you should do so before another day passes. Any still pool or water-filled wash basin or tub will do to watch the circular waves speed outward, reflect from rocks or walls, run through each other, and finally die out.

Dip your finger tip into and out of the water quickly, or let a drop of water fall from your finger into the water. Now watch the circular wave approach and then bounce off a straight wall or a board. The long side of a tub is fine as a straight barrier.

Figure 12.25 pictures what you will see, where S is the point of the waves' source. Three crests are shown in the sketches. You may see more or fewer than three good ones, but that does not matter. In the upper sketch, the outer crest is approaching the barrier at the right. The next two sketches show the positions of the crests after first one and then two of them have been reflected. The dashed curves in the last sketch are an attempt to show that the reflected wave appears to originate from a nonexistent source S′ behind the barrier, S′ being as far behind the barrier as S is in front of it. The imaginary source at the point S′ is called the image of the source S.

We looked at the reflection of circular waves first, because that is what one is likely to notice first when studying water waves. But it is easier to see a general principle in operation when we look at a straight wave front reflecting from a straight barrier.

If we push a half-submerged ruler or any straightedge quickly back and forth parallel to the water surface, we will generate a wave in which the crests lie along straight lines over a large fraction of the ruler's length. Figure 12.26 (a) shows what happens when this wave reflects obliquely from a straight barrier. The first sketch shows three crests approaching the barrier; the last shows the same crests as they move away from the barrier after the encounter. The two sketches between show the reflection process at two different intermediate instants.



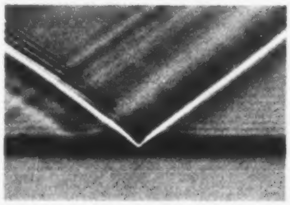


Fig. 12.26 (a)

In connection with light, "ray" is often used loosely as synonymous with "narrow beam". A ray, even with respect to light, is a geometrical construct. Consistent use of "ray" as a geometrical line, as is begun on this page, will avoid some awkwardness or physical misconception later.

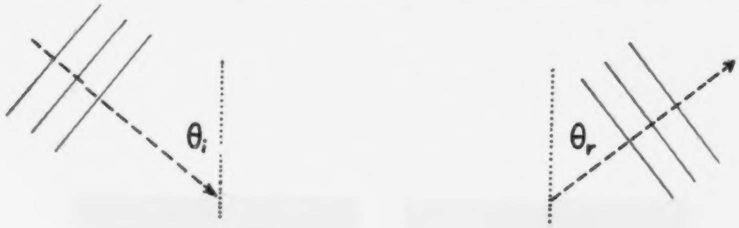


Fig. 12.26(b)

The sketches in Fig. 12.26 (b) include dashed construction lines which were drawn so that they are perpendicular to the wave's crest lines. Imaginary lines of this kind are called rays, and they are often helpful when describing wave behavior. The important feature of a ray is that the wave's velocity at any point lies along the ray at that point.

The behavior of the wave upon reflection, as pictured in Fig. 12.26, is described easily in terms of rays. As shown in the first sketch, a representative ray for the incident wave makes the angle θ_i with a line drawn perpendicular to the reflecting surface. (The perpendicular is the dotted line in the figure.) A representative ray for the reflected wave makes the angle θ_r with the same perpendicular line. The experimental fact is that these two angles are equal; the angle of incidence θ_i is equal to the angle of reflection θ_r. You can prove it for yourself in the laboratory.

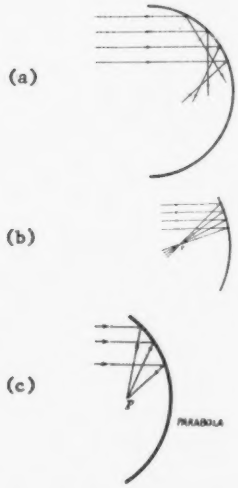
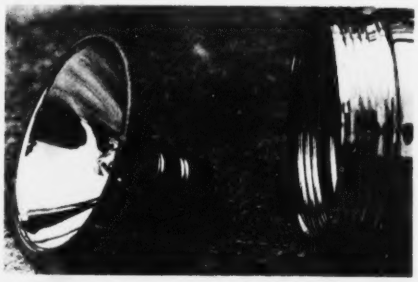


Fig. 12.27 The reflection of parallel wave fronts from circular and parabolic boundaries.

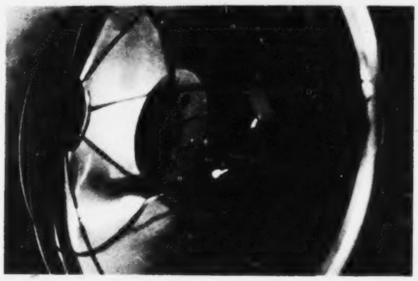
Reflection also occurs from other kinds of surfaces than plane reflectors or mirrors. Many other kinds of wave reflectors are in use today, such as radar antennae and infrared heaters. Figure 12.27 (a) & (b) shows how straight-line waves reflect from two circular reflectors. A few incident and reflected rays are shown. The dotted lines are perpendiculars to the barrier surface; in these cases the perpendiculars are along radius lines of the two circles. While rays reflected from the half circle are headed off in all directions, the rays reflected from the small segment of a circle come close to converging toward a single point. However, a parabolic curve will focus straight-line waves precisely; similarly, a parabolic surface[*] will reflect plane waves to a sharp focus. An impressive example of this is the radio telescope, where huge reflectors are used to detect faint radio waves from space. Conversely, spherical waves originating at the focus will become plane waves on reflection from a parabolic reflector. The automobile headlamp and the flashlight reflector are familiar examples.

[*] More precisely, a "paraboloid of revolution", a surface generated by rotating a parabola on its axis.

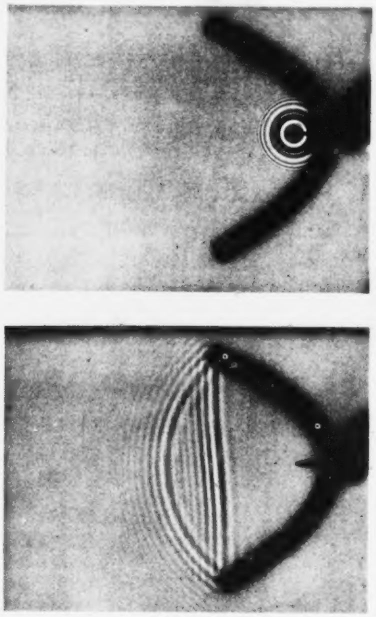
A variety of parabolic reflectors.



Flashlight



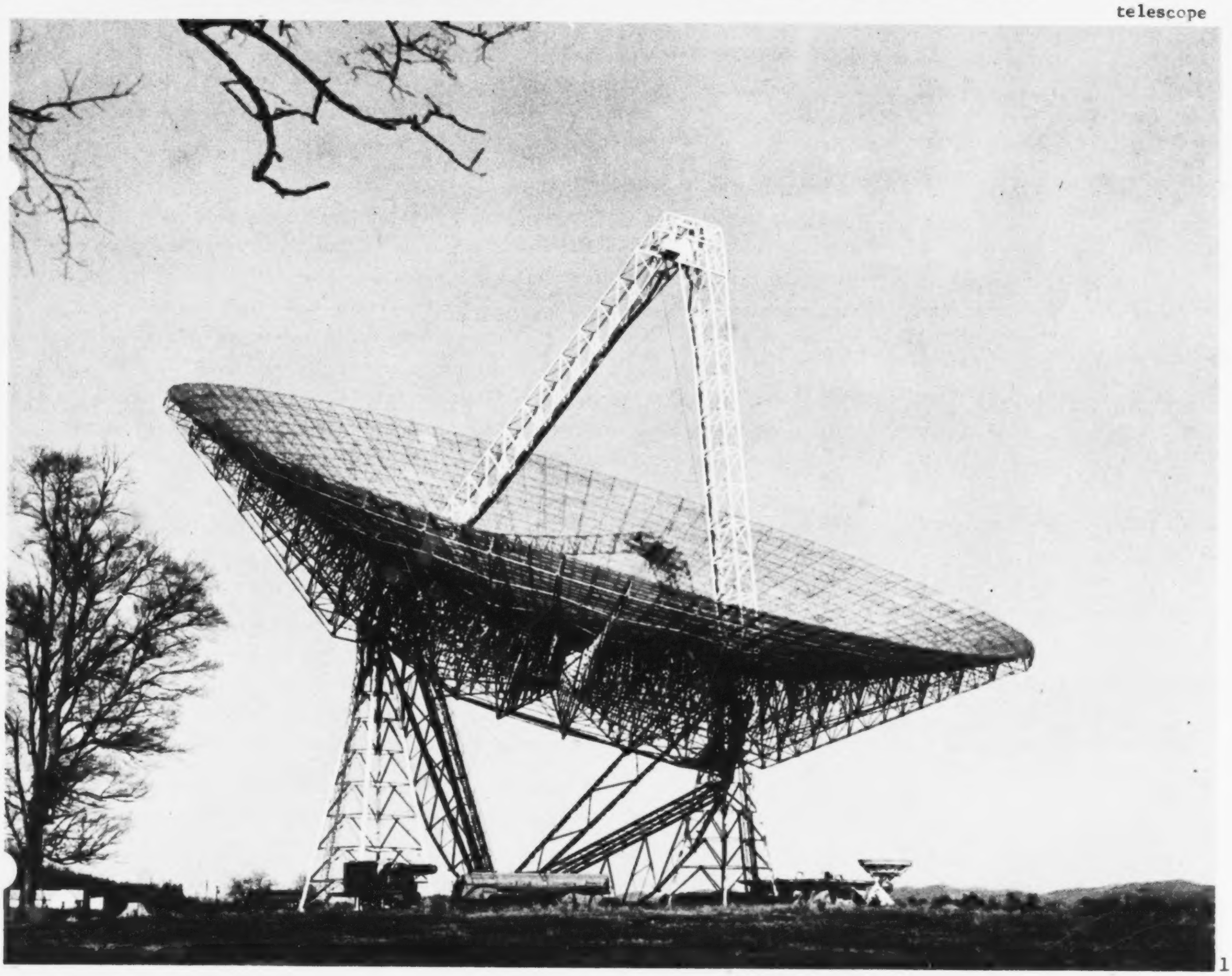
Radiant heater



The pulse is
made with a
pencil held
in the hand

Reflection of a circular pulse in ripple tank.

Radio telescope



Q26 Why is a "ray" an imaginary line?

Q27 What is the relationship between the angle at which a wave front strikes a barrier and the angle at which it leaves?

Q28 What shape of reflector can converge parallel wave fronts to a sharp focus?

Q29 What happens to wave fronts originating at the focus of such a reflecting surface?

D42: Refraction

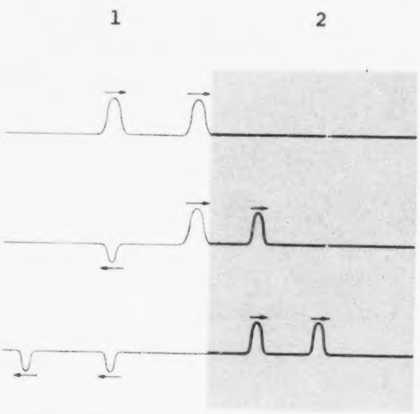


Fig. 12.28. A pair of pulses encountering a medium in which the propagation speed is reduced.

12.10 Refraction. What happens when a wave propagates from one medium to another medium in which its speed of propagation is different? We begin with the simple situation pictured in Fig. 12.28 where two one-dimensional pulses approach a boundary separating two media. We are assuming that the speed of the propagation in medium 1 is greater than it is in medium 2. We might imagine the pulses to be in a light rope (medium 1) tied to a relatively heavy rope or slinky (medium 2). Part of each pulse is reflected at the boundary with the reflected component flipped upside down relative to the incident pulse. (We might have expected this, since the heavier rope is trying to hold the boundary point fixed in a way analogous to the rope reflection at a fixed point discussed earlier.) But we are not particularly interested here in the reflected wave. We want to see what happens to that part of the wave which continues into the second medium.

As shown in Fig. 12.28, the transmitted pulses are closer together in medium 2 than they are in medium 1. Is it clear why that is so? The speed of the pulses is less in the heavier rope, so that the second pulse is catching up during the time it is still in the light rope and the first pulse is already in the heavy rope. In the same way that the two pulses come closer and stay closer, each separate pulse is itself squeezed into a narrower spatial form. That is, while the front of the pulse begins to enter the region of lesser speed, the back of the pulse is still moving with the greater speed, and thus crowding the pulse into a narrower space.

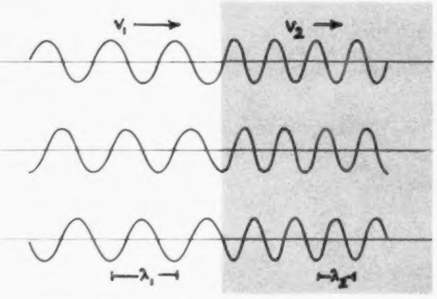


Fig. 12.29. A continuous wave-train entering a medium in which the wave speed is reduced. The effects of the reflected wave train have been neglegted.

Something of the same sort happens to a periodic wave at such a boundary. Figure 12.29 pictures this situation where, for the sake of simplicity, we have assumed that all of the wave is transmitted, and none of it reflected. Just as the two transmitted pulses were brought closer and each pulse was squeezed into a narrower region in Fig. 12.28, so here the spatial pattern is squeezed tighter too. That means that the wavelength λ_2 of the transmitted wave is shorter than the wavelength λ_1 of the incoming, or incident wave.

Although the wavelength changes when the wave passes across the boundary, the frequency of the wave certainly cannot. If the rope is to be unbroken at the boundary, then the pieces immediately adjacent to the boundary and on either side of it must certainly go up and down together. The frequencies of the incident and transmitted waves must, then, be equal. So, since there is no point to labeling them f_1 and f_2, we shall simply label both of them f.

We can write down our wavelength, frequency and speed relationship for both the incident and transmitted waves separately:

$$\lambda_1 f = v_1 \text{ and } \lambda_2 f = v_2.$$

If we divide one of these by the other, cancelling the f's, we get

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2},$$

which tells that the ratio of the wavelengths in the two media equals the ratio of the speeds.

We can make the same sort of boundary for water ripples. Experiments show that the waves move more slowly in shallower water. A boundary can be created by laying a piece of plate glass on the bottom of a ripple tank to make the water shallower there. Figure 12.30 shows the case where the boundary is parallel to the crest lines of the incident wave.

Figure 12.30(a) is a three dimensional picture of what happens at the boundary. Figures 12.30(b) and (c) are cross-section and top views. The photograph Fig. 12.30(d) shows what is actually seen in a ripple tank. The results are the same as those we have already described when a heavy rope is tied to the end of a light one. In fact, the edge view shown in Fig. 12.30(b) could as well be a representation of the rope case.

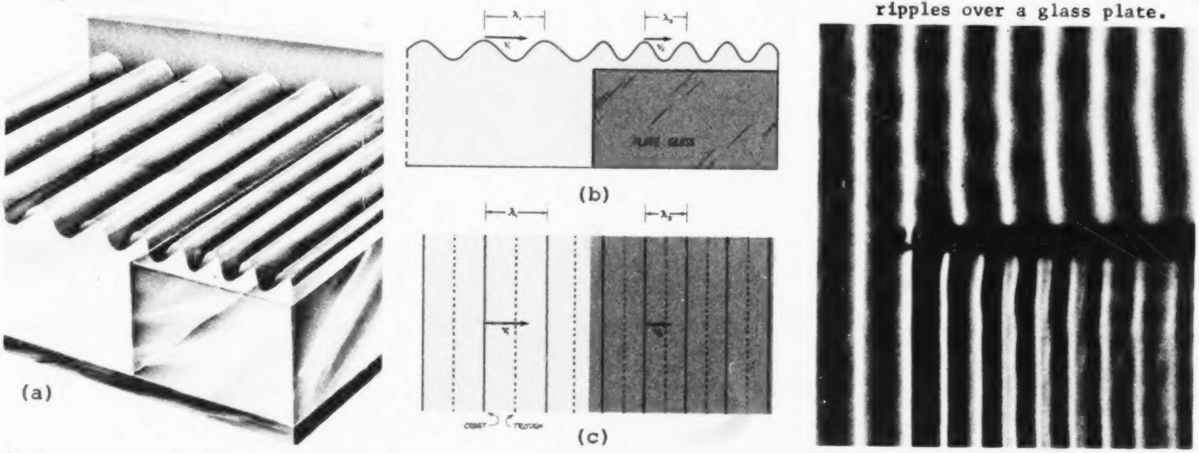
Summary 12.10

1. A wave passing from a medium where its speed is V_1 into a medium where its speed is V_2 undergoes a change in wavelength but not in frequency. This relation is expressed as

$$\frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2}$$

2. When a wave passes into a medium in which the wave speed is reduced, the wave fronts are turned more nearly parallel to the boundary.

Fig. 12.30 Refraction of water ripples over a glass plate.



(a) (b) (c) (d)

D 41 : Wave trains

12.10

Water waves offer a possibility not present for rope waves. We can arrange to have the crest lines approach the boundary at an angle. That is, we can see what happens when the boundary is not parallel to the crest lines. The photograph in Fig. 12.31 shows what happens when a ripple tank wave approaches the boundary at the angle of incidence θ_1. Not only do the wavelength and speed change as the wave passes through the boundary, but the direction of the wave propagation changes too. Figure 12.32 illustrates the way this comes about. As each part of a crest line in medium 1 enters medium 2, its speed lessens, and, thus, the crest lines in medium 2 are turned from the orientation they had in medium 1.

When a wave passes into a medium in which the wave velocity is reduced, the wave fronts are turned so that they are more nearly parallel to the boundary. This is what is pictured in Fig. 12.31, and it accounts for something that you may have noticed if you have been at a beach at an ocean shore. No matter in what direction the waves are moving far from the shore, when they near the beach their crest-lines are practically parallel to the shoreline. A wave's speed is steadily being reduced as it moves into water that gets gradually more shallow. So the wave is being refracted continuously as if it were always crossing a boundary between different media, as indeed it is. The refraction of sea waves is so pronounced that wave crests can curl around a small island with an all-beach shoreline and provide surf on all sides.

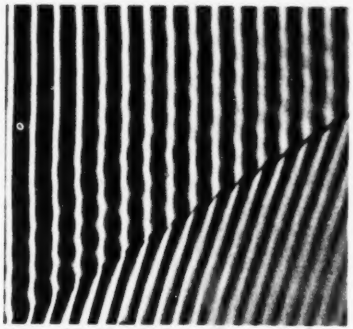


Fig. 12.31 Refraction of water ripples over glass plate.

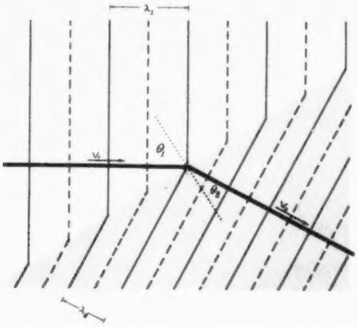
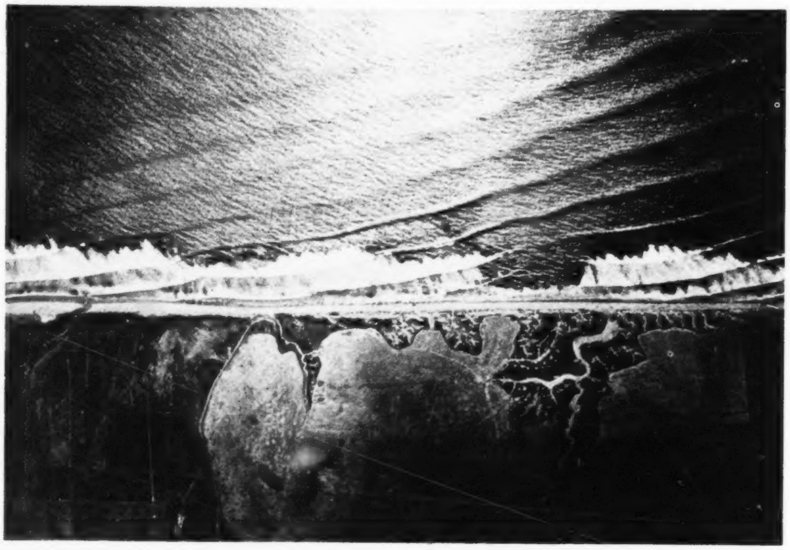


Fig. 12.32. Diagram of rays for refracting wavefronts. Θ_1 and Θ_2 are the angles the rays make with a line (dotted) perpendicular to the boundary.

The ratio Θ_1/Θ_2 is roughly the same as v_2/v_1. The exact relationship is $\sin\Theta_1/\sin\Theta_2 = v_2/v_1$. (See Study Guide 12.3.)



Refraction of ocean waves approaching shore.

Q30 If a periodic wave slows down on entering a new medium, what happens to (1) its frequency? (2) its wavelength? (3) its direction?

Q31 Sketch roughly what happens to a wave train which enters a new medium where its speed is greater.

12.11

12.11 Sound waves. Sound waves are mechanical disturbances that propagate through an elastic medium, such as the air, as longitudinal or compressional displacements of particles in that medium. Whether the substance of the medium is in a solid, liquid, or gaseous state, longitudinal waves of pressure from some vibrating source will move through the cooperating particles of that material medium. If these waves strike the ear, they can produce the sensation of hearing.

Summary 12.11
This section is a collection of information about sound which exemplifies the wave phenomena described in this chapter.

The biology and psychology of hearing is as important to the science of acoustics as is the physics of sound. But for now you may also be interested in the ways in which sound waves exhibit all the properties of wave motion that we have considered thus far in this chapter.

E 31*: Sound

L 40: Standing waves in a gas

F 29: Sound waves in air

Sound waves can be used to demonstrate all but one of the properties of wave motion in general. The frequencies, velocities, wavelengths and amplitudes of periodic sound waves can be measured, and they can be shown to produce reflection, refraction, diffraction, interference and absorption. Only the property of polarization is missing, because sound waves are longitudinal compression waves.

Vibrating sources for sound waves may be as simple as a tuning fork or as complex as the human larynx with its vocal cords. Sound energy may be transferred through materials as diverse as a quartz crystal and the earth's crust, water of the sea or extremely thin helium gas; in short, any material medium may transmit sound vibrations.

Pulses of sound waves may meet our ears as "clicks" or "dits" or "bams." But when we hear steady tones or buzzes or hums that do not fluctuate, you may be sure that a periodic vibration is generating a periodic wave or set of waves. Audible simple harmonic motion is best illustrated by the steady "pure tone" given off by a tuning fork, although today electronic devices can generate far more "pure" single-frequency sounds than can a tuning fork. The "pitch" of a sound we hear depends on the frequency of the wave.

See "Musical Instruments and Scales" and "Founding a Family of Fiddles" in Project Physics Reader 3.

People can hear waves with frequencies between about 20 cycles per second and 20,000 cycles per second. Dogs can hear over a much wider range (15-50,000 cps), and, as men have only recently learned, bats and porpoises hear and feel frequencies up to about 120,000 cps.

A: Determination of velocity of sound in air

12.11

Loudness or volume is a subjective estimate of intensity, and the latter is defined in terms of power, that is, as the number of watts per square centimeter falling on a surface perpendicular to a wavefront. So adaptable is the human ear that an exponential scale is used to measure loudness of sound. Figure 12.38 illustrates the wide range of intensities of familiar sounds, relative to a threshold level of 10^{-16} watts per square centimeter.

Many sense phenomena are conveniently represented by exponential (or "logarithmic") scales. This implies that it is the fractional change which is significant for perception. A mathematically related phenomena of constant fractional change is radioactive decay, which is described by an exponential curve.

See "Silence Please" in Project Physics Reader 3.

Relative Intensity	Examples
1	Threshold of hearing (in anechoic chambers)
10^{1}	Normal breathing
10^{2}	(Rustling) leaves in a breeze
10^{3}	Empty movie house
10^{4}	Residential neighborhood at night
10^{5}	Quiet restaurant
10^{6}	Two-person conversation
10^{7}	Busy traffic
10^{8}	Vacuum cleaner
10^{9}	Water at foot of Niagara Falls
10^{10}	Subway train
10^{11}	
10^{12}	Propeller plane at takeoff
10^{13}	Machine-gun fire, close range
10^{14}	Military jet at takeoff
10^{15}	
10^{16}	Wind tunnel (test facility)
10^{17}	Future space rocket (at lift-off)

Fig. 12.38 Levels of noise intensity above 10^{12} times threshold intensity can be felt as a tickling sensation in the ear; beyond 10^{13} times threshold intensity, the sensation changes to pain and may damage the unprotected ear.

That sound waves take time to travel from source to receiver has always been fairly obvious. Sights and sounds are often closely associated in the same event (lightning and thunder, for instance), and sound is usually late compared to the sight. In 1640 the French mathematician Marin Mersenne first computed the speed of sound in air by timing echoes over a known distance.

It took another seventy years of refinements before William Derham in England, comparing cannon flashes and noises across 12 miles, came close to the modern measurements: sound in air at 68° F moves at 1,125 feet per second.

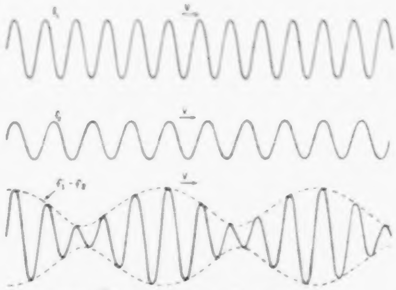
As for all waves, the speed of sound waves depends on the properties of the medium—the temperature, the density and the elasticity. Sounds travel faster in liquids than in gases, and faster still in solids, because in each state the elasticity of the medium is greater. In sea water, its speed is about 4,800 ft/sec; in steel, it is about 16,000 ft/sec; its fastest known speeds occur in quartz, about 18,000 ft/sec.

•But the density is also much greater in liquids and solids than in gases; it isn't obvious which factor will dominate.

Beats are not very important in this course, but they appear occasionally in the students' world, are entertaining, and are a good example of the superposition principle.

12.11

Interference of sound waves is evident in the acoustically "dead" spots found in many large halls, such as railroad terminals, ballrooms, stadiums and auditoriums. Another interesting example of sound interference is the phenomenon known as beats.● When two notes of slightly different frequency are heard simultaneously, they interfere to produce beats, or an intermittent hum. Piano tuners are able to make very fine adjustments in pitch by listening to beats.



An example of "beats." Two wave trains of different frequencies f_1 and f_2 interfere to produce a wave which fluctuates in amplitude at a frequency ($f_1 - f_2$).

Diffraction is perhaps the most distinctive property of sound waves. Sound waves readily turn around all corners and bend around all barriers to bathe the listener anywhere within range and within the same media. This behavior of sound waves is consistent with Huygens' principle. Any point on a sound wave front meeting an obstacle may act as a new source for a new series of waves radiating in all directions from that point. The ability of sound to diffract through narrow openings or diffract around several sharp corners is surprisingly potent. Sound reflects, like rope or water waves, wherever it encounters a different medium or a boundary to its confinement. Echo chamber effects, often artificially produced by electronics, have become familiar to listeners who enjoy popular music. The architectural accidents called "whispering galleries" (there is one under the dome of the U. S. Capitol) show vividly how sound waves can be reflected to a focus. The weirdly "live" sound of a bare room results from the multiple reflections of waves which are usually absorbed by furniture, rugs and drapes. Scientists have recently devised rooms which maximize echoe called reverberation chambers, and others which minimize them, called anechoic chambers. Both are special laboratories of great value to the study of acoustics.

An anechoic chamber being used for research in acoustics.

Refraction of sound accounts for the fact that we can sometimes see lightning without hearing thunder. It also accounts for most of the shortcomings of sonar devices (sound navigation and ranging) used at sea. Sonic refraction is used for a variety of purposes today, among them the study of the earth's deep structure and seismic prospecting for fossil fuels and minerals.

Adjustable ceiling surface at the Lincoln Center for the Performing Arts are used to control the acoustical properties of the hall.

In 1877 the third Lord Rayleigh wrote The Theory of Sound, which is often considered the culmination of Newtonian mechanics applied to energy transfer. Only a decade earlier the great German physicist-physiologist Hermann von Helmholtz had written The Sensations of Tone, a detailed study of music and hearing. Together these two scientists established the science of sound on a firm Newtonian basis. Acoustics was a marvelous integration of Newtonian science.

●Beats in the superposition of two waves of electric potential can be shown beautifully and simply by connecting two audio oscillators to the same oscilloscope input, or connecting one oscillator tuned close to 60 cycles/sec. to the oscilloscope with a long lead and a capacitor (about 0.001 uf) to pick up 60-cycle hum.

12.11

All this happened before electromagnetic and atomic physics had progressed very far. With the advent of radio waves, of the vacuum tube and electronics, of ultrasonic and infrared research, and of quantum and relativistic physics, the science of acoustics gradually came to seem a very restricted field of study—which might be called "small amplitude fluid mechanics." The wave viewpoint, however, continued to grow in importance.

Q32 List five wave behaviors that can be demonstrated with sound waves.

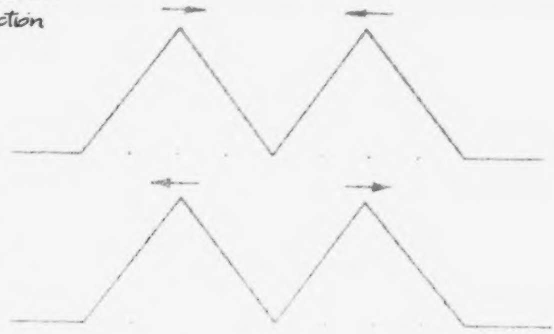
Q33 Why can't sound waves be polarized?

Two charts that show the range of frequencies of sound sources and sound receivers can be found in Scientific American July 1948; Life Science Library's Sound and Hearing, Time, Inc, NYC 1965, pp. 194, 195.

Study Guide

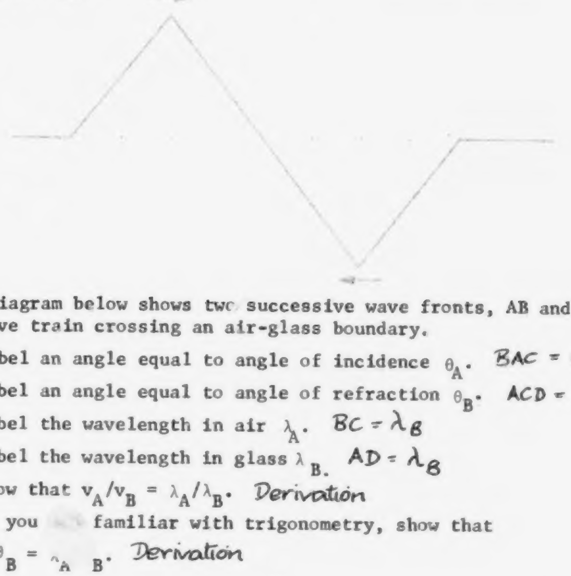
12.1 Pictured are two idealized rope waves at the instants before and after they overlap. Divide the elapsed time into four equal intervals and plot the shape of the rope at the end of each interval.

Construction



12.2 Repeat Exercise 12.1 for the two waves pictured below.

Construction



12.3 The diagram below shows two successive wave fronts, AB and CD, of a wave train crossing an air-glass boundary.

a) Label an angle equal to angle of incidence θ_A. $BAC = \theta_A$

b) Label an angle equal to angle of refraction θ_B. $ACD = \theta_B$

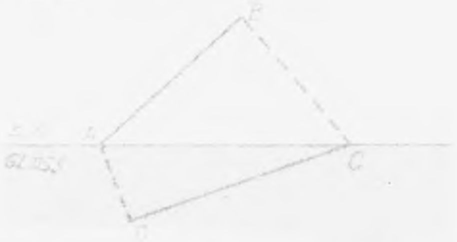
c) Label the wavelength in air λ_A. $BC = \lambda_B$

d) Label the wavelength in glass λ_B. $AD = \lambda_B$

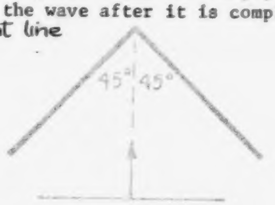
e) Show that $v_A/v_B = \lambda_A/\lambda_B$. *Derivation*

f) If you [illegible] familiar with trigonometry, show that $\sin\theta_A/\sin\theta_B = \lambda_A/\lambda_B$. *Derivation*

The trigonometric relation was first stated by Willebrord Snell (1591 – 1626) of the University of Holland in an unpublished paper.

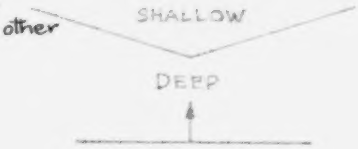


12.4 A straight-line wave approaches a right-angle reflecting barrier as shown in the figure. Find the shape, size and direction of propagation of the wave after it is completely reflected by the barrier. *Straight line*



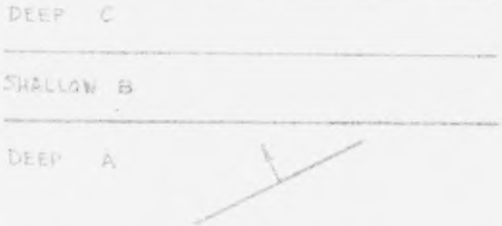
12.5 A straight-line wave in a ripple tank approaches a boundary between deep and shallow water as shown. Describe the shape of the wave as it passes through the boundary and then as it continues in the shallow water.

Two straight line waves inclined toward each other

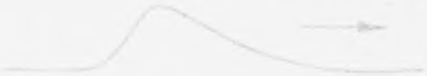


12.6 A periodic ripple-tank wave passes through a straight boundary between deep and shallow water. The angle of refraction is 30°. The propagation speed in the deep water is 0.35 m/sec, and the frequency of the wave is 10 cycles per sec. Find the wavelengths in the deep and shallow water. $\lambda_D = .035\, m$ $\lambda_S = .025\, m$

12.7 A straight-line ripple-tank wave approaches a narrow region B of shallow water as shown. Prove that the crest line of the wave when in region C is parallel to the crest line shown in region A when regions A and C have the same water depth. Derivation



12.8 The wave below propagates to the right along a rope. What is the shape of the wave propagating to the left that could for an instant cancel this one completely? Reflection



12.9 The velocity of a portion of rope at some instant is the superposition of the velocities of waves passing through that portion. Is the kinetic energy of a portion of the rope the superposition of the kinetic energies of waves passing through that region? No

12.10 Trace the last three curves of Fig. 12.12 and add them graphically to obtain the original curve. Construction

12.11 What kind of interference pattern would you expect to see if the separation between two in-phase sources were less than the wavelength λ? Where would the nodal and and antinodal lines be if the two in-phase sources were separated by the distance λ? By λ/2? Convince yourself that one additional nodal line appears on each side of the central antinodal line whenever the separation between the two in-phase sources is increased by one wavelength. Construction

12.12 Estimate the wavelength of a 1000 cycles per second sound wave in air; in water; in steel (refer to data in text). Do the same if f = 10,000 cps. Design the dimensions of an experiment to show two-source interference for 1000 cps sound waves. Discussion

12.13 If you were to begin to disturb a stretched rubber hose or slinky with a frequency that precisely matched a standing wave frequency, would standing waves appear immediately? If not, what factors would determine the time delay? No, discussion

12.14 A megaphone directs sound along the megaphone axis if the wavelength of the sound is small compared to the diameter of the opening. Estimate the upper limit of frequences which are diffracted at a cheerleader's megaphone opening. Can you hear what a cheerleader shouts even though you are far off the axis of his or her megaphone? 100 and 1000 cps Yes

12.15 Suppose that straight-line ripple waves approach a thin straight barrier which is a few wavelengths long and which is oriented with its length parallel to the wavefronts. What do you predict about the nature of the diffraction pattern along a straight line behind the barrier which is perpendicular to the barrier and passes through the center of the barrier? Why do people who design breakwaters need to concern themselves with diffraction effects? Maximum

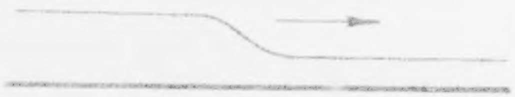
12.16 By actual construction with ruler and compass show that the sketches in Fig. 12.25 appear to be rays originating at S'. Construction

12.17 Sketch the "image" wave for the wave shown in each of the sketches in Fig. 12.26 (a). What relationship exists between the incident image wave and the real reflected wave? Construction

12.18 With ruler and compass reproduced Fig. 12.27 (b) for yourself and find the distance from the circle's center to the point P in terms of the radius of the circle R. Make the radius of your circle much larger than the one in the figure. $R/2$

12.19 Convince yourself that a parabolic reflector will actually bring parallel wavefronts to a sharp focus. Draw a parabola and some parallel rays along the axis as in Fig. 12.27 (c). Construct line segments perpendicular to the parabola where the rays hit it, and draw the reflected rays at equal angles on the other side of these lines. Construction

12.20 Recalling that water surface waves travel slower in shallow water, what would you expect to happen to the shape of the following wave as it continues to the right? Pay particular attention to the region of varying depth. Discussion



Can you use the line of reasoning used above to give at least a partial explanation of the cause of breakers near a beach?

12.21 Look at Fig. 12.32. Convince yourself that if a wave were to approach the boundary between medium 1 and medium 2 from below, along the same direction as the refracted ray in the figure, it would be refracted along the direction of the incident ray in the figure. This is another example of a general rule: if a wave follows a set of rays in one direction, then a wave can follow the same set of rays in the opposite direction. In other words, wave paths are reversible. Discussion

12.22 Can you explain how sound waves are being used to map the floors of oceans? $2d = vt$

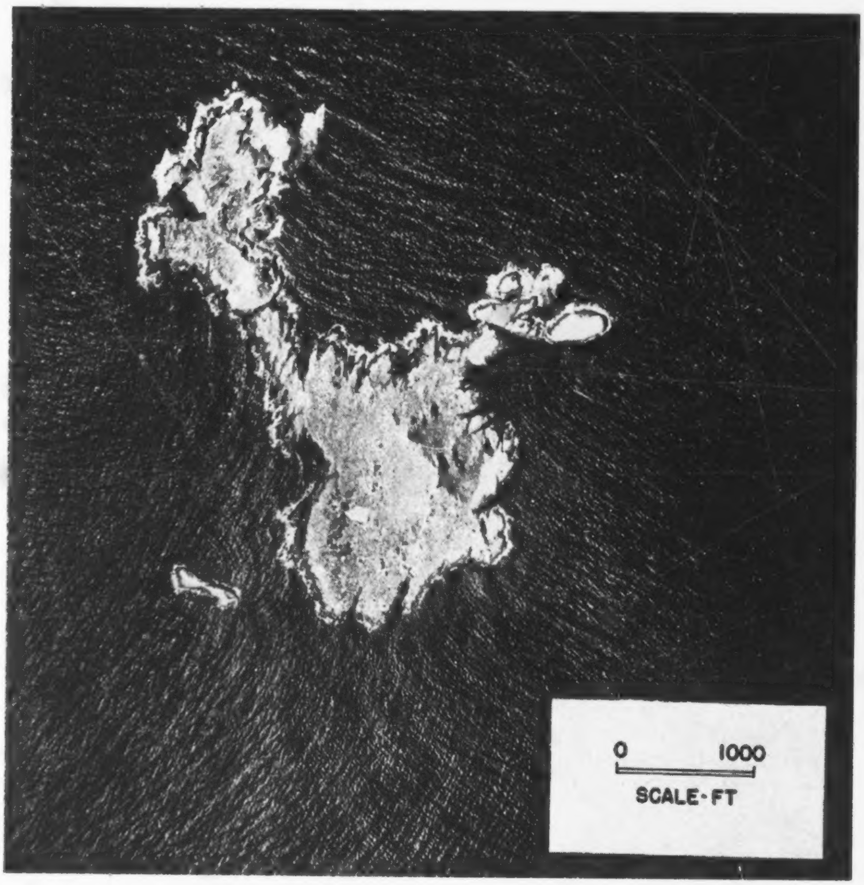
12.23 When a sound source passes us, whether it be a car horn, a train whistle, or a racing car motor, the pitch we hear goes from high to low. Why is that? Doppler effect

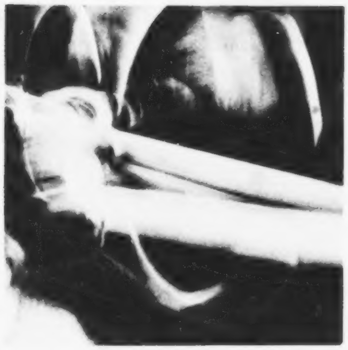
12.24 Directed reflections of waves from an object occur only when the wavelength is small compared to the dimensions of the object. This is true for sound waves as well as for any other. What does this tell you about the sound frequencies a bat must generate if it is to catch a moth or fly? Actually some bats can detect the presence of a wire about 0.12 mm in diameter. What frequency does that require? 3×10^4 cps 2.5×10^6 cps

12.25 Suppose you can barely hear in an extremely quiet room a buzzing mosquito at a distance of one meter. What is the sound power output of the mosquito? How many mosquitoes would it take to supply the power for one 100-watt reading lamp? If the swarm were at ten meters' distance, what would the sound be like?

1.27×10^{-11} watts
8×10^{12} mosquitoes
wind-tunnel test

Refraction, reflection, and diffraction of waves around Farallon Island, California. There are breakers all around the coast. The swell coming from top right rounds both sides of the island, producing a crossed pattern below. The small islet 'radiates' the waves away in all directions. (U.S.Navy photograph.)





Epilogue Seventeenth-century scientists thought they would eventually be able to explain all physical phenomena by reducing them to matter and motion. This mechanistic viewpoint later became known as the Newtonian cosmology, since its most impressive success was Newton's theory of planetary motion. Newton and his contemporaries proposed to apply similar methods to other problems (see the Prologue to this unit).

The early enthusiasm for this new approach to science is vividly expressed by Henry Power in his book Experimental Philosophy (1664). Addressing his fellow natural philosophers (or scientists, as we would now call them), he wrote:

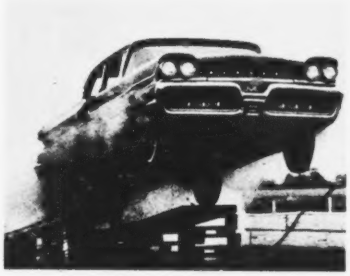
> You are the enlarged and elastical Souls of the world, who, removing all former rubbish, and prejudicial resistances, do make way for the Springy Intellect to flye out into its desired Expansion...
>
> ...This is the Age wherein (me-thinks) Philosophy comes in with a Spring-tide...I see how all the old Rubbish must be thrown away, and carried away with so powerful an Inundation. These are the days that must lay a new Foundation of a more magnificient Philosophy, never to be overthrown: that will Empirically and Sensibly canvass the Phaenomena of Nature, deducing the causes of things from such Originals in Nature, as we observe are producible by Art, and the infallible demonstration of Mechanicks; and certainly, this is the way, and no other, to build a true and permanent Philosophy.

In Power's day there were still many people who did not regard the old Aristotelian cosmology as rubbish to be thrown away. For them, it provided a reassuring sense of unity and interrelationship among natural phenomena which was liable to be lost if everything was reduced simply to atoms moving randomly through space. The poet John Donne, in 1611, lamented the change in cosmology which was already taking place:

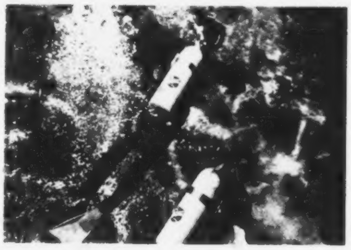
And new Philosophy calls all in doubt,
The Element of fire is quite put out;
The Sun is lost, and th'earth, and no man's wit
Can well direct him where to looke for it.
And freely men confesse that this world's spent,
When in the Planets, and the Firmament
They seeke so many new; then see that this
Is crumbled out againe to his Atomies.
'Tis all in peeces, all coherence gone;
All just supply, and all Relation...

Newtonian physics provided powerful methods for analyzing the world, and for uncovering the basic principles that govern the motions of individual pieces of matter. But could it also deal successfully with the richness and complexity of processes that take place in the real world, as well as idealized frictionless processes in a hypothetical vacuum? Can colors, sounds and smells really be reduced to nothing but matter and motion? In the seventeenth century, and even in the eighteenth century, it was too soon to expect Newtonian physics to answer these questions; there was still too much work to be done in establishing the basic principles of mechanics and applying them to astronomical problems. A full-scale attack on the properties of matter and energy had to wait until the nineteenth century.

We have seen in this unit some of the successful generalizations and applications of Newtonian mechanics which were accomplished by the end of the nineteenth century: the conservation laws, new explanations of the properties of heat and gases, and estimates of some properties of molecules. We have introduced the concept of energy to link mechanics to heat, to sound and (in Unit 4) to light, electricity and magnetism. We have also noted that the application of mechanics on a molecular level requires statistical ideas and leads to interesting questions about the direction of time.



Throughout most of the unit we have emphasized the application of mechanics to separate pieces or molecules of matter. But using a molecular model was not the only way to understand the behavior of matter. Without departing from the basic viewpoint of the Newtonian cosmology, scientists could also interpret many phenomena (such as sound and light) in terms of wave motions in continuous matter. By the middle of the nineteenth century it was generally believed that all physical phenomena could be explained by either a particle theory or a wave theory. In the next unit, we will discover how much validity there was in this belief, and we will begin to see the emergence of a new viewpoint in physics, based on the field concept. Then, in Unit 5, particles, waves and fields will come together in the context of twentieth century physics.



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Brief Answers to Unit 3 Study Guide

Chapter 9

9.1 Discussion
9.2 Yes
9.3 No
9.4 Discussion
9.5 (a) 220.2g
(b) 20.2g
9.6 (a) Discussion
(b) Yes
(c) Yes
9.7 0.8 m/sec
9.8 Derivation
9.9 3.33×10^{-6} kg
9.10 Discussion
9.11 Discussion
9.12 Discussion
9.13 1200 kg-m/sec
400 nts
30 m
9.14 (a) 200 kg-m/sec
(b) 10 nts
9.15 Yes
9.16 Derivation
9.17 (a) 0
(b) 0
$M_c\vec{V}_c + M_s\vec{V}_s$
(c) $M_c\vec{V}_c = -M_s\vec{V}_s$
(d) $V_s/V_c = M_c/M_s$
(e) 10 m/sec
9.18 Derivation
9.19 Left car
(1) speed of one car or
(2) mass of cars
Distance
Force
9.20 Discussion
9.21 Derivation
9.22 One-half initial speed in opposite directions
9.23 (a) No
(b) Continues forward
(c) Continues backward
9.24 .8 kg-m/sec forward
.8 kg-m/sec away
1.6 kg-m/sec away
No
9.25 4.2 kg-m/sec, 1260 kg-m^2/sec^2
0.171 kg, 427.5 kg-m^2/sec^2
0.075 kg-m/sec, 0.1125 kg-m^2/sec^2
111.0 m/sec, 1.62×10^4 kg-m/sec
10^{-4} kg, 0.2 m/sec
9.26 Discussion
9.27 No, discussion

Chapter 10

10.1 1.82×10^{-18} j
5.5×10^{17} electrons
10.2 (a) 67.5 j
(b) 4.5×10^9 j
(c) 3750 j
(d) 2.7×10^{33} j
10.3 Discussion
10.4 (a) .00225 j
(b) .056 j
10.5 Discussion
10.6 (a) 5m/sec^2
19 sec
95 m/sec
(b) 95 m/sec
10.7 Derivation
10.8 22°
10.9 9.3 min
10.10 Derivation
10.11 (a) 9.6×10^9 j
(b) 4.8×10^5 nt
(c) 7.68×10^6 watts
(d) 880 m
(e) 1 & 2 increase
3 decreases
(f) change direction
10.12 (a) 1046 times
(b) increase weight
decrease amount of water
increase height of fall
lower specific heat
10.13 0.12°C
10.14 Discussion
10.15 3 weeks
10.16 1.5 kg
10.17 Discussion
10.18 Discussion
10.19 Discussion

10.20 Undamaged rocket
10.21 1800 nts
90 j
10.22 Discussion
10.23 Discussion

Chapter 11

11.1 Pressure, density,
temp., viscosity
11.2 Discussion
11.3 Discussion
11.4 Working model,
Theoretical model
11.5 Discussion
11.6 No, discussion
11.7 (a) probable speed
(b) average speed
(c) no negative speeds
11.8 $P \propto T$
11.9 (a) 10^{-9}m
(b) 10^{-9}m
11.10 Discussion
11.11 Discussion
11.12 Density changes
11.13 50 atmospheres
11.14 5 atmospheres
11.15 Discussion
11.16 Discussion
11.17 Discussion
11.18 Discussion
11.19 Discussion
11.20 Discussion
11.21 Discussion

Chapter 12

12.1 Construction
12.2 Construction
12.3 (a) BAC = θ_A
(b) ACD = θ_B
(c) BC = λ_A
(d) AD = λ_B
(e) Derivation
(f) Derivation
12.4 Straight-line
12.5 Two straight-line waves inclined
toward each other
12.6 $\lambda_D = .035$ m
$\lambda_s = .025$ m
12.7 Derivation
12.8 Reflection
12.9 No
12.10 Construction
12.11 Construction
12.12 Discussion
12.13 No, discussion
12.14 100 and 1000 cps
yes
12.15 Maximum
12.16 Construction
12.17 Construction
12.18 R/2
12.19 Construction
12.20 Discussion
12.21 Discussion
12.22 2d = vt
12.23 Doppler Effect
12.24 3×10^4 cps
2.5×10^6 cps
12.25 1.27×10^{-11} watts
8×10^{12} mosquitoes
wind-tunnel test

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Prologue

Pp. 0, 3 Albert B. Gregory, Jr.

P. 1 Swiss Federation of Watch Manufacturers.

P. 2 Whitworth Art Gallery, University of Manchester.

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P. 8 Pictorial Parade, N.Y.C.

P. 9 Dr. Harold E. Edgerton, Massachusetts Institute of Technology

P. 10 (thunderhead) Peter M. Saunders, Woods Hole Oceanographic Institute; (bonfire) Colgate University.

P. 12 (Lavoisier portrait) painted by Jacques Louis David. Courtesy of The Rockefeller University.

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P. 43 (guitar string) Albert B. Gregory, Jr.; (golf ball) Dr. Harold E. Edgerton, M.I.T.

P. 44 Dimitri Kessel, LIFE MAGAZINE, © Time Inc.

P. 46 (camel & waterwheel) C.W. Kirkland, LIFE MAGAZINE, © Time Inc, (boy on waterwheel) LIFE MAGAZINE, © Time Inc; (reversible overshot waterwheel) Agricola, Georgius, *De Re Metallica*, 1556, Houghton Library, Harvard University, (windmill) H.G. Walker, LIFE MAGAZINE, © Time Inc.

P. 47 (sprinkler) Dr. Harold E. Edgerton. M.I.T.; (aeolipile) Science Museum, London.

P. 50 Newcomen Society, London.

P. 51 (Watt in his laboratory) Figuier, Louis, *Les Merveilles de la Science*, Furne, Jouvet et Cie, Paris; (portrait) engraved by S.W. Reynolds after the oil portrait by C.F. von Breda, 1793, Science Museum, London.

P. 54 ("Rocket") Figuier, Louis, *Les Merveilles de la Science*, Furne, Jouvet et Cie, Paris; Science Museum, London.

P. 55 (train) Boston & Maine Corporation; (steamboat) Science Museum, London.

Pp. 56, 57 (blacksmith) W. Fitzenthaler, Photography Collection, Carpenter Center for the Visual Arts, Harvard University; (steel workers) Jones & Laughlin Steel Corp., Pittsburgh; (steam turbine rotor) Franco Tosi, S.p.A. Legnano.

P. 58 American Institute of Physics.

P. 60 Professor Keith R. Porter, Dept. of Biology, Harvard University.

P. 63 "The Repast of the Lion" by Henri Rousseau, courtesy of The Metropolitan Museum of Art, bequest of Samuel A. Lewisohn, 1951.

P. 66 American Institute of Physics.

P. 67 (Goethe) painting by Angelica Kauffmann, 1787; (Schelling) pastel by Friedrich Tieck, 1801, collection of Dr. Hermann von Schelling.

P. 68 Koenigsberger, Leo, *Hermann von Helmholtz*, Braunschweig Druck und Verlag, von Friedrich Vieweg und Sohn.

P. 71 Ron MacNeil, Boston.

P. 73 from "The Last Great Cruise" advertisement by Kenyon & Eckhardt, Inc., for the Diners Club Inc. and Fugazy Travel Bureau, Inc., General Sales Agents for the City of Long Beach, California.

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P. 76 Courtesy AMF-Voit.

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P. 81 Burndy Library, Norwalk, Conn.

P. 82 Courtesy of Professor Eric Mendoza.

P. 104 (1) Courtesy of Trinity College Library, Cambridge; (2) Burndy Library; (3) Thompson, Silvanus P., *Life of William Thomson*, vol. 1, MacMillan and Co., Ltd., London, (4) American Institute of Physics, (5) Deutsches Museum, Munich.

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P. 107 P.E. Genereux, E. Lynn.

P. 109 Greek National Tourist Office, N.Y.C.

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P. 112 Magnum Photos Inc., N.Y.C. Werner Bischof.

P. 117 Union Pacific Railroad.

P. 122 (portrait) Oeuvres de Fourier, vol. 2, Gauthier et Fils Paris, 1890.

Figs. 12.13, 12.14, 12.30(d), 12.31, and pp. 124, 131, 133, 135, (ripple tank photos) p. 137. Courtesy, Film Studio Educational Development Center.

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Answers to End of Section Questions

Chapter 9

Q1 No. Don't confuse mass with volume.

Q2 a) No; its mass increases by 2000 tons a year.
b) The solar system.
c) The earth is very nearly a closed system; 2,000 tons is a very small fraction of the earth's mass.

Q3 Answer (c).

Q4 All three quantities are not in general conserved; it is the product of mass and velocity which is conserved.

Q5 In cases (a), (c) and (d) the two carts will stop after the collision, since their momenta before collision are equal in magnitude and opposite in direction.

Q6 Least momentum: a pitched baseball (very small mass and fairly small speed).
Greatest momentum: a jet plane in flight (very large mass and high speed).

Q7 a) about 4 cm/sec. Faster ball delivers more momentum to girl.
b) about 4 cm/sec. More massive ball delivers more momentum to girl.
c) about 1 cm/sec. With some gain in momentum, more massive girl gains less speed.
d) about 4 cm/sec. Momentum change of ball (a vector!) is greater if its direction reverses.
(These answers assume the mass of the ball is much less than the mass of the girl.)

Q8 a) $\Delta(mv) = F\,(\Delta t) = (35 \times 10^6 \text{ newtons}) \times (150 \text{ sec.}) = 5.2 \times 10^9 \text{ kg m/sec.}$
b) Mass of rocket changes as it burns fuel.

Q9 In cases (a), (b) and (d), Δt is lengthened, thereby decreasing F. In case (c), Δt is short, making F large.

Q10 No, not if the girl and skateboard are an isolated system. (In fact, a skillful skateboarder can use frictional forces to make sharp turns.)

Q11 Answer (c).

Q12 In cases (a) and (b) the earth exerts a net force on the system. In case (c) the sun exerts a net force on the system.

Q13 Answer (c); vis viva is conserved only if the colliding objects are very hard.

Q14 False. Huygens explained the demonstration.

Q15 With parallel strings, the balls will collide horizontally.

Q16 Answer (c); the square of any number is positive.

Q17 Answer (c).

Q18 He said that the vis viva was "dissipated among the small parts" of the colliding bodies.

Chapter 10

Q1 Answer (a).

Q2 Answer (e).

Q3 about .2 meters.

Q4 Answer (c).

Q5 Answer (c). An increase in potential energy equals the work you do on the spring.

Q6 Answer (e). You must do work on the objects to push them closer together.

Q7 Answer (e). Kinetic energy increases and gravitational potential energy decreases. Their sum stays the same (if air resistance is negligible).

Q8 Kinetic energy is greatest at midpoint, where string is unstretched. Potential energy is greatest at extreme position, where the speed of the string is zero.

Q9 Both will gain the same amount of kinetic energy (conservation of mechanical energy). The less massive treble string will gain more speed, however.

Q10 No. The force on Tarzan is inward along the radius while he moves along a circular arc.

Q11 Answer (c).

Q12 Some increase in gravitational potential energy in both cases.

Q13 Answer (c).

Q14 False. It was the other way around.

Q15 Savery's patent prevented Newcomen from profiting much from his engine.

Q16 Answer (c).

Q17 Answer (c).

Q18 Answer (e).

Q19 Answer (b).

Q20 Nearly all. A small amount was transformed in kinetic energy of the slowly descending weights.

Q21 Answer (a).

Q22 Answer (e).

Q23 Answer (c).

Q24 Answer (a).

Q25 Answer (d).

Q26 Answer (c).

Q27 Joule's approach was experimental, whereas Mayer's was theoretical (although he used data which had resulted from the experiments of others).

Chapter 11

Q1 Answer (c).

Q2 It is assumed that Newton's laws do apply.

Q3 False. Quantum mechanics is needed to treat the motion of atoms within molecules.

Q4 Answer (b).

Q5 In gases the molecules are far enough apart that the rather complicated intermolecular forces can safely be neglected.

Q6 Answer (b).

Q7 Answer (c).

Q8 Answer (d).

Q9 Answer (c).

Q10 Answer (c). Greater speed means greater momentum change in each collision and also more collisions per second.

Q11 Molecules will bounce off the piston with greater speed than before, so the total kinetic energy will increase.

Q12 Answer (a) is the ideal gas law and answer (b) is the prediction of kinetic theory. When combined they result in the prediction that $\overline{KE}$ is proportional to T.

Q13 Answers (a), (b) and (c) are consistent with the second law of thermodynamics.

Q14 Answer (a).

Q15 a) An unbroken egg is more ordered.
b) A glass of ice and warm water is more ordered.

Q16 a) True
b) False
c) False
Maxwell's demon is an imaginary, hypothetical device.

Q17 Answer (b).

Q18 Answer (c).

Q19 Answer (a). The ancient philosophers postulated recurrence but did not "discover" it as a theorem in mechanics.

Chapter 12

Q1 Transverse, longitudinal, and torsional waves.

Q2 Only longitudinal waves. Fluids can be compressed, but they are not stiff enough to be bent or twisted.

Q3 Transverse waves.

Q4 No. The movement of the bump in the rug depends on the movement of the mouse; it does not go on by itself.

Q5 Energy. (Particles of the medium are not transferred along the direction of wave motion.)

Q6 Stiffness and density.

Q7 1) Wavelength, amplitude, polarization.
2) Frequency, period.

Q8 Wavelength is the distance between two points of the wave that are moving in the same way. The wave does not have to be a sine wave; any repeating wave pattern has a wavelength.

Q9 1) 100 cycles per second.
2) 0.01 second.
3) $\text{wavelength} = \frac{\text{speed}}{\text{frequency}} = \frac{10 \text{ m/sec}}{100 \text{ cycles/sec}}$
$= 0.1$ meter (per cycle).

Q10 Answer (b).

Q11 Greatest displacement of point P is $(A_1 + A_2)$.

Q12 Wave amplitudes are positive or negative quantities; they add algebraically.

Q13 Nodal lines are regions of cancellation.

Q14 Antinodal lines are regions of reinforcement; the amplitude there is greatest.

Q15 Answer (a)

Q16 Waves from two in-phase sources arrive at a point out of phase if the point is one-half wavelength (or 3/2, 5/2, 7/2, etc.) farther from one source than the other.

Q17 1) No motion at the nodes.
2) Greatest motion at the antinodes.

Q18 Distance between nodes is $\lambda/2$.

Q19 Wavelength = 2L, so that one-half wavelength just fits on the string.

Q20 No. The frequency must be one for which the corresponding wavelength is such that 1 or 2 or 3 or ... half-wavelengths fit between the boundaries of the medium.

Q21 All points on a wave front have the same phase; that is, they are in the same state of motion.

Q22 Every point on a wave front may be considered to behave as a point source for waves generated in the direction of the wave's propagation.

Q23 At no point in space is the distance to one edge of the slit one-half wavelength farther than the distance to the other edge.

Q24 As the wavelength increases, the diffraction pattern becomes more spread out and the number of nodal lines decreases.

Q25 Answer (b). Waves with short wavelength are less noticeably diffracted than waves with long wavelength.

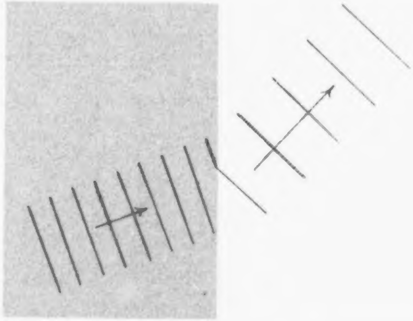
Q26 Because light diffracts, it is impossible actually to produce a "ray of light."

Q27 The angles are equal.

Q28 Parabolic.

Q29 The reflected wave fronts are parallel wave fronts.

Q30 1) Frequency does not change.
2) Wavelength decreases.
3) Its direction of propagation becomes closer to the perpendicular to the boundary between the media.



Q32 Superposition, reflection, refraction, diffraction, interferrence.

Q33 Sound waves are longitudinal; only transverse waves can be polarized.

Project Physics **Teacher's Guide**

An Introduction to Physics **3** **The Triumph of Mechanics**

Authorized Interim Version 1968-69

Distributed by **Holt, Rinehart and Winston, Inc.** New York Toronto

This teacher guide is the authorized interim version of one of the many instructional materials being developed by Harvard Project Physics, including text units, laboratory experiments, and readers. Its development has profited from the help of many of the colleagues listed at the front of the text units.

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The work of Harvard Project Physics has been financially supported by the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education and Harvard University.

03-080018-8

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Overview of Unit 3

In Units 1 and 2 we have developed the basic principles of Newtonian (or "classical") physics and shown their successful application to the astronomy of the solar system. Historically, this success led physicists in the eighteenth and nineteenth centuries to use these same principles in explaining many other natural phenomena.

In Unit 3 we will focus on the generalization of Newtonian mechanics by means of conservation laws for mass, momentum and energy, and the application of Newtonian mechanics to collisions of objects, heat, gas theory and waves.

In presenting the conservation laws, we stress the metaphysical and semi-theological origin of these laws in the seventeenth century: the idea that the world is like a machine, which God has created with a fixed amount of matter and motion. Again, in the discussion of the generalized law of conservation of energy, we point out connections with steam-engine technology and other historical and philosophical factors that lay in the background of the simultaneous discovery of this law by several scientists in the middle of the nineteenth century. Our purpose here is to make students aware of the relationships between physics and other human activities.

In selecting certain applications for detailed presentation, while ignoring many others that are parts of the traditional physics course, we have been guided by two main criteria: (1) to give the bare minimum of technical detail that is needed to illustrate the main principles; (2) to develop material that will be needed in later units. The climax of the unit is really in Chapter 11, where the concepts of momentum, energy and heat are combined in the kinetic theory of gasses to yield the first definite information about molecular speeds and sizes (as well as an explanation of the macroscopic properties of gases). This is about as far as Newtonian mechanics can take us into the atomic world; we will have to wait for the quantum concepts introduced in Unit 5 before we can go farther. Chapter 12 begins the study of wave phenomena, still in a mechanical framework, but breaking the ground for the study of light and electromagnetic waves in Unit 4.

Experiments

E22	Collision in one dimension
E23	Collisions in two dimensions
E24	Conservation of mechanical energy
E25	Speed of a bullet
E26	Hotness, thermometers and temperature
E27	Calorimetry
E28	Monte Carlo experiment on molecular collisions
E29	Behavior of gases
E30	Introduction to waves
E31	Sound

Demonstrations

D33	An inelastic collision
D34	Range of a slingshot
D35	Diffusion of gases
D36	Brownian motion
D37	Wave propagation
D38	Energy transport
D39	Superposition
D40	Reflection
D41	Wave trains
D42	Refraction
D43	Interference patterns
D44	Diffraction
D45	Standing waves
D46	Turntable oscillator

Note Concerning Polaroid Picture Taking:

When you are taking Polaroid pictures with the Model 002 camera, remove the film pack after picture no. 6. This will leave 2 films, one of which will be exposed and ruined; the other one will remain covered. Save several of these film packs for later use in Units 4, 5 and 6, for x rays and autoradiographs.

Teaching Aid Perspective

Loops

L 4	A matter of relative motion
L18	One-dimensional collisions
L19	Further examples of one-dimensional collisions
L20	Perfectly inelastic one-dimensional collisions
L21	Two-dimensional collisions
L22	Further examples of two-dimensional collisions
L23	Perfectly inelastic two-dimensional collisions
L24	Scattering of a cluster of objects
L25	Explosion of a cluster of objects
L26	Finding the speed of a rifle bullet—method I
L27	Finding the speed of a rifle bullet—method II
L28	Recoil
L29	Colliding freight cars
L30	Dynamics of a billiard ball
L31	A method of measuring energy—nails driven into wood
L32	Gravitational potential energy
L33	Kinetic energy
L34	Conservation of energy I—pole vault
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L36	Kinetic theory—gas model
L37	Reversibility of time
L38	Superposition of waves
L39	Standing waves on a string
L40	Standing waves in a gas
L42	Vibrations of a rubber hose
L43	Vibrations of a wire
L44	Vibrations of a drum
L45	Vibrations of a metal plate

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T19	One-dimensional collisions
T20	Equal mass two-dimensional collisions
T21	Unequal mass two-dimensional colsions
T22	Inelastic two-dimensional collisions
T23	Slow collisions
T24	The Watt engine
T25	Superposition
T26	Square wave analysis
T27	Standing waves
T28	Two-slit interference
T29	Interference pattern analysis

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P 9	Kinetic-molecular theory
P10	Waves

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F28	Stationary transverse waves (McGraw Hill) 12.7
F29	Sound waves in air (PSSC) 12.11

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CHAPTER 10

R2 The Steam Engine Comes of Age (Forbes & Dijksterhuis) (10.5)

R18 The Seven Images of Science (Holton) (10.8)

R19 Scientific Cranks (Gardner) (10.8)

R3 Energy (Thomson & Tait) (10.9)

CHAPTER 11

R8 The Law (Coates) (11.3)

R5 The Great Molecular Theory of Gases (Rogers) (11.5)

R6 On The Kinetic Theory of Gases (Maxwell) (11.5)

R4 The Barometer Story (Calandra) (11.6)

R7 The Law of Disorder (Gamow) (11.7)

R10 James Clerk Maxwell (Newman) (11.7)

R11 Maxwell's Demon (Gamow) (11.7)

R9 The Arrow of Time (Bronowski) (11.8)

R12 Randomness and the Twentieth Century (Bork) (11.8)

CHAPTER 12

R13 Introduction to Waves (PSSC) (12.1)

R14 What is a Wave (Einstein & Infeld) (12.2)

R16 Musical Instruments and Scales (White) (12.2)

R15 Wave Motion & Acoustics (Lindsay) (12.11)

R17 Founding a Family of Fiddles (Hutchins) (12.11)

MULTI-MEDIA DAILY PLAN UNIT III Details on following pages.

LAB STATIONS 1: Conservation of Mass Text: Unit 3 Prologue and 9.1	STUDENT REPORTS from day 1. Text 9.2	TEACHER DEMONSTRATION: Inelastic Collisions Text 9.3-9.4	LAB STATIONS 2: Conservation of Momentum Text 9.5
EXPERIMENT 22: Collisions in One Dimension Text: 9.6, 9.7 Analyze strobe photo	STUDENT DEMONSTRATION Collisions in Two Dimensions Text: 10.1 and 10.2	PROBLEM SOLVING DAY Conservation of Energy Text 10.3	FILM: Energy and Work SMALL-GROUP DISCUSSION Text 10.4
LAB STATIONS 3: Kinetic and Potential Energy Text: 10.5	DISCUSS DAY 9 FILM: Elastic Collisions and Stored Energy or: TEACHER DAY Text: 10.6 – 10.10	TEACHER DEMONSTRATION Conservation of Energy in Inelastic Collisions Text: 11.1— 11.2	FILM: Mechanical Energy and Thermal Energy SGD Text: 11.3 and 11.4
FILM: Conservation of Energy DEMONSTRATIONS Reader: Energy, Thomson	LAB STATIONS 4: Traditional Heat and	Experiments in Calorimetry Reader: The Arrow of Time, Bronowski	EXPERIMENT 28: Monte Carlo Experiment on Molecular Collisions Text: 11.5
EXPERIMENT 29: Behavior of Gases Program: Kinetic-Molecular Theory, 30 m	STUDENT ACTIVITY DAY Program, KMT, 30 min. Text: 11.6 and 11.7	TEACHER DISCUSSION Second Law of Thermodynamics Reader: Maxwell's Demon, Gamow Finish KMT Program	REVIEW QUIZ Programmed Instruction Waves, Parts 1 and 2 Text: 11.8, 12.1
LAB STATIONS 5: Waves and Wave Behavior Text: 12.2	Text: 12.3-12.7	FILM: Sound Waves in Air Text: 12.8-12.11	SMALL-GROUP Review of Unit 3 25) Unit 3 EXAM

Details of the Multi-Media Schedule

Day 1

Lab stations I (Conservation of Mass):

(Students do one only.)

1. Alchemical. Lead filings are heated in an open test tube. (The system gains weight.)
2. Boyle. Lead filings are heated in open tube as before. After heating, the tube is sealed, cooled and weighed.
3. Lavoisier. Lead filings are heated in a closed tube.
4. Alka Seltzer. Weigh heavy two-liter flask, stopper, water and tablets before and after interaction. Use sensitive balance (0.1 g).
5. Precipitate. Put 20 g lead nitrate dissolved in water in one-liter flask; 11 g of potassium dichromate in water in a small test tube inside. Stopper and weigh. Invert to mix and reweigh.

Assignment: Prepare to give the class a report on your experiment (see Day 2).

Day 2

Student reports

After a ten-minute rehearsal, students explain to class the experiments they did on the previous day. (i.e., Alchemical: explains what has happened as an alchemist would.)

Day 3

Teacher demonstration (Inelastic Collisions):

Use lab carts with bricks to show one-dimensional inelastic collisions. Use double sticky "Bear" tape on carts.

Day 4

Qualitative lab stations (Conservation of Momentum):

Students experience one- and two-dimensional collisions using balloon pucks, dylite beads, disk magnets and lab carts. In each case, they are to look for conservation of momentum.

Day 5

Experiment 22: Collisions in One Dimension

A quantitative measurement is made of momentum exchange in a collision. An airtrack or colliding lab carts may be used. Data is recorded with strobe-camera. Each student will need a record of a collision for analysis.

Day 6

Student presentation E23: Collisions in Two Dimensions

Have two better students do E23 and analyze conservation of momentum in two dimensions for the class.

Day 7

Problem solving day (Conservation of Energy):

Use photos taken in Day 5 to check for conservation of kinetic energy.

Day 8

Film: Energy and Work (PSSC #0311, 28 min.)

Small group discussions to follow film. Provide groups with questions to discuss.

Day 9

Lab stations (Kinetic and Potential Energy):

1. simple pendulum
2. Galileo's pendulum
3. ball on inclined place
4. energy stored in compressed spring
5. Film loops 32, 34 or 35

Students are to look for changes in PE and KE.

Day 10

Discussion:

Discuss results of previous day's experiment—10 minutes.

Film: Elastic Collisions and Stored Energy (PSSC #0318, 27 min.)

Follow with small group discussions. Teacher to provide questions.

Day 11

Teacher demonstration (Conservation of Energy in Inelastic Collisions):

Mechanical equivalent of heat (see Student Handbook, Chapter 11),

or

D33 An inelastic collision (see Teacher Guide).

Day 12

Film: Mechanical Energy and Thermal Energy (PSSC #0312, 22 min.)

Follow with small group discussion. Teacher to provide questions that cover both text and film.

Day 13

Film: Conservation of Energy (PSSC #0313, 27 min.)

Energy traced from coal to electrical output in a power plant.

Demonstrate energy transformations, photocells, thermocouples, heat engines, transducers, batteries, chemoluminescence, etc.

Day 14 and 15

Lab stations (Heat, Calorimetry and Thermodynamics):

1. Measure linear expansion of a rod
2. Measure heat of fusion of ice
3. Measure heat of vaporization of water
4. Measure specific heat by method of mixtures
5. Others

Students spend 20 minutes (approximately) at each station making measurements and doing calculations.

Day 16

Experiment 28: Monte Carlo Experiment on Molecular Collisions

Day 17

Experiment 29: Behavior of Gases

Day 18

Student activity day

Students pick activities from Handbook or other sources. Make arrangements in advance for needed materials.

Day 19

Teacher discussion (The Second Law of Thermodynamics, 35 min.):

Read: Gamow, Maxwell's Demon, Mr. Tompkin's Version (15 min.)

Day 20

Review quiz

Programmed instruction, Waves, Parts I and II

Days 21 and 22

Lab stations (Waves):

1. pulses on a rope or rubber tube
2. pulses on a slinky
3. pulses in a ripple tank
4. sound waves in air
5. ultra-sound
6. microwaves
7. continuous waves in ripple tank

Students are asked to look for and make observations of velocity of propagation, wavelength, frequency, diffraction, absorption, reflection, superposition, energy transfer, standing waves, etc.

Day 23

Film: Sound Waves in Air (PSSC #0207, 35 min.)

The wave characteristics of sound transmission are investigated with large scale equipment using frequencies up to 5000 Hz.

Class discussion of film.
Review of Chapter 12.

Day 24

Small group review

Review for Unit III exam. Guide questions may be issued or selected from study guide.

Day 25

Unit III exam.

Chapter 9 Schedule Blocks

Each block represents one day of classroom activity and implies a 50-minute period. The words in each block indicate only the basic material under consideration.

Chapter 9 Conservation of Mass and Momentum

Read Prologue, 9.1

Conservation of mass and closed system

Read 9.2, 9.3

Lab
Collisions in one dimension

Analyze Lab

Post-Lab and/or problem seminar

Read 9.4

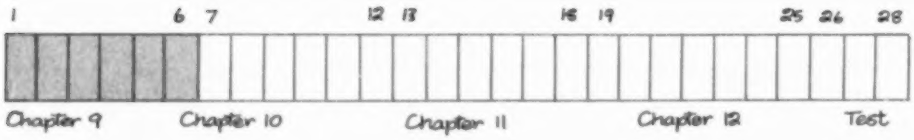
Momentum conservation is consistent with Newton's laws

Read 9.5, 9.6, 9.7

Need for a second conservation law

Review

Tests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Chapter 9
Chapter 10
Chapter 11
Chapter 12
Test

Unit 3 Resource Chart

Text	Schedule	Problems			Experiments [illegible] Demonstrations
Prologue					
9.1 Conservation of mass		1 2	3 4 5		
9.2 Collisions					E 22* Collision in one dimension
9.3 Conservation of momentum		7	6	8 9	E 23 Collision in two dimensions
9.4 Momentum and Newton's laws of motion		10 11 12 19	16	13 14 15 17 18	
9.5 Isolated systems			20		
9.6 Elastic collisions		23	21 22		
9.7 Leibniz and the conservation of vis viva		24			

Unit 3 Resource Chart

R1 Silence, Please	
F17 Elements, compounds and mixtures F18 The perfection of matter	Is mass conserved?
T19 One-dimensional collisions L18 One-dimensional collisions L19 Further examples of one-dimensional collisions L20 Perfectly inelastic one-dimensional collisions	Stroboscopic photographs of one-dimensional collisions
T20 Equal mass two-dimensional collisions T21 Unequal mass two-dimensional collisions L23 Perfectly inelastic two dimensional collisions	Interesting exchange of momentum devices Interesting case of elastic impact
L28 Recoil	Stroboscopic photographs of two-dimensional collisions
L24 Scattering of a cluster of objects L25 Explosion of a cluster of objects	
L21 Two-dimensional collisions L22 Further examples of two dimensional collisions F19 Elastic collisions and stored energy (PSSC)	

Chapter 9 Experiment Summaries

E22* Collisions in One Dimension

In this "discovery" experiment straight-line collisions between known masses are observed in three different settings and the velocities of objects before and after each collision are measured. Whichever of the three procedures a student follows, he then goes on to the the discovery that the only quantity that is conserved is mass-times-velocity.

The following Experiment 23 on collisions in two dimensions may be done simultaneously if apparatus and student preparation permit.

Equipment

Procedure I

Air track and two or three gliders
Blower for air track
Polaroid camera and tripod
either Motor strobe with slotted disc
or Xenon strobe lamp
White or metal straws or cardboard pointers to be attached to gliders as markers for photographic measurements
10X magnifier with scale

Procedure II

Dynamics carts with a steel exploder spring for each pair
10X magnifier with scale
Weights for changing masses of carts
either Bell-timers with batteries and ticker tape for each cart
or Polaroid camera and
Motor strobe with 12-slotted disc or
Xenon strobe lamp

Procedure III

Film loop showing collision in 1-D
Kodachrome loop projector
(graph paper, masking tape, ruler)

E23 Collisions in Two Dimensions

In this "verification" experiment two-dimensional collisions between known masses are observed in three different settings, and (as in Experiment 22) the speeds and directions of objects before and after each collision are measured. Whichever of the three procedures a student follows, he then goes on to the discovery that mass-times-speed is not conserved but mass-times-velocity is conserved.

This experiment assumes familiarity with the law of conservation of momentum beforehand. Treated as a verification of the law, it may be done simultaneously with Experiment 22.

Equipment

Procedure I

Ripple tank
Plastic (Dylite) spheres
Three or four balloon pucks without balloons, but with small (∿1/2") white styrofoam hemisphere glued to center of bottom of two of them, as markers
Polaroid camera
Mount for camera vertically above center of the ripple tank, e.g., a stepladder
either Xenon strobe lamp
or Motor strobe with 12-slotted disc and strong spotlight
10X magnifier with scale

Procedure II

Exactly the same equipment and set-up as Procedure I except that the air pucks are replaced by three or four disc magnets, two of which have their centers marked by 1/2" white styrofoam hemispheres.

Procedure III

Film loop showing collision in 2-D
Kodachrome loop projector
(graph paper, masking tape, ruler)

Chapter 10 Schedule Blocks

Each block represents one day of classroom activity and implies a 50-minute period. The words in each block indicate only the basic material under consideration.

Chapter 10: Energy

Read 10.1, 10.2, 10.3

Work produces energy changes

Read 10.4 and Lab

Lab: E24*
Conservation of mechanical energy

Analyze Lab

Post-Lab and/or problem seminar

Read 10.5, 10.6, 10.7, 10.8.

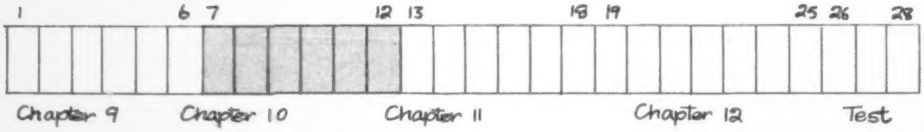
Different forms of energy

Read 10.9

Another conservation law

Review

Test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Chapter 9
Chapter 10
Chapter 11
Chapter 12
Test

Unit 3 Resource Chart

Text	Schedule	Problems			Experiments and Teacher Demonstration
10.1 Work and kinetic energy		1	2		E25 Speed of a bullet D33 An inelastic collision
10.2 Potential energy			4	3	D34 Range of a slingshot
10.3 Conservation of mechanical energy		5	6 10	11	E24* Conservation of mechanical energy
10.4 Forces that do no work				7	
10.5 Heat energy and the steam engine			8 9		E26 Hotness, thermometers and Temperature
10.6 James Watt and the Industrial Revolution					
10.7 The experiments of Joule		12 13		14	E27 Calorimetry
10.8 Energy in biological systems		15 16	17	18	
10.9 The law of conservation of energy		20	19 21	22 23	

Unit 3 Resource Chart

T22 Inelastic two-dimensional collisions L31 A method of measuring energy – nails driven into wood L33 Kinetic energy F20 Energy and work (PSSC)	
T23 Slow collisions L32 Gravitational potential energy	Bouncing ball determination of a_g
L34 Conservation of energy I — pole vault L35 Conservation of energy II — aircraft takeoff L26 Speed of rifle bullet — method I L27 Speed of rifle bullet — method II	Predicting the range of a bow
	Measuring a_g by a whirling object
R2 The Steam Engine Comes of Age	Heron's engine Savery's steam pump Small ball-valves (visible action)
T24 The Watt engine	Measuring the sun's power Crook's radiometer
	Steam powered boat
R18 The Seven Images of Science R19 Scientific Cranks	One student = ? horsepower
L29 Colliding freight cars L30 Dynamics of a billiard ball L4 A matter of relative motion F21 Conservation of energy (PSSC) R3 Energy	Rubber band engines Energy analysis of a pendulum swing Ice calorimetry

Chapter 10 Experiment Summaries

E24 Conservation of Mechanical Energy

Four procedures are provided: different student groups will be working on different experiments.

I. Students photograph a "slow collision" between two dynamics carts. Analysis of the results introduces the idea of potential energy. Whereas momentum is conserved during the collision, i.e., while the carts are in contact and the springs are compressed, there is a marked decrease in kinetic energy, which is later recovered as the carts move apart again.

II. Interaction between disc magnets on a low-friction plastic bead surface does not involve physical contact between the interacting bodies. Photographs of the two-dimensional process are analyzed for conservation of kinetic energy. The kinetic energy minimum "during" the interaction can be detected here too by careful analysis; some of the kinetic energy is temporarily converted to (magnetic) potential energy when the magnets are close to each other.

III. Students analyze the change of kinetic energy to gravitational potential energy as a glider moves up an inclined airtrack. With a less than perfect system $\Delta(PE) = -\Delta(KE)$ may not be true for all their observations and students are asked to account for any apparent breakdown in the conservation law.

IV. A fourth group of students can watch and take measurements on some of the nine film loops on collisions and energy conservation.

Equipment

I. One pair of dynamics carts with weak (.010 inch) springs
Four 1 kg masses
Two light sources
Polaroid camera and film
Motor strobe unit
Masking tape
10X magnifier and scale and/or mm ruler
Graph paper

II. Disc magnets
Plastic (Dylite) beads
Ripple tank
Polaroid camera and film
Camera tripod, or other means of supporting camera above ripple tank
Xenon strobe (or photo-flood lamps together with motor strobe unit)
Steel or styrofoam balls
10X magnifier and scale and/or mm ruler
Graph paper

III. Airtrack and glider, blower
Meter stick
Stopwatch
(Optional—camera, light source, disc strobe, graph paper)

IV. Film loop projector
Film loops—any or all of numbers 18-35
Stopwatch, or strip chart recorder ("dragstrip")

E25 The Speed of a Bullet

Two different procedures are provided:

I. The projectile is fired into a block or can mounted on an airtrack glider. After the inelastic collision, the glider plus block plus bullet move off slowly: the velocity can be measured with a stop watch. This method involves conservation of momentum only.

II. In the classical method (ballistic pendulum) the projectile is fired into a suspended block or can. Two conservation laws are needed to calculate the bullet's speed: conservation of momentum and conservation of mechanical energy.

Whichever method is used, students should check the value for v_{bullet} by an independent method.

Equipment

I. Airtrack and glider
Gun and projectile
Juice can plus cotton wadding
Stopwatch
Meter stick

II. Can plus wadding, or soft block
Ceiling hooks, or other very firm support for pendulum
Thread to support pendulum
Gun and projectile, or bow and arrow, etc.
Tube and straw device for measuring displacement of pendulum (see description of experiment for details)
Ruler

E26 Hotness, Thermometers, Temperature

The experiment tries to convince students that any device that responds in a repeatable way to changes in "hotness" could be used as a thermometer and to define a temperature scale. The choice is apparently an arbitrary one. Students construct their own thermometers (gas volume, mercury in glass, thermocouple, electrical resistance, alcohol in glass, etc.) and compare the temperature readings given by different thermometers when they are placed in the same water bath. The temperature readings do not agree. The fact that different gases, unlike liquids, do expand at the same rate suggests that a gas-expansion (or gas-pressure) thermometer should be used to define temperature, and that it may be possible to construct a simple model of gases to explain their behavior.

Equipment

As many as possible of:
Uncalibrated mercury in glass thermometer
Gas-pressure thermometer. (Source of these items is listed in the Teacher's Notes to the experiment)
Gas-volume thermometer
Thermistor* plus amplifier/power supply* plus meter*
Thermocouple
Large baths of boiling water, ice water and four or five water-baths at intermediate temperatures
Millimeter scales (not plastic) to attach to uncalibrated thermometers
As many different gases (e.g., CO_2, N_2, O_2, N_2O, etc.) as possible

E27 Calorimetry

The experiment introduces students to the idea that heat flow can be measured by observing the change in temperature of some standard substance (for example, liquid water). Using this method, students learn that measured amounts of heat produce different temperature changes in different substances. They measure the latent heat of ice and/or the specific heat of a metal and investigate the law of cooling.

Equipment

Foam plastic (styrofoam) cups
Thermometer 0-100° C
Hot and cold water
Ice—cracked or small cubes
Balance
Metal samples

Chapter 11 Schedule Blocks

Each block represents one day of classroom activity and implies a 50-minute period. The words in each block indicate only the basic material under consideration.

Chapter 11: The kinetic Theory of Gases

Read 11.1, 11.2, 11.3

Heat and molecular motion

Read 11.4 and Lab

Lab: E29*
Behavior of gases

Analyze Lab

Post-Lab
and/or ideal gas
law derivation

Read 11.6

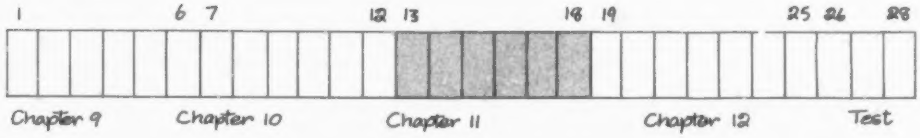
Entropy and
irreversible
processes

Read 11.7, 11.8

Statistical nature
of the kinetic
theory

Review

Test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Chapter 10
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Chapter 12
Test

Unit 3 Resource Chart

Text	Schedule	Problems			Experiments and Teacher Resource [illegible]
11.1 Heat and mechanical energy					
11.2 A model for the gaseous state		1 2	3 4	5 10	E29 Behavior of gases D35 Diffusion of gases
11.3 The speeds of molecules		7	6		D 36 Brownian motion E28* Monte Carlo experiment on molecular collisions
11.4 The sizes of molecules		9	12	11	
11.5 Optional		13 14 15	16	8	
11.6 The second law of thermodynamics and the dissipation of energy			15	17	
11.7 Maxwells demon and the statistical view of the second law of thermodynamics			18		
11.8 Time's arrow and the recurrence paradox.			19 20	21	

Unit 3 Resource Chart

F22 Mechanical energy and thermal energy (PSSC)	Stirling hot air engine Mechanical equivalent of heat
F23 Demonstrating the gas laws (Coronet) F24 Gas laws and their applications (EBF)	Gas laws How the amateur can experiment with films only one molecule thick
L36 Kinetic theory — gas model R8 The Law	Molecular speed distribution
F25 Molecular theory of matter (EBF)	Wie gross ist ein Molekül?
P9 Kinetic - molecular theory R5 The Great Molecular Theory of Gases R6 On the Kinetic Theory of Gases	Studying the properties of gases experimentally Drinking duck How to weigh a car with a pressure guage
R4 The Barometer Story	Rockets
R1 The Law of Disorder R10 James Clerk Maxwell R11 Maxwell's Demon	Perpetual motion mechanics
L37 Reversibility of time R9 The Arrow of Time R12 Randomness and the Twentieth Century	Problems of scientific and technological growth

E28 Monte Carlo Experiment on Molecular Collisions

Part A. By counting the hits and misses of a bombarding marble (molecule) rolling into a random array of target marbles (molecules), the diameter of the marble is obtained.

Part B. By tracing the path of a test particle graphically through a random array of squares (molecules), data for the mean free path is obtained.

Equipment

Part A.

12 marbles
Board studded with nails
Board marked off with coordinates
Some sticky wax

Part B.

Large piece of graph paper on a drawing board

E29 Behavior of Gases

There are two parts to the experiment:

A. Pressure and volume at constant temperature (Boyle's law)

B. Temperature and volume at constant pressure (law of Charles and Gay-Lussac)

Both could be done as quantitative lecture experiments or as short qualitative demonstrations, instead of as student experiments.

These classical experiments give students an opportunity to see the first steps in the development of a satisfactory model for the behavior of gases. As well as supplementing the text, the experiments should teach students the usefulness of graphical methods of presenting and analyzing data.

Equipment

A. Boyle's law apparatus—conventional J-tube, or simple syringe type (e.g., Macalaster #30220 $3.00; Linco #6250 $3.50; Damon #99129 $2.50)
Set of weights for use with syringe—hooked or flat
Other gases—e.g., CO_2, N_2, O_2, N_2O, etc.—if possible.

B. Capillary tubes
Mercury
Silicone rubber sealant (G.E. "Bathtub Seal" about $2.00)
Millimeter scales (metal or wood, not plastic)
Beaker of water
Bunsen burner or hot plate
Thermometers 0-100°C

Chapter 12 Schedule Blocks

Each block represents one day of classroom activity and implies a 50-minute period. The words in each block indicate only the basic material under consideration.

Chapter 12: Waves

Read 12.1, 12.2, 12.3, 12.4

Nature of waves

Read 12.5 and Lab

Lab: E30*
Introduction to waves

Analyze Lab

Post-Lab
and/or problem seminar

Read 12.6, 12.7, 12.8, 12.9, 12.10.

Wave phenomena

Read 12.11 and Lab

Lab: E31*
Sound

Analyze Lab

Post-Lab
and/or problem seminar

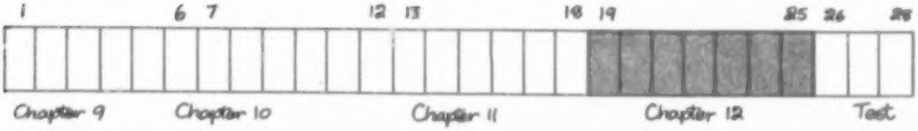
Chapter Review

Chapter test

Unit review

Unit test

Go over test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Unit 3 Resource Chart

Text	Schedule	Problems			Experiments and Further Documentation
12.1 Introduction					
12.2 Properties of waves		1 2			E 30* Introduction to waves
12.3 Wave propagation					D 37 Wave propagation D 38 Energy transport
12.4 Periodic waves					
12.5 When waves meet: the superposition principle		8 10	9		D39 Superposition D46 Beats and harmonic synthesis
12.6 A two-source interference pattern			12	11	D43 Interference patterns
12.7 Standing waves				13	D45 Standing waves
12.8 Wave fronts and diffraction			15	14	D44 Diffraction
12.9 Reflection		16 17	18 19	4	D40 Reflection
12.10 Refraction		3	6 7	5 20 21	D41 Wave trains D42 Refraction
12.11 Sound waves		22	23 25	24	E31* Sound

R13 Introduction to Waves		"Least time" or "least energy" situations
P10 Wave R14 What is a Wave? R16 Musical Instruments and Scales F26 Progressive waves, Transverse and longitudinal (McGraw Hill)		Mechanical wave machines
T25 Superposition T26 Square wave analysis L38 Superposition of waves		Graphical addition of waves
T28 Two-slit interference		Moiré patterns
T27 Standing waves T29 Interference pattern analysis L39 Standing waves on a string L42 Vibrations of a rubber hose	L43 Vibrations of a wire F27 Stationary longitudinal waves F28 Stationary Transverse waves	Standing waves on drum and violin
L44 Vibrations of a drum L45 Vibrations of a metal plate		
L40 Standing waves in a gas F29 Sound waves in air R15 Wave Motions and Acoustics	R17 Founding a Family of Fiddles	Determination of the velocity of sound in air Music and Speech activities Measurements of the speed of sound.

E30 Introduction to Waves

The properties of one-dimensional waves are observed on springs and ropes stretched out on the floor. These properties are then studied in a ripple tank in two dimensions. Finally, the wavelength of standing waves in the ripple tank is measured in order to verify that $v = f\lambda$.

Equipment

Waves in springs

"Slinky" spring
20-30 feet of rope (clothesline) or a different spring that can be stretched to this length

Waves in a ripple tank

Ripple tank set-up, complete with light source, beaches and variable-speed wave generator with straight wave and two point sources
Paraffin blocks for wave-barriers
Rubber tubing (~ 1½ feet) for use as a wave "mirror"
Dowel (3/4 inch) or broomstick handle (1 to 1½ feet long) to generate straight pulses
Sheet of glass with one edge 1-1½ feet long to fit in tank, with washers as corner supports to adjust height (for refraction)
Hand-driven stroboscope (or motor stroboscope to be driven by hand)
Meter stick
Clock or watch with second hand.
Large beaker or jar for filling and emptying ripple tank

E31 Sound

While one group of students studies the properties of sound, a second group studies ultrasound, and a third group performs Young's two-source experiment with waves in a ripple tank.

During a subsequent class period the sound and ultrasound groups compare results and discuss evidence for sound as a wave phenomenon. The Young's experiment group thereupon provides a method for verifying wave character, and in the following lab exercise they join the sound and ultrasound groups as advisors for the application of Young's method to sound and ultrasound and the measurement of their wavelengths.

Equipment

Sound

Amplifier/power supply
Oscillator plug-in unit
2 small loudspeakers
Funnel (or thistle tube) and 1½ feet of rubber tubing to fit it—for ear trumpet
Various sheets of styrofoam, metal, glass, paraffin, masonite, wood, celotex, etc. for absorption tests
Spherical balloon and CO_2 source
3 ringstands with adjustable clamps
Meter stick

Ultrasound

Amplifier/power supply
Oscillator plug-in unit
3 ultrasonic transducers
Oscilloscope or
Amplifier/power supply and microameter and diode
(100,000 ohm resistor, optional)
Sheets of test materials as listed under Sound above.
Meter stick

Ripple tank

Ripple tank set-up, complete with light source, beaches, and variable-speed wave generator with two point sources
Meter stick
Large beaker or jar for filling and emptying ripple tank

Brief Answers to Unit 3 Study Guide

Chapter 9

9.1 Discussion

9.2 Yes

9.3 No

9.4 Discussion

9.5 (a) 220.2g
(b) 20.2g

9.6 (a) Discussion
(b) Yes
(c) Yes

9.7 0.8 m/sec

9.8 Derivation

9.9 3.33×10^{-6}kg

9.10 Discussion

9.11 Discussion

9.12 Discussion

9.13 1200 kg-m/sec
400 nts
30 m

9.14 (a) 200 kg-m/sec
(b) 10 nts

9.15 Yes

9.16 Derivation

9.17 (a) 0
(b) 0
$M_c\vec{V}_c + M_s\vec{V}_s$
(c) $M_c\vec{V}_c = -M_s\vec{V}_s$
(d) $V_s/V_c = M_c/M_s$
(e) 10 m/sec

9.18 Derivation

9.19 Left car
(1) speed of one car or
(2) mass of cars
Distance
Force

9.20 Discussion

9.21 Derivation

9.22 One-half initial speed in opposite directions

9.23 (a) No
(b) Continues forward
(c) Continues backward

9.24 .8 kg-m/sec forward
.8 kg-m/sec away
1.6 kg-m/sec away
No

9.25 4.2 kg-m/sec, 126.0 kg-m^2/sec^2
0.171 kg, 427.5 kg-m^2/sec^2,
0.075 kg-m/sec, 0.1125 kg-m^2/sec^2
111.0 m/sec, 1.62×10^4 kg-m/sec
10^{-4}kg, 0.2 m/sec

9.26 Discussion

9.27 No, discussion

Chapter 10

10.1 1.82×10^{-18} j
5.5×10^{17} electrons

10.2 (a) 67.5 j
(b) 4.5×10^9 j
(c) 3750 j
(d) 2.7×10^{33} j

10.3 Discussion

10.4 (a) .00225 j
(b) .056 j

10.5 Discussion

10.6 (a) 5m/sec^2
19 sec
95 m/sec
(b) 95 m/sec

10.7 Derivation

10.8 22°

10.9 9.3 min

10.10 Derivation

10.11 (a) 9.6×10^9 j
(b) 4.8×10^5 nt
(c) 7.68×10^6 watts
(d) 880 m
(e) 1 and 2 increase
3 decreases
(f) change direction

10.12 (a) 1046 times
(b) increase weight
decrease amount of water
increase height of fall
lower specific heat

10.13 0.12°C

10.14 Discussion

10.15 3 weeks

10.16 1.5 kg

10.17 Discussion

10.18 Discussion

10.19 Discussion

10.20 Undamaged rocket

10.21 1800 nts
90 j

10.22 Discussion

10.23 Discussion

Chapter 11

11.1 Pressure, density, temp., viscosity

11.2 Discussion

11.3 Discussion

11.4 Working model, theoretical model

11.5 Discussion

11.6 No, discussion

11.7 (a) probable speed
(b) average speed
(c) no negative speeds

11.8 $P \sim T$

11.9 (a) 10^{-9}m
(b) 10^{-9}m

11.10 Discussion

11.11 Discussion

11.12 Density changes

11.13 50 atmospheres

11.14 5 atmospheres

11.15 Discussion

11.16 Discussion

11.17 Discussion

11.18 Discussion

11.19 Discussion

11.20 Discussion

11.21 Discussion

Chapter 12

12.1 Construction

12.2 Construction

12.3 (a) $BAC = \theta_A$
(b) $ACD = \theta_B$
(c) $BC = \lambda_A$
(d) $AD = \lambda_B$
(e) Derivation
(d) Derivation

12.4 Straight-line

12.5 Two straight-line waves inclined toward each other

12.6 $\lambda_D = .035$ m
$\lambda_s = .025$m

12.7 Derivation

12.8 Reflection

12.9 No

12.10 Construction

12.11 Construction

12.12 Discussion

12.13 No, discussion

12.14 100 and 1000 cps
yes

12.15 Maximum

12.16 Construction

12.17 Construction

12.18 R/2

12.19 Construction

12.20 Discussion

12.21 Discussion

12.22 $2d = vt$

12.23 Doppler Effect

12.24 3×10^4 cps
2.5×10^6 cps

12.25 1.27×10^{-11} watts
8×10^{12} mosquitoes
wind-tunnel test

Solutions for Chapter 9 Study Guide

9.1 Discuss the law of conservation of mass as it applies to the following situations.

a) a satellite "soft-landing" on Venus

b) a rifle firing a bullet

c) the manufacture of styrofoam

d) a person drinking a Coke

ANSWER:

The discussion should emphasize that one must consider an isolated system.

9.2 Would you expect that in your lifetime, when more accurate balances are built, you will see experiments which show that the law of conservation of mass is not entirely true for chemical reactions?

ANSWER:

If experiments are limited to ordinary chemical reactions the answer is yes.

9.3 Dayton C. Miller was a renowned expermenter at Case Institute of Technology. He was able to show that two objects placed side by side on a pan balance did not balance two identical masses placed one on top of the other. The reason is that the pull of gravity decreases with distance from the center of the earth. Would Lavoisier have said this experiment contradicted the law of conservation of mass?

ANSWER:

No. Lavoisier would have been able to distinguish between mass and weight.

9.4 A children's toy known as a snake consists of a tiny pill of mercuric thiocyanate. When the pill is ignited, a large, serpent-like foam curls out almost from nothingness. Devise and describe an experiment by which you could test the law of conservation of mass for this demonstration.

ANSWER:

Ignite the pill within a closed bottle and weigh before and after. The weights should be the same within experimental error.

9.5 Consider the following chemical reaction, which was studied by Landolt in his tests of the law of conservation of mass. A solution of 19.4 g of potassium chromate in 100.0 g of water is mixed with a solution of 33.1 g of lead nitrate in 100.0 g of water. A bright yellow solid precipitate forms and settles to the bottom of the container. When removed from the liquid, this solid is found to have a mass of 32.3 g and is found to have properties different from either of the reactants.

a) What is the mass of the remaining liquid? (Assume the combined mass of all substances in the system is conserved.)

b) If the remaining liquid (after removal of the yellow precipitate) is then heated to 95 °C, the water it contains will evaporate, leaving a white solid. What is the mass of this solid? (Assume that the water does not react with anything, either in (a) or in (b).)

ANSWER:

a) Liquid remaining = initial mass minus precipitate. Liquid remaining = (19.4 + 100 + 33.1 + 100) - 32.3 = 220.2 grams.

b) Mass of solid = initial solids minus precipitate. Mass of solid = (19.4 + 33.1) - 32.3 = 20.2 grams.

9.6 If powerful magnets, so-called radar magnets obtainable from surplus stores, are placed on top of each of two carts, and the magnets are so arranged that like poles face each other as one cart is pushed toward the other, the carts bounce away from each other without actually making contact.

a) In what sense can this be called a collision?

b) Does the law of conservation of momentum apply?

c) Does the law of conservation of momentum hold for any two times during the interval when the cars are approaching or receding (the first called before and the second called after the "collision")?

ANSWER:

a) The cars exert forces on each other.

b) Yes, to the system as a whole, there is no friction between cars and track.

c) Yes. The system of two cars is isolated at all times, as far as momentum in the direction of motion is concerned.

9.7 A freight car of mass 10^5 kg travels at 2.0 m/sec and collides with a motionless freight car of mass 1.5×10^5 kg. The two cars lock and roll together after impact. Find the final velocity of the two cars after collision.

HINTS:

a) Equation (9.1) states

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

What factors in this equation are given in the problem?

b) Rearrange terms to get an expression for $\vec{v}'_A$.

c) Find the value of $\vec{v}'_A$ ($\vec{v}'_A = \vec{v}'_B$).

ANSWER:

$10^5 \times 2 = \vec{v}'_B\ (10^5 + 1.5 \times 10^5)$.
$\vec{v}'_B = 0.8$ m/sec.

9.8 From the equation

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

show that the change in momentum of object A is equal and opposite to the change of momentum of object B. (Write momentum changes in Δ form.)

ANSWER:

$$m_A\vec{v}'_A - m_A\vec{v}_A = \Delta(m_A\vec{v}_A) = m_B\vec{v}_B - m_B\vec{v}'_B = - \Delta(m_B\vec{v}_B).$$

9.9 Benjamin Franklin, in correspondence with his friend James Bowdoin (founder and first president of the American Academy of Arts and Sciences), objected to the corpuscular theory of light by saying that a particle traveling with such immense speed (3×10^8 m/sec) would have the impact of a 10 kg ball fired from a cannon at 100 m/sec. What mass did Franklin assign to the light particle?

ANSWER:

Impact proportional to momentum = 10 kg × 100 m/sec = m × 3 × 10^8 m/sec m = 1000/3 × 10^8 = 3.33 × 10^{-6} kg.

9.10 In a baseball game, both the bunt and the long outfield fly are sacrifice hits. Contrast the collision processes which make them.

ANSWER:

For the fly the initial velocity of the bat is opposite to that of the ball, for the bunt the velocity is in same direction.

9.11 In the light of your knowledge of the relationship between momentum and force, comment on reports about unidentified flying objects (UFO) turning sharp corners.

ANSWER:

A UFO is described as an isolated system. It must conserve its momentum. A sharp turn would change the direction of its velocity and hence its momentum. There is no evidence of the discharge of a rocket.

9.12 A hunter fires a gun horizontally at a target fixed to a hillside. Describe the changes of momentum of the hunter, the bullet, the target and the earth. Is momentum conserved in each case?

ANSWER:

Assume the hunter fixed to the earth. While the bullet is in flight, hunter, target and earth collectively have the same backward momentum as the bullet forward. Upon the bullet's impact on the target, the total momentum returns to zero.

9.13 A girl on skis (mass of 60 kg including skis) reaches the bottom of a hill going 20 m/sec. What is her momentum? She strikes a snowdrift and stops within 3 seconds. What force does the snow exert on the girl? How far does she penetrate the drift?

ANSWER:

$p = mv = 60 \times 20 = 1200$ kg-m/sec
$F = p/t = 1200/3 = 400$ newtons
$KE = Fd \qquad d = 1/2\ mv^2/F =$

$$\frac{60 \times 20^2}{2 \times 400} = 30 \text{ meters}$$

9.14 A horizontal conveyor belt is used to transport grain from a bin to a truck. A 50.0 kg bag of grain falls straight from a chute onto the belt every 20 seconds. The velocity of the conveyor belt is 4.0 m/sec.

a) What is the momentum gained or lost by a bag of grain just as it is placed on the belt?

b) What is the average additional force required to drive the belt when carrying grain?

ANSWER:

a) Momentum gained in direction of motion of belt $50 \times 4 = 200$ kg-m/sec.

b) F = rate of change of momentum = $200/20 = 10$ newtons.

9.15 The text derives the law of conservation of momentum for two bodies from Newton's third and second laws. Is the principle of the conservation of mass essential to this derivation? If so, where does it enter?

ANSWER:

Yes. In equating $m(\Delta\vec{v})$ to $\Delta(m\vec{v})$

9.16 If mass remains constant, then $\Delta(mv) = m(\Delta v)$. Check this relation for the case where $m = 3$ units, $v' = 6$ units and $v = 4$ units.

ANSWER:

$\Delta v = 6 - 4 = 2 \qquad m = 3$
$p' = (3 \times 6) = 18, \quad p = 3 \times 4 = 12$

$m\Delta\vec{v} = 6$
$\Delta p = p' - p = 18 - 12 = 6$

9.17 Equation (9.1), $m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A' + m_B\vec{v}_B'$, is a general equation. For a loaded cannon where the subscripts c and s refer to cannon and shell respectively,

a) What are the values of v_c and v_s before firing?

b) What is the value of the left hand side of the equation before firing? After firing?

c) Compare the magnitudes of the momenta of cannon and shell after firing.

d) Compare the ratios of the speeds and the masses of shell and cannon after firing.

e) A 10 kg shell has a speed of 1000 m/sec. What is the recoil speed of 1000 kg cannon?

ANSWER:

a) $\vec{v}_c = \vec{v}_s = 0$

b) before 0; after $m_c\vec{v}_c + m_s\vec{v}_s$

c) conservation of momentum gives $m_c\vec{v}_c = -m_s\vec{v}_s$

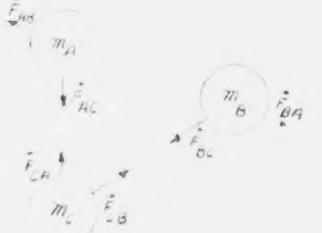
d) from (c) $v_s/v_c = m_c/m_s$

e) $v_c = 10 \times 1000/1000 = 10$ m/sec

9.18 Newton's laws of motion lead to the law of conservation of momentum not only for two-body systems, as was shown in Sec. 9.4, but for systems consisting of any number of bodies. In this problem you are asked to repeat the derivation of Sec. 9.4 for a three-body system.

The figure shows three bodies, with masses m_A, m_B and m_C, exerting forces on one another. The force exerted on body A by body B is $\vec{F}_{AB}$; the force exerted on body C by body A is $\vec{F}_{CA}$, etc.

Using the symbol $\vec{p}$ to represent momentum, we can write Newton's second law as:



$$\vec F_{net}\,\Delta t = \Delta(m\vec v) = \Delta\vec p.$$

Applied to body A, the second law says:

$$(\vec F_{AB} + \vec F_{AC})\,\Delta t = \Delta\vec p_A.$$

a) Copy the last equation above and write corresponding equations for body B and body C.

b) According to Newton's third law, $\vec F_{AB} = -\vec F_{BA}$, $\vec F_{AC} = -\vec F_{CA}$ and $\vec F_{BC} = -\vec F_{CB}$. Combine these relations with the three equations of (a) to obtain $\Delta\vec p_A + \Delta\vec p_B + \Delta\vec p_C = 0$.

c) Show that the result of (b) is equivalent to $\vec p_A + \vec p_B + \vec p_C = \vec p\,'_A + \vec p\,'_B + \vec p\,'_C$.

The last equation says that the momentum of the three-body system is constant during the time interval Δt.

ANSWER:

Write the three equations using Newton's third law.

$$(\vec F_{AB} + \vec F_{AC})\,\Delta t = \vec p_A \qquad (1)$$

$$(-\vec F_{AB} + \vec F_{BC})\,\Delta t = \vec p_B \qquad (2)$$

$$(-\vec F_{CA} - \vec F_{BC})\,\Delta t = \vec p_C \qquad (3)$$

Sum (1), (2) and (3) to obtain

$$0 = \Delta\vec p_A + \Delta\vec p_B + \Delta\vec p_C \qquad (4)$$

But $\Delta\vec p_A = \vec p\,'_A - \vec p_A$; $\Delta\vec p_B = \vec p\,'_B - \vec p_B$;

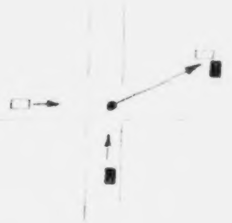
$$\Delta\vec p_C = \vec p\,'_C - \vec p_C \qquad (5)$$

Substitute (5) in (4) and transpose. Obtain

$$\vec p\,'_A + \vec p\,'_B + \vec p\,'_C = \vec p_A + \vec p_B + \vec p_C,$$

showing momentum of three-body system is conserved.

9.19 A police report of an accident describes two vehicles colliding (inelastically) at a right-angle intersection. The cars skid to a stop as shown. Suppose the masses of the cars are approximately the same. Which car was traveling faster at collision? What information would you need in order to calculate the speed of the automobiles?



ANSWER:

The car to left was traveling faster.

a) The speed of one car, or

b) Mass of the cars, the distance they tracked after the collision and the retarding or stopping force of the ground.

9.20 Two pucks on a frictionless horizontal surface are joined by a spring. Can they be considered an isolated system? How do gravitational forces exerted by the earth affect your answer? What about forces exerted on the planet earth? Do you wish to reconsider your answer above? How big would the system have to be in order to be considered completely isolated?

ANSWER:

They are isolated in the horizontal plane.

The downward gravitational attraction of the earth for the pucks is balanced by the upward force of the surface of the ice. The system, even if the earth is accelerated, may be considered isolated in the horizontal plane. To be completely isolated and to accurately define a horizontal surface you would have to include the solar system, our galaxy or perhaps the universe.

9.21 Two balls, one of which has three times the mass of the other, collide head-on, each moving with the same speed. The more massive ball stops, the other rebounds with twice its original speed. Show that both momentum and vis viva are conserved.

ANSWER:

$3\ mv - mv = 2mv$ momentum

$3mv^2 + mv^2 = 4mv^2$ vis viva

9.22 If both momentum and vis viva are conserved and a ball strikes another three times its mass at rest, what is the velocity of each ball after impact?

ANSWER:

$m\vec{v}_A = m\vec{v}\,'_A + 3m\vec{v}\,'_B$ momentum conserved

$mv_A{}^2 = mv'^2_A + 3mv'^2_B$ vis viva conserved

Cancel out m. Square the momentum equation and subtract the vis viva equation from it, obtaining

$$0 = 6\ v'_A v'_B + 6\ v'^2_B.$$

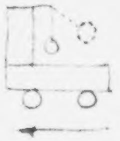
From which either $v_B = 0$, meaningless or $\vec{v}\,'_A = -\vec{v}\,'_B$, i.e., equal and opposite. From the momentum equation, $\vec{v}_A = -2v'_A$, i.e., both balls rebound with one-half the initial speed.

9.23 A student in a physics class, having learned about elastic collisions and conservation laws, decides that he can make a self-propelled car. He proposes to fix a pendulum on a cart, using a "super-ball" as a pendulum bob. He fixes a block to the cart so that when the ball reaches the bottom of the arc, it strikes the block and rebounds elastically. It gives the cart a series of bumps that propel it along.

a) Will his scheme work? (Assume that the "super-ball" is perfectly elastic.) Give reasons for your answer.

b) What would happen if the cart had an initial velocity in the forward direction?

c) What would happen if the cart had an initial velocity in the backward direction?



ANSWER:

a) No. Momentum is conserved. The center of gravity of the system remains fixed. As the ball swings forward the cart moves backward and vice versa as the ball swings back.

b) Continues to move forward with the slight oscillation described in (a).

c) Continues to move backward with the slight oscillation described in (a).

9.24 A billiard ball moving 0.8 m/sec collides with the cushion along the side of the table. The collision is head-on and perfectly elastic. What is the momentum of the ball before impact? After impact? What is the change in momentum? Is momentum conserved? (Pool sharks will say that it depends upon the "English" [spin] that the ball has, but the problem is much simpler if you neglect this condition.)

ANSWER:

.8 (mass of ball) kg-m/sec toward the cushion

.8 (mass of ball) kg-m/sec away from the cushion

1.6 (mass of ball) kg-m/sec away from the cushion

Momentum is not conserved as the cushion exerts a force on the ball.

9.25 Fill in the blanks:

OBJECT	MASS (kg)	VELOCITY (m/sec)	MOMENTUM kg-m/sec	VIS VIVA $kg\text{-}m^2/sec^2$
baseball	0.14	30.0		
hockey puck		50.0	8.55	
super-ball	0.050	1.5		
Corvette	1460			1.79×10^6
mosquito			2.0×10^{-5}	4.0×10^{-6}

ANSWER:

OBJECT	MASS (kg)	VELOCITY (m/sec)	MOMENTUM kg-m/sec	VIS VIVA $kg\text{-}m^2/sec^2$
baseball	0.14	30.0	4.2	126.0
hockey puck	.171	50.0	8.55	427.5
super-ball	0.050	1.5	0.075	0.1125
Corvette	1460	111.0	1.62×10^4	1.79×10^6
mosquito	10^{-4}	0.2	2.0×10^{-5}	4.0×10^{-6}

9.26 You have been given a precise technical definition for the word momentum. Look it up in the dictionary and record its various uses. Can you find anything similar to our definition in these more general meanings? Why was the word momentum selected as the name for quantity of motion?

ANSWER:

Webster's International Dictionary gives 3 for momentum and 8 for moment. Derivation from the Latin movere, to move, and suffix ment which is a general term for movement, attachment, concrete result, action, process, state or condition. Used in sense of "matter of great moment" i.e., importance of matter in motion.

9.27 Descartes defined the quantity of motion of an object as the product of its mass and its speed. Is his quantity of motion conserved as he believed it was? If not, how would you modify his definition so it would be conserved?

ANSWER:

No. Change "speed" to velocity. Momentum is a vector quantity.

Solutions for Chapter 10 Study Guide

10.1 An electron of mass 9.1×10^{-31} kg is traveling 2×10^6 meters per second toward the screen of a television set. What is its kinetic energy? How many electrons like this one would it take to make up a joule of energy?

ANSWER:

$$KE = mv^2/2 = 9.1 \times 10^{-31} \times (2 \times 10^6)^2/2$$

$$= 1.82 \times 10^{-18} \text{ joules}$$

$$1/1.82 \times 10^{-18} = 5.5 \times 10^{17} \text{ electrons}$$

10.2 Estimate the kinetic energy of each of the following:

a) a pitched baseball

b) a jet plane

c) a sprinter in a 100-yard dash

d) the earth in its motion around the sun

ANSWER:

Estimates may have a wide range.

	m	v	v^2	$KE = mv^2/2$	
a)	.15 kg	30 m/sec	900	67.5	joules
b)	10^5 kg	300 m/sec	9×10^4	4.5×10^9	joules
c)	75 kg	10 m/sec	10^2	3750	joules
d)	6×10^{24}	3×10^4 m/sec	9×10^8	2.7×10^{33}	joules

10.3 As a home experiment, hang weights on a rubber band and measure its elongation. Plot the force vs. stretch on graph paper. How could you measure the stored energy?

ANSWER:

Determine the value of a square of the graph paper in joules. It is the product of the value of one side in newtons and the other side in meters. Count the squares under the curve and multiply by value of a square in joules.

10.4 A penny has a mass of about 3.0 grams and is about 1.5 millimeters thick. You have 50 pennies which you pile one above the other.

a) How much more gravitational potential energy has the top penny than the bottom one?

b) How much more have all 50 pennies together than the bottom one?

ANSWER:

Height of top penny 1.5 × 50 = 75 millimeters. Height of center of group 1.5 × 25 = 37.5 millimeters

$PE = mgh = .003 \times 10 \times .075 = .00225$ joules of top.
$PE = 50 \times .003 \times 10 \times .0375 = .056$ joules of pile.

10.5 Discuss the following statement: all the chemical energy of the gasoline used in your family automobile is used only to heat up the car, the road and the air.

ANSWER:

You have an isolated system. Unless you pick up speed or slow down and therefore gain or lose kinetic energy, the energy of the system is conserved.

10.6 A 200-kilogram iceboat is supported by a smooth surface of a frozen lake. The wind exerts on the boat a constant force of 1000 newtons, while the boat moves 900 meters. Assume that frictional forces are negligible, and that the boat starts from rest. Find the speed after 900 meters by each of the following methods:

a) Use Newton's second law to find the acceleration of the boat. How long does it take to move 900 meters? How fast will it be moving then?

b) Find the final speed of the boat by equating the work done on it by the wind and the increase in its kinetic energy. Compare your result with your answer in (a).

ANSWER:

a) $a = F/m = 1000/200 = 5 \text{ m/sec}^2$

$\bar{v} = at/2 = d/t$ or $d = at^2/2$

$t^2 = 2 \times 900/5 = 360$

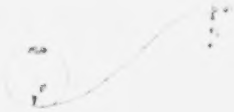
$t = 19$ sec

$v = at = 5 \times 19 = 95.$ m/sec

b) work = Fd = $mv^2/2$

$v^2 = 2 \times 1000 \times 900/200 = 9000$

$v = 95$ m/sec



10.7 The figure shows a model of a carnival "loop-the-loop." A car starting from a platform above the top of the loop coasts down and around the loop without falling off the track. Show that, to successfully traverse the loop, the car need start no higher than one-half a radius above the top of the loop. Neglect frictional forces.

HINT: The centripetal force required at the top of the loop must be greater than the weight of the car.

ANSWER:

$KE = PE = mv^2/2 = mgh$ or $h = v^2/2g$

$F = mg = mv^2/r$ or $v^2 = gr$

Hence $h = gr/2g = r/2$

10.8 A cardboard tube a meter long closed at both ends contains some lead shot. It is turned end for end 300 times, each time allowing the shot to fall the length of the tube and come to rest before inverting it again. If the shot loses a negligible amount of heat to the tube, what is the increase in temperature? Try it.

ANSWER:

Mass for mass, lead takes 3 per cent of the heat that water takes to raise it a given temperature.

$PE = mgh \times 300 = m \times 10 \times 1 \times 300$ m joules

3000 m joules $= \frac{3000}{4184}$ m = .72 mkg cal

$.72^\circ$ if m is water or 24° if lead.

10.9 An electric coffee pot holds a kilogram (about a quart) of water and is rated at 600 watts. Starting from room temperature (20° C), estimate how long it will take the water to reach boiling temperature.

ANSWER:

$(100-20) \times 4184 = 600\ t$

t = 558 sec = 9.3 min., assuming only a small fraction of heat warms pot and surroundings.

10.10 Show that if a constant propelling force F keeps a vehicle moving at a constant speed v, the power required is equal to Fv.

ANSWER:

Power = work/t = Fd/t = Fv

10.11 The Queen Mary, one of Britain's largest steamships, has been retired to a marine museum on our west coast after completing 1,000 crossings of the Atlantic. Her mass is 81,000 tons (75 million kilograms) and her maximum engine power of 234,000 horsepower (174 million watts) gives her a maximum speed of 30.63 knots (16 meters per second).

a) What is her kinetic energy at full speed?

b) What constant force would be required to stop her from full speed within 10 nautical miles (20,000 meters)?

c) What power would be required to keep her going at full speed against this force?

d) Assume that at maximum speed all the power output of her engines goes into overcoming water drag. If the engines are suddenly stopped, how far will the ship coast before stopping? (Assume water drag is constant.)

e) The assumptions made in (d) are not valid for the following reasons:

1) Only about 60% of the power delivered to the propellor shafts results in a forward thrust to the ship; the rest results in turbulence, eventually warming the water.

2) Water drag is less for lower speed.

3) If the propellors are not free-wheeling, they add an increased drag. Which of the above reasons tend to increase, which to decrease the coasting distance?

f) Explain why tugboats are important for docking big ships.

ANSWER:

a) $75 \times 10^6 \times (16)^2/2 = 9.6 \times 10^9$ joules

b) 4.8×10^5 newtons

c) 7.68×10^6 watts

d) F = power/v

$$d = KE/F = \frac{9.6 \times 10^9 \times 16}{1.74 \times 10^8} = 880 \text{ meters}$$

e) (1) and (2) to increase; (3) to decrease

f) To change direction of ship after she has lost steering way.

10.12 Consider the following hypothetical values for a paddle wheel experiment like Joule's: a 1 kilogram weight descends through a distance of 1 meter, turning a paddle wheel immersed in 5 kilograms of water.

a) About how many times must the weight be allowed to fall in order that the temperature of the water will be increased by 1/2 Celsius degree?

b) List ways you could modify the experiment so that the same temperature rise would be produced with fewer falls of the weight? (There are at least four possible ways.)

ANSWER:

a) Needed 4184 × 5/2 = 10460 joules or 1046 times.

b) 1) increase the weight

2) decrease the amount of water

3) increase height of fall

4) use a liquid with smaller specific heat

10.13 On his honeymoon in Switzerland, Joule attempted to measure the difference in temperature between the top and the bottom of a waterfall. Assuming that the amount of heat produced at the bottom is equal to the decrease in gravitational potential energy, calculate roughly the temperature difference you would expect to observe between the top and bottom of a waterfall about 50 meters high, such as Niagara Falls.

ANSWER:

$mgh = m\Delta t \, 4184$

$\Delta t = 10 \times 50/4184 = 0.12^\circ$ C

10.14 Devise an experiment to measure the power output of

a) a man riding a bicycle

b) a motorcycle

c) an electric motor

ANSWER:

Support each so that a pulley attached to the rotating part may lift a weight at the end of a rope. Divide the work done by the time. Another method would be to ride up and down a hill noting the speed in each case. Assume power and friction the same in each case. With the greater speed the component of the weight along the road aids up the hill it opposes. A brake horsepower test consists of a brake band pressed against a pulley on the motor shaft so that the difference of tensions on the two sides of the band and the speed at which the pulley slips over the band gives the power.

10.15 When a person's food intake supplies less energy than he uses, he starts "burning" his own stored fat for energy. The oxidation of a pound of animal fat provides about 4,300 kilocalories of energy. Suppose that on your present diet of 4,000 kilocalories a day you neither gain nor lose weight. If you cut your diet to 3,000 kilocalories and maintain your present physical activity, about how many weeks would it take to reduce your mass by 5 pounds?

ANSWER:

Loss required/amount per week =

$$\frac{4300 \times 5}{1000 \times 7} = 3 \text{ weeks}$$

10.16 About how many kilograms of boiled potatoes would you have to eat to supply the energy for a half-hour of swimming? Assume that your body is 20% efficient.

ANSWER:

Energy required 9 × 30 = 270 kg cal

Food × 20% 890 × 0.2 = 178 cal

270/178 = 1.5 kilograms

10.17 In order to engage in normal light work, an average native of India needs about 1,950 kilocalories of food energy a day, whereas an average West European needs about 3,000 kilocalories a day. Explain how each of the following statements makes the difference in energy need understandable.

a) The average adult Indian weighs about 110 pounds; the average adult West European weighs about 150 pounds.

b) India has a warm climate.

c) The birthrate in India is higher than in Western Europe.

ANSWER:

The average food needs are roughly proportional to average body weights. The slight discrepancy could easily be explained by the warmer climate. The birthrate has nothing to do with individual food needs, but is probably the cause of lack of food.

10.18 Show how the laws of the conservation of energy and momentum apply to a rocket lifting off.

ANSWER:

The upward momentum of the rocket equals the downward momentum of the exhaust gases. The chemical energy of the rocket's fuel is accounted for by the gain in kinetic energy of the rocket plus the heat energy generated and the sound and light energy radiated.

10.19 In each of the following, trace the chain of energy transformations from the sun to the final form of energy:

a) A pot of water is boiled on an electric stove.

b) An automobile accelerates from rest on a level road, climbs a hill at constant speed and comes to stop at a traffic light.

c) A windmill in Holland pumps water out of a flooded field.

ANSWER:

a) Assume hydroelectric plant. Sun's radiation evaporates water which is condensed in upland lakes. Water gives its potential energy to hydraulic turbines which rotate generators of electric power. Electric current in resistances is converted to heat which boils water.

b) Radiation from sun was stored in plants which, after millennia, became fossil fuels such as gasoline, which in the automobile gives mechanical energy, which increases kinetic energy, given potential energy as it climbs a hill and the remaining kinetic energy becomes heat as the car is braked to a stop.

c) Radiation from sun causes temperature and pressure differences in atmosphere which give rise to winds. These in turn give kinetic energy to windmills which is converted to potential energy as water is lifted.

10.20 The two identical space vehicles shown here were drifting through interstellar space. Each was struck by a 10-kilogram meteor traveling at 100 m/sec. Which rocket was knocked further off course? Explain.

ANSWER:

Conservation of momentum demands that the larger the change in momentum of the incident meteor, the larger will be the change in momentum of the object which is struck. Since momentum is a vector quantity, the meteor which bounces undergoes the greater momentum change. Hence, the undamaged rocket will experience a larger change also.

10.21 A 2-gram bullet is shot into a tree stump. It enters at a speed of 300 m/sec and comes to rest after having penetrated 5 cm in a straight line. Compute the average force on the bullet during the impact, and the work done.

ANSWER:

Equate kinetic energy of bullet to work done.

$$KE = mv^2/2 = 0.002\ (300)^2/2 = 90 \text{ joules}$$

$$F = 90/.05 = 1800 \text{ newtons}$$

$$\text{Work done} = 1800 \times 0.05 = 90 \text{ joules}$$

10.22 In the prologue to Unit 1 of this course, it was stated that Fermi used materials containing hydrogen to slow down neutrons. Explain why collisions with light atoms would be more effective in slowing down neutrons than collisions with heavy atoms.

ANSWER:

A neutron hitting elastically a hydrogen nucleus of equal mass gives up its momentum and the proton picks it all up. For collision with heavy nuclei the neutron bounces back.

The greatest loss of kinetic energy occurs when the mass of the target atom is the same as the mass of the incident neutron. If the collision is head-on, the incident neutron would stop in such a case. A hydrogen atom has a mass almost precisely equal to that of a neutron, and therefore materials containing hydrogen are effective ones for slowing neutrons.

If the target atoms are much heavier than the incident neutrons, the neutrons would undergo numerous changes in direction but little energy loss.

10.23 The actual cost of moving furniture and individuals various distances is shown in the following tables. Using these tables, discuss the statement, "the cost of moving is approximately proportional to the amount of work that has to be done on it, using the physicist's definition of work."

Truck Transportation (1965)

Weight	Moving rates (including pickup and delivery) from Boston to:		
	Chicago 967 miles	Denver 1969 miles	Los Angeles 2994 miles
100 lbs	$ 18.40	$ 24.00	$ 27.25
500	92.00	120.00	136.25
1000	128.50	185.50	220.50
2000	225.00	336.00	406.00
4000	384.00	606.00	748.00
6000	576.00	909.00	1122.00

Air Cargo Transportation (1965)

Weight	Moving rates (including pickup and delivery) from Boston to:		
	Chicago	Denver	Los Angeles
100 lbs	$ 13.95	$ 25.57	$ 29.85
500	70.00	127.50	149.25
1000	129.00	216.50	283.50
2000	248.00	413.00	527.00
4000	480.00	796.00	1024.00
6000	703.00	1164.00	1255.00

Personal Transportation (1965)

	One way fare from Boston to:		
	Chicago	Denver	Los Angeles
Bus	$33.75	$ 58.40	$ 88.75
Train	47.85	79.31	115.24
Airplane (jet coach)	53.39	107.36	167.58

ANSWER:

The distances in the table are in the ratios 1:2:3. Moving costs do not show those ratios. So the costs do not follow the work principle. In a single column where the distance is constant, costs should be proportional to weights. For truck transportation the law holds for the first 500 pounds, after that it fails. For air cargo it holds except for weights above 4000 pounds. For jet coach the distance principle holds but there is no discrimination against a 250 lb man over a 125 lb woman.

Solutions for Chapter 11 Study Guide

11.1 List some of the directly observable properties of gases.

ANSWER:

Pressure, density, temperature, viscosity.

11.2 What could kinetic theorists explain about gases?

ANSWER:

Relations between observable properties.

Predict new relations.

Infer sizes and speeds of molecules.

11.3 Where did Newtonian mechanics run into difficulties in explaining the behavior of molecules?

ANSWER:

Explaining motions of atoms inside molecules and the radiations given off by atoms, and apparent irreversibility of processes.

11.4 Distinguish between two uses of the word "model" in science.

ANSWER:

A working model which copies the real thing on a reduced scale; a theoretical model which idealizes the actual situation to facilitate mathematical analysis.

11.5 Randomness can be used in predicting the results of flipping a large number of coins. Give some other examples where randomness is useful.

ANSWER:

Conducting political polls, t.v. ratings, and gambling casinos.

11.6 The speed of sound in a gas is about the same as the average speed of the gas molecules. Is this a coincidence? Discuss.

ANSWER:

No sound is transmitted by the forward and back motion of the molecules.

11.7 Consider the curves showing Maxwell's distribution of molecular speeds.

a) All show a peak.

b) The peaks move towards higher speed at higher temperatures.

c) They are not symmetrical like normal distribution curves.

Explain these characteristics on the basis of the kinetic model.

ANSWER:

a) There is a most probable speed for the molecules.

b) Average speed is proportional to absolute temperature.

c) No molecules can have negative speeds.

11.8 Many products are now sold in spray cans. Explain in terms of the kinetic theory of gases why it is dangerous to expose the cans to high temperatures.

ANSWER:

Much of the material in the can is in the gaseous phase under pressure. The kinetic theory shows that increase in temperature means increase in speed of molecules and hence an increase in pressure. The can may burst!

11.9 Benjamin Franklin in 1765 observed that not more than a teaspoonful of oil covered half an acre of a pond. Suppose that one cubic centimeter of oil forms a continuous layer one molecule thick that just covers an area on water of 1000 square meters.

a) How thick is the layer?

b) What is the diameter of a single molecule of oil?

ANSWER:

a) $V = hA$; $h = V/A = 10^{-6}\text{m}^3/1000\text{m}^2$

$= 10^{-9}$meters. (Volume V of oil equals area A in contact with water times thickness h of layer.)

b) Thickness of layer, namely 10^{-9} meters as layer is one molecule thick.

11.10 How did Clausius modify the simple "model" for a gas? What was this new model able to explain?

ANSWER:

By giving the "particle" of the simple theory an appreciable size. It explained the "mean free path" and the constancy of viscosity with density change.

11.11 How did Josef Loschmidt estimate the size of a molecule?

ANSWER:

By assuming the molecules closely packed in a liquid, he saw that the $V = Nd^3$ and then using the value for the viscosity of a gas which employs Nd^2, he was able to compute both N and d.

11.12 The atmospheric pressure of air is balanced by a column of mercury of height 0.76 meters or by 10.5 meters of water. Air is approximately a thousand times less dense than water. Why can you not say the atmosphere is only 10,000 meters deep?

ANSWER:

The simple pressure-height relation requires constancy of density. The density of air falls off with height.

11.13 The spike heel on a girl's shoe is a square, one centimeter on an edge. If her mass is 50 kilograms, how many atmospheres of pressure are exerted when she balances on one heel?

ANSWER:

$p = F/A = mg_a/A = (50 \times 10)/10^{-4}$

$= 5 \times 10^6\ \text{N/m}^2 = 50$ atmospheres.

If a light particle rebounds from a massive, stationary piston with almost no loss of speed, then, according to the principle of Galilean relativity, it would still do so from a moving piston in the frame of reference of the moving piston. Show that the rebound speed as measured in the laboratory would be less from a retreating piston, as is claimed at the top of p. 95. (Hint: Express the speed of the particle relative to the piston in terms of their speeds in the laboratory frame.)

ANSWER:

The speed of the particle relative to the piston is the same before and after the collision. In the laboratory frame this would be expressed as follows: If u is the speed of the piston and v is the speed of the particle before collision, then the relative speed before collision is $v - u$. If v' is the speed of the particle after collision, then the relative speed after collision is $v' + u$. Since the relative speeds are equal, $v - u = v' + u$. Therefore $v' = v - 2u$; the speed after collision is less than the speed before by 2u.

11.15 Clausius' statement of the second law of thermodynamics is: "Heat will not of its own accord pass from a cooler to a hotter body." Show in words how a refrigerator can operate.

ANSWER:

Heat is driven from the cold inside to the hot outside by the organized energy of an electric motor. It does not pass of its own accord.

11.16 When a gas is compressed by pushing in a piston, its temperature increases. Explain this fact in two ways:

a) By using the first law of thermodynamics, and

b) By using the kinetic theory of gases.

ANSWER:

Work is done on the gas by the piston. By the first law of thermodynamics the internal energy of the gas must increase and so the temperature rises. From the kinetic theory point of view a molecule striking the moving piston has its momentum and hence its speed increased. The resulting increase in mean kinetic energy of the molecules gives a higher temperature.

11.17 Why, if there is a tendency for heat to flow from hot to cold, will not the universe eventually reach absolute zero?

ANSWER:

Eventually all temperatures will become equalized above the absolute zero. Energy cannot be destroyed.

11.18 How did Maxwell's demon hope to circumvent the second law of thermodynamics?

ANSWER:

He hoped to hold the speedier molecules in the colder gas and the slower ones in the hotter.

11.19

a) Explain what is meant by the statement that Newton's laws of motion are time-reversible.

b) Describe how a paradox arises when the time reversibility of Newton's laws of motion is combined with the second law of thermodynamics.

ANSWER:

a) Newton used a mathematical time. His laws are equations of the first degree (linear) in time. Reverse the sign of the time and the process runs backwards. Every purely mechanical process is reversible.

b) When heat and temperature are introduced, the law of the increase of entropy with time gives time a definite direction and processes no longer are reversible.

11.20 Since molecular motions are random, one might expect that any given arrangement of molecules would recur if he waited long enough. Explain how a paradox arises when this prediction is combined with the second law of thermodynamics.

ANSWER:

The paradox lies in what is meant by "long enough." The chances of return to a former state are very slight on account of the large number of molecules. The second law, on the other hand, takes an overall point of view and says it never happens.

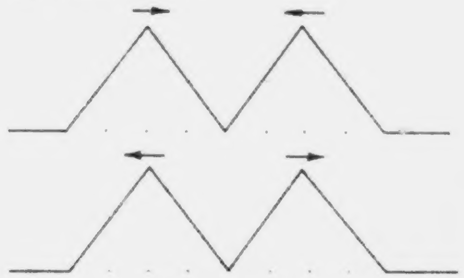
11.21 Many philosophical and religious systems of the Far East and the Middle East include the ideas of eternal return and resurrection. Read about some of these philosophies and discuss them in the light of your knowledge of the second law of thermodynamics.

ANSWER:

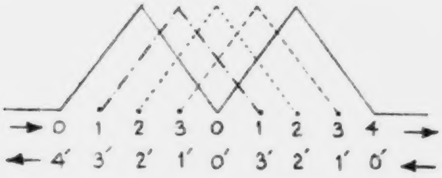
In discussing this as a problem for a physicist, one must take an anthropomorphic or materialistic view of religion. Population increase precludes a cyclical process for individuals. The number of individuals, although numbered in terms of billions, is infitisimal compared with billions of billions of molecules. You probably contain within your body, a few molecules of Julius Caesar!

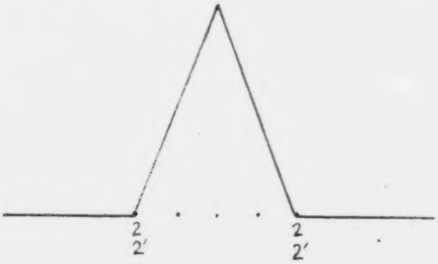
Solutions for Chapter 12 Study Guide

12.1 Pictured are two idealized rope waves at the instants before and after they overlap. Divide the elapsed time into four equal intervals and plot the shape of the rope at the end of each interval.



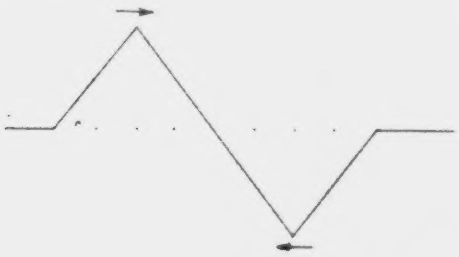
ANSWER:



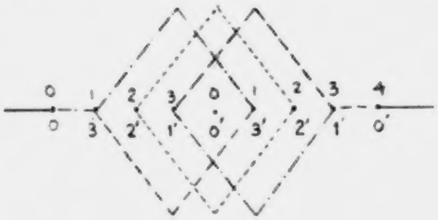


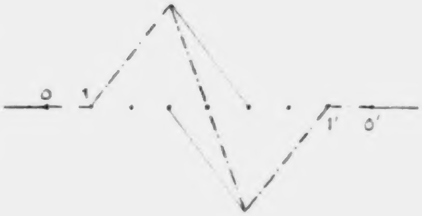
The shape at the end of the first and third intervals are the same, a truncated triangle; at the central time a triangle of twice the height. Use the principle of superposition.

12.2 Repeat Exercise 12.1 for the two waves pictured below.



ANSWER:





The wave shapes at end of first and third intervals are reflections of each other. Waves cancel at the central time.

12.3

The diagram shows two successive wave fronts meeting an air-glass boundary.

a) Label an angle equal to angle of incidence θ_A.

b) Label an angle equal to angle of refraction θ_B.

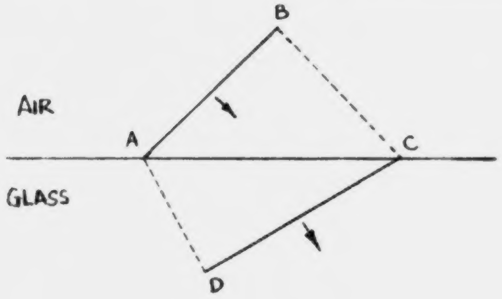
c) Label the wavelength in air λ_A.

d) Label the wavelength in glass λ_B.

e) Show that $\sin \theta_A / \sin \theta_B = \lambda_A / \lambda_B$.

f) Show that $V_A / V_B = \lambda_A / \lambda_B$.

The need for a trigonometric relation was first stated by Willebrord Snell (1591-1626) of the University of Leyden in Holland in an unpublished paper. If you are familiar with trigonometry try (e).



ANSWER:

a) BAC = θ_A

b) ACD = θ_B

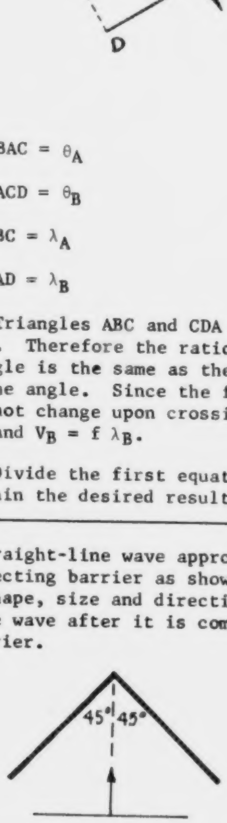
c) BC = λ_A

d) AD = λ_B

e) Triangles ABC and CDA have the common hypotenuse. Therefore the ratio of sides opposite an angle is the same as the ratio of the sines of the angle. Since the frequency of the wave does not change upon crossing the boundary, $V_A = f\,\lambda_A$ and $V_B = f\,\lambda_B$.

f) Divide the first equation by the second to obtain the desired result.

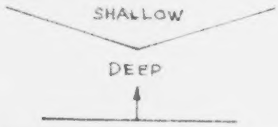
12.4 A straight-line wave approaches a right-angle reflecting barrier as shown in the figure. Find the shape, size and direction of propagation of the wave after it is completely reflected by the barrier.



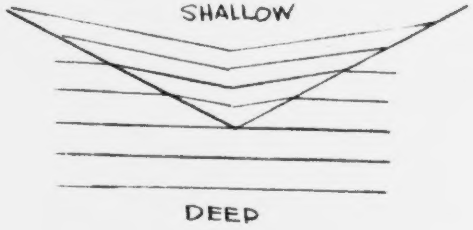
ANSWER:

The wave should return as a straight-line wave that goes in just the direction opposite to that along which it started.

12.5 A straight-line wave in a ripple tank approaches a boundary between deep and shallow water as shown. Describe the shape of the wave as it passes through the boundary and then as it continues in the shallow water.



ANSWER:



It splits into two straight-line waves which are inclined toward each other and are propagated more slowly.

This is a simple example of the way that focussing can be accomplished by using refraction. Eager students might be asked to find the "focal point" in the shallow water.

12.6 A periodic ripple-tank wave passes through a straight boundary between deep and shallow water. The angle of refraction is 30°. The propagation speed in the deep water is 0.35 m/sec, and the frequency of the wave is 10 cycles/sec. Find the wavelengths in the deep and shallow water.

ANSWER:

We can use

$$\lambda f = v$$

to find the wavelength λ_1 in the deep water

$$(\lambda_1)\ (10) = 0.35$$

or $\lambda_1 = .035$ m. Now use

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2}$$

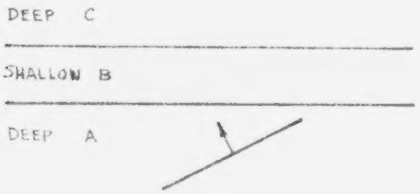
to find the wavelength λ_2 in shallow water, i.e.,

$$\frac{1/\sqrt{2}}{1/2} = \frac{.035}{\lambda_2}$$

or

$$\lambda_2 = (.035)/\sqrt{2} \approx .025\text{m}.$$

12.7 A straight-line ripple-tank wave approaches a narrow region B of shallow water as shown. Prove that the crest line of the wave when in region C is parallel to the crest line shown in region A when regions A and C have the same water depth.

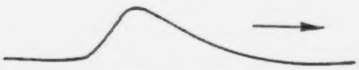


ANSWER:

The angle of refraction in B from A equals the angle of incidence from B into C as the sides of B are parallel. $V_A/V_B = \sin\theta_A/\sin\theta_B$ and $V_B/V_C = \sin\theta_B/\sin\theta_C$. Multiply the two equations together obtaining $V_A/V_C = \sin\theta_A/\sin\theta_C$.

But $V_A = V_C$; hence $\sin\theta_A = \sin\theta_C$, or $\theta_A = \theta_C$, and there is no net bending of wave crest.

12.8 The wave below propagates to the right along a rope. What is the shape of the wave propagating to the left that could for an instant cancel this one completely?



ANSWER:

The reflection of this wave form in the undisturbed rope.

12.9 The velocity of a portion of rope at some instant is the superposition of the velocities of waves passing through that portion. Is the kinetic energy of a portion of the rope the superposition of the kinetic energies of waves passing through that region?

ANSWER:

No. Consider a velocity node for example. Kinetic energy does not obey the superposition principle. There are two reasons why KE does not obey the superposition principle. First, the mass of the medium in motion may change as two waves superpose. Second, while the velocities superpose (i.e.,

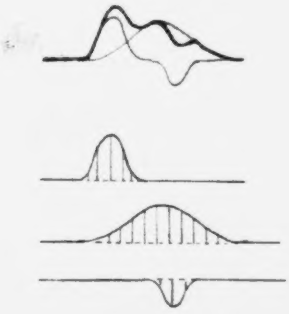
$$\vec{v}_1 + \vec{v}_2$$

_is_ the velocity of the superposed wave), the square of the speeds does not superpose (i.e.,

$$v_1^2 + v_2^2 \neq (v_1 + v_2)^2).$$

12.10 Trace the last three curves of Fig. 12.12 and add them graphically to obtain the original curve.

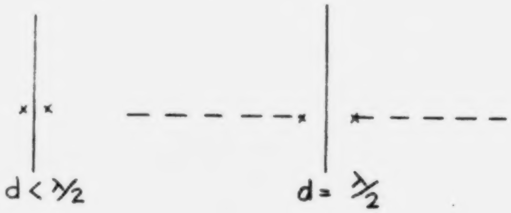
ANSWER:



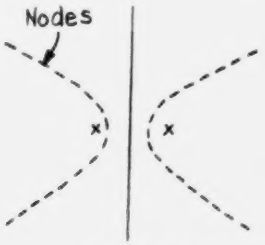
Eight amplitudes were combined to give proper wave form.

12.11 What kind of interference pattern would you expect to see if the separation between two in-phase sources were less than the wavelength λ? Where would the nodal and antinodal lines be if the two in-phase sources were separated by the distance λ? By λ/2? Convince yourself that one additional nodal line appears on each side of the central antinodal line whenever the separation between the two in-phase sources is increased by one wave-length.

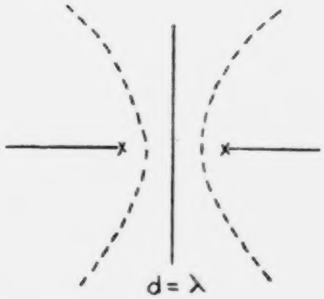
ANSWER:



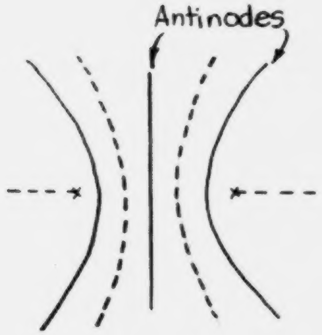
The perpendicular bisector of the line joining the sources is always an antinode because the path difference to each source is zero. For a separation less than λ, this is the only antinodal line. For a separation equal to λ, other antinodal lines are the continuations of the line of sources beyond the line joining them. On them the path difference is just λ.



$\lambda > d > \lambda/2$



$d = \lambda$



$d = 3\lambda/2$

For separation of sources less than λ/2 there are no nodal lines. For a separation equal to λ/2 the nodal line is the same as the additional antinodal line for a separation λ. Along these lines the path difference is λ/2. For a separation equal to λ, there are two nodal lines of the usual form which meet the line between sources at points equidistant from source and midpoint. The series of sketches in which dotted lines are nodes and solid lines antinodes should convince you how nodal lines are born as the distance between sources is increased.

12.12 Estimate the wavelength of a 1000-cycles-per-second sound wave in air, in water, and in steel (refer to data in text). Do the same if f = 10,000 cps. Design the dimensions of an experiment to show two-source interference for 1000 cps sound waves.

ANSWER:

The text (page 142) gives the speed of sound waves in air as 1,125 feet per second, in sea water as 4800 feet per second and in steel as 16,000 feet per second. The relation between frequency, speed and wavelength is $v = \lambda f$ where v is the speed, λ the wavelength and f the frequency. Solving for λ we obtain $\lambda = v/f$. Thus for air $\lambda = 1125/1000 = 1.125$ ft for sea water $\lambda = 4.8$ ft and for steel $\lambda = 16$ ft. At the higher frequency the wavelengths are one tenth of these, namely 0.1125 ft, 0.48 ft and 1.6 ft, respectively. Place the sound source at one end of a closed, well padded box about 6 feet long. At the other end cut two vertical slits about eight inches long and half an inch wide. Examine the region outside this end of the box with a microphone, connected to an amplifier. The output of the amplifier is fed into a cathode-ray oscilloscope. Movement of the microphone will show interference fringes.

12.13 If you were to begin to disturb a stretched rubber hose or slinky with a frequency that precisely matched a standing wave frequency, would standing waves appear immediately? If not, what factors would determine the time delay?

ANSWER:

No. (a) You would have to wait for the reflected wave to return. (b) If the wave were appreciably damped, you would have to wait until a final steady amplitude is reached. Then the power supplied equals that dissipated.

12.14 A megaphone directs sound along the megaphone axis if the wavelength of the sound is small compared to the diameter of the opening. Estimate the upper limit of frequencies which are diffracted at a cheerleader's megaphone opening. Can you hear what a cheerleader shouts even though you are far off the axis of his or her megaphone?

ANSWER:

Sound of frequencies might vary between 100 and 1000 with wavelengths from 10 to 1 foot. None are small compared with the opening of the megaphone. You can hear the megaphone well off the axis. The quality of the sound will be altered because the higher frequencies will be less diffracted.

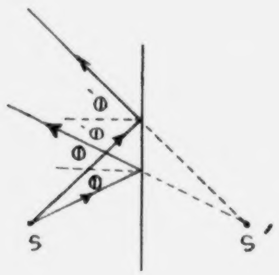
12.15 Suppose that straight-line ripple waves approach a thin straight barrier which is a few wavelengths long and which is oriented with its length parallel to the wavefronts. What do you predict about the nature of the diffraction pattern along a straight line behind the barrier which is perpendicular to the barrier and passes through the center of the barrier? Why do people who design breakwaters need to concern themselves with diffraction effects?

ANSWER:

There will be a maximum of disturbance along this line fed by diffracted waves from both ends. Long ocean swells are diffracted more than short choppy waves. The amplitude along this line may be quite large. The energy developed there could cause destruction of boats, docks, etc.

12.16 By actual construction with ruler and compass show that the reflected rays in Fig. 12.25 appear to be rays originating at S'.

ANSWER:



Two rays coming from S are sufficient to locate S' using the law of regular reflection.

12.17 Sketch the "image" wave for the wave shown in each of the sketches in Fig. 12.26(a). What relationship exists between the incident image wave and the real reflected wave?

ANSWER:

The "image" wave is a continuation of the real reflected wave back behind the mirror. They have the same direction and speed of propagation.

12.18 With ruler and compass reproduce Fig. 12.27(b) for yourself and find the distance from the circle's center to the point P in terms of the radius R of the circle. Make the radius of your circle much larger than the one in the figure.

ANSWER:

The distance from the center of the circle to the point P is about R/2. This is just a cross section of a spherical mirror, for which we get approximate focusing at R/2.

12.19 Convince yourself that a parabolic reflector will actually bring parallel wavefronts to a sharp focus. Draw a parabola and some parallel rays along the axis as in Fig. 12.27(c).

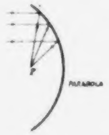


Fig. 12.27 (c)

Construct line segments perpendicular to the parabola where the rays hit it, and draw the reflected rays at equal angles on the other side of these lines.

ANSWER:

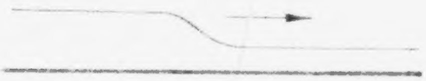
Plot the equation

$$y^2 = 4x$$

to get a parabola. A careful construction gives the evidence.

Another solution is to draw the reflected rays through the focus F(1,2). Bisect the angle between incident and reflected ray and see that the bisector is perpendicular to the parabola.

12.20 Recalling that water surface waves travel slower in shallow water, what would you expect to happen to the shape of the following wave as it continues to the right? Pay particular attention to the region of varying depth.



Can you use the line of reasoning used above to give at least a partial explanation of the cause of breakers near a beach?

ANSWER:

The wave front will steepen as it moves forward more slowly. On a sloping beach the depths change rapidly and the wave front becomes so steep it "topples over" or breaks. The propagation speed in deep water is greater than it is in shallow water. Thus, the left side of the sloping water level continues to catch up with the shallower right side, making the slope steeper and steeper. Finally, the higher part of the wave will have a greater speed than the lower part while the slope is exceedingly steep. Then the upper part will just "spill over" and result in a breaker.

12.21 Look at Fig. 12.31. Convince yourself that if a wave in medium 2 were to approach the boundary along the ray labeled "refracted ray" in the figure, it would be refracted so as to travel along the ray labeled "incident ray" in medium 1. This is another example of the general rule: if a wave follows a set of rays in one direction, then a wave can follow the same set of rays in the opposite direction.

ANSWER:

The refraction depends upon the ratio of the relative speeds in the two media. If the media are interchanged, so are the speeds and, from Snell's law, so are the angles of incidence and refraction.

12.22 Can you explain how sound waves are being used to map the floors of oceans?

ANSWER:

The speed in water is known. The time interval between the sending of a signal and the reception of its echo enables one to compute twice the depth. The process is automatic using magnetostriction oscillators and cathode-ray tube display.

12.23 When a sound source passes us the pitch we hear goes from high to low, whether it be a car horn, a train whistle, or a racing car motor. Why is that?

ANSWER:

Coming toward us the waves are jammed together, shortening the wave length. Receding from us the waves are stretched out, lengthening the waves. Since the speed of propagation is constant, there is a drop in frequency or pitch.

12.24 Directed reflections of waves from an object occur only when the wavelength is small compared to the dimensions of the object. This is true for sound waves as well as for any other. What does this tell you about the sound frequencies a bat must generate if it is to catch a moth or fly? Actually some bats can detect the presence of a wire about 0.12 mm in diameter. What frequency does that require?

ANSWER:

If we assume that the moth is approximately a centimeter long and the velocity of sound 300 m/sec, then from $v = f\lambda$,

$$f = \frac{300}{0.01} = 3 \times 10^4 \text{ cycles per sec.}$$

For the wire,

$$f = \frac{300}{0.00012} = 2.5 \times 10^6 \text{ cycles per sec.}$$

This is a very high frequency. It is doubtful if the bat "sees" the wire. He avoids the wire by a sort of sonar system and estimates its distance.

12.25 Suppose you can barely hear in an extremely quiet room a buzzing mosquito at a distance of one meter. What is the sound power output of the mosquito? How many mosquitos would it take to supply the power for one 100-watt reading lamp? If the swarm were ten meters from your ear, what would the sound be like?

ANSWER:

The least audible sound above no background noise is 10^{-16} watts per cm^2. The sound from the mosquito is spread over the surface of a sphere one meter in radius. $A = 4\pi r^2 = 1.27 \times 10^5$ cm^2. The power of the mosquito is 1.27×10^{-11} watts. It would take $100/1.27 \times 10^{-11} = 8 \times 10^{12}$ mosquitoes. From the inverse square law the power would be 1 watt/cm^2 or 10^{16} times the least audible sound. The swarm would sound like a wind-tunnel test facility.

Summary of Chapter 9

The law of conservation of mass expresses accurately the ancient conviction that the amount of matter in the universe does not change. A sound experimental basis for this belief was provided in the eighteenth century by the experiments of Lavoisier. It has since been demonstrated with great precision that the mass of any closed system remains always the same.

Seventeenth-century philosophers believed also that the "quantity of motion" in the universe remained always the same; the problem was to formulate a definition of "quantity of motion" such that it would be conserved. Descartes proposed the product of mass and speed as the correct measure of quantity of motion. By considering collisions between two carts, however, it is seen that Descartes was wrong: it is the product of mass and velocity which is conserved. This vector quantity is called "momentum."

The law of conservation of momentum is perfectly general. The resultant momentum of any number of bodies exerting any kind of forces on one another is conserved, so long as the net force on the entire system of bodies is zero.

The law of conservation of momentum is a logical consequence of Newton's second and third laws of motion. Yet it allows us to solve problems which could not be solved by application of Newton's laws directly. In particular, use of the law of conservation of momentum enables one to find out about the motion of interacting objects even when the details of the forces of interaction are unknown.

A system on which the net force is zero (and the momentum of which therefore is conserved) is called an isolated system. It is nearly impossible to arrange a system which is truly isolated; many systems can be regarded as very nearly isolated, however, and the law of conservation of momentum applied to them with little error.

A description of a demonstration witnessed by the Royal Society in 1666 in which two hard pendulum bobs repeatedly collided with one another leads to the introduction of another quantity which Huygens showed to be conserved in collisions between very hard objects (perfectly elastic collisions). It was called vis viva. The German philosopher Leibniz was convinced that vis viva was always conserved, even in collisions between soft objects (inelastic collisions). Just as Descartes' "quantity of motion" mv developed into the modern concept of momentum, mv, so vis viva is closely related to the modern kinetic energy.

Sec. 9.1: Conservation of Mass

The main point of Sec. 9.1 is the faith that natural philosophers had in conservation of mass, even in the absence of good experimental verification. The work of Lavoisier which finally gave acceptable experimental support to the conservation of mass was not a source of surprise or even of much information; it was rather somewhat a source of relief— an end to the vague, lingering worry about having taken it for granted. (This situation can be recalled in Unit 6, where similar attitudes are discussed in connection with the "discovery" of the neutrino.)

There are any number of visually appealing demonstrations which could be done in a closed flask to show conservation of mass (see Student Handbook). The one using Alka Seltzer tablets is written up in some detail and can be used to make the more subtle point of accuracy limitations to conservation laws.

The figure and the reading on page 3 may seem to be inconsistent concerning changes in mass during burning. Burning will lead to an increase in mass as described in the paragraph, if you catch everything. But if the burning is in an open pan as in the photograph, initial heating can drive off gases, and smoke can escape, leading to a net loss in mass. If time permits, this would be a good point to bring in more chemistry by discussing which reactions increase mass and which decrease it because of the escape of gaseous products from an open pan

Although substances seem to disappear, dissolving them does not violate the law of conservation of matter. For example, during World War II, Niels Bohr, the physicist who developed the atomic theory which we discuss in Chapter 19, had to flee his laboratory in Denmark. Two of his colleagues gave him their gold Nobel Prize medals, which he dissolved in a jar of acid and left in his office. Believing in the conservation of matter, he was certain that the acid would only dissolve the gold and he knew that the Nazis would never suspect the jar of acid to be valuable. When he returned after the war, he regained the gold from the acid and had the medals recast.

Weight-mass distinction was made in Unit 1, but it might be well to treat it briefly here again. Weight changes with distance from the earth were treated extensively in Unit 2. Because mass conservation was studied (until recently) only with analytical balances, an excursion might be taken into the operation of balances and what they measure.

The ratio of the weights of two bodies is equal to the ratio of their masses if they are weighed at the same place or at places where a_g is the same. (The proportionality has been experimentally established to better than 1 part in 10^{10}. In general relativity, the proportionality is assumed, on the basis that acceleration and gravitational fields are mathematically indistinguishable.)

Precision chemical experiments were made about the beginning of the 20th century that established conservation of mass to 1 part in 10^6. In some nuclear reactions observed since then, the mass of all the particles is detectably different after from what it was before: but if the inertia of the energy involved in the reaction is included in the mass, the total mass is still conserved.

Sec. 9.2: Collisions

This entire section is designed to show how people were thrashing about using incomplete and vague concepts in their search for order and fundamental principles. It may well represent the sort of thing that always goes on when one is trying to find order amid the seemingly chaotic; a sort of private, pre-physics stage in which guesswork, hunch, and intuition are stirred in about equal parts with experimental observation, good existing theory, and hope. Looking back from our present vantage point, it seems extraordinary that anything came from it all. One hesitates to draw current parallels in other areas of study for fear of being misunderstood, but to some degree the same sort of thrashing goes on today in physics as we try to understand the role of fundamental particles. Of course, now there is a good bit less reference to what is on God's mind than there was in Descartes' time.

Note that while the text treats inelastic collisions, the experiments and demonstrations deal primarily with elastic collisions. This should present no problem if no distinction is made at this time. Section 9.6 deals with this matter.

Beginning with an equal-mass elastic collision (one body initially at rest), students can be led to suggest that speed is conserved. Then to establish the generality of a "conservation of speed" principle, the principle must be confirmed in other collisions. An unequal-mass elastic collision will show that "conservation of speed" does not hold. If the masses are in simple ratio, like 2:1, it will be fairly obvious that the product of mass and speed is conserved. Again, to claim the generality of the "conservation of mass x speed" principle, other types of collisions must be tried. Note that the product of mass and speed squared is also conserved, but not m^2v, mv^3, m/v, $m\sqrt{v}$, etc. In the two types of collisions considered thus far, mv and mv^2 have both been conserved. Students can be advised that the simpler relation will be taken up first, and that mv^2 will be taken up later. The linear "explosion" or head-on inelastic collision will show that even mass x speed is not conserved. Then the conservation principle can be saved only if the direction of the speed is considered. A "conservation of mass x velocity" principle holds in these two cases as well as in the others. The use of the vector velocity should suggest that non-linear collisions might be tried. The collisions of pucks on plastic beads or of air-pucks can be photographed stroboscopically with very satisfactory results. The film loops on two-dimensional collisions give excellent results. An inexpensive air-table (Macalaster Scientific Corp.) provides excellent two-dimensional analyses.

Sec. 9.3: Conservation of Momentum

A wealth of material—experiments and demonstrations, film loops, stroboscopic photographs, overhead transparencies, are available for teaching conservation of momentum. The topic is, however, not so important to warrant the use of all parts of all these media. What is important is that students get some intuitive feel for the kind of outcome to be expected in a collision and that they get this feel before the corresponding bald claim about conservation is made in the text. The collection, or examination, of data in the laboratory could be coordinated to allow the progressive "discovery" of conservation of momentum independent of (and preferably before) the reading of the text treatment.

This is an appropriate time to introduce the summation symbol Σ, if it has not been done before, or to review it. But here Σ means vector addition. The use of different objects and subscript numbers 1 and 2 to mean before and after collision is not very generalizable. If we let subscript numbers identify the different objects and let unprimed and primed $\vec{v}$'s mean before and after respectively, then we can write in general

$$\sum_i m_i\vec{v}_i = m_i\vec{v}_i,$$

or

$$\Delta\sum_i m_i\vec{v}_i = 0$$

Notice that the summation is vectorial. Actually it might be more satisfactory to write algebraic summation for each of the three components of momentum:

$$\Delta \sum_i m_i v_{x_i} = 0$$

$$\Delta \sum_i m_i v_{y_i} = 0$$

$$\Delta \sum_i m_i v_{z_i} = 0$$

These principles are true generally, both independently and all together.

Sec. 9.4: Momentum and Newton's Laws of Motion

Attention should be given to the question of how does "momentum enter or leave a system?" The only way we can change the linear momentum of a system is to exert a force on part or on all of the system from outside. The force that one part of a system exerts on another part cannot change the system's total momentum: it might redistribute the momentum among the parts, but it cannot change it.

The operation of a rocket engine may be discussed in terms of conservation of linear momentum. The analogy of the recoil of a gun, suggests that the forward momentum of the bullet as it leaves the muzzle of the gun is equal to the recoil momentum of the gun in the opposite direction. If we then picture the rocket as a continuously firing gun, we can derive the "rocket equation." This is done in the Reader 5 article, "Space Travel: Problems of Physics and Engineering."

Several kinds of inexpensive toy rockets are available in stores or from hobby companies. The least trouble is one that is propelled with water put under pressure by pumping air into the rocket. Avoid the explanation that the water leaving the nozzle *causes* the rocket to move upwards. It is more consistent with the approach taken in this course to say that the unbalanced air pressure up on the top of the rocket causes it to accelerate upward.

Sec. 9.5: Isolated Systems

An isolated system need not be *completely* isolated, only isolated with respect to what we are interested in. "Entering" or "leaving" a system implies passing through a spatial boundary surrounding the system. It should be made clear that something created or destroyed within the boundary of a closed system is not entering or leaving the system.

A conservation *law* must hold true for *any* closed system. We do not speak of "the conservation-of-energy-in-an-insulated-calorimeter law." It is not enough to conserve something in a *particular* isolated system; that something must be conserved in *every* isolated system. Otherwise we have no law.

It is very difficult indeed to think of good conservation laws from outside of physics. That is one reason it is so remarkable that they occur in physics at all. Since all around us we see change--change-change, it is surprising and wonderful to find that amidst all the turmoil some things sometimes do not change.

Physicists cling to the conservation laws, sometimes in the face of seemingly contrary evidence. At first the conservation laws of linear momentum and energy seemed to be violated in beta decay of nuclei. Physicists then invented the neutrino in order to preserve the conservation laws.

The students have already seen a very good conservation law: Kepler's second law (Unit 2, Section 7.2). The text statement on page 53 of Unit 2 is, "the line from the sun to the moving planet sweeps over areas that are proportional to the time intervals." Of course, for this to be true, it must be that the line sweeps over an area at the same rate no matter where the planet is in its motion, i.e. the rate at which the line sweeps out area is conserved throughout all the changes in the planet's velocity and distance from the sun.

Since we do not discuss angular momentum in the text, the student will not recognize this as just a special case of the conservation of angular momentum. But it does have the advantage of being a physical example in which what is being conserved is not a substance. It also has the advantage that the quantity which is conserved has different values for different sun-planet systems.

Sec. 9.6: Elastic Collisions

The main point here is the necessity of introducing another conservation law in order to successfully account for the behavior of the motion of the bobs. Since we have an interaction involving two unknown velocities, we must have two equations to arrive at their final values.

Summary of Chapter 10

The concept of work, defined as the product of the force on an object and the distance the object moves while the force is exerted on it, is interpreted as a measure of energy transformed from one form to another. With this interpretation expressions can be derived for the kinetic energy, $\frac{1}{2}mv^2$, of an object and for the change in gravitational potential energy, $F_{grav}d$, of an object of weight F_{grav} which moves through a vertical distance d. Other forms of potential energy are mentioned. If there is negligible friction, the sum of the kinetic energy and the potential energy does not change; this is the law of conservation of mechanical energy.

Work is more accurately defined as the product of the component of the force on the object in the direction of motion of the object and the distance the object moves while the force is being applied. Thus, if an object moves in a direction perpendicular to the direction of the force on it, the force does no work.

The present-day view of heat as a form of energy was established in the nineteenth century, partly because of knowledge of heat and work gained in the development of the steam engine. The first practical steam engine was invented by Savery to pump water from flooded mines. A considerably better engine was that of Newcomen, which was widely used in Britain and other European countries in the eighteenth century.

The invention of the separate condenser by Watt in 1765 resulted in a vastly superior steam engine, one which could do more work than the Newcomen engine with the same amount of fuel. Watt charged a fee for use of his engines which depended on the rate at which they could do work; that is, on their power. The Watt engine was quickly adapted to a variety of tasks and was instrumental in ushering in the Industrial Revolution.

One of the scientists who helped establish the idea that heat is a form of energy was Joule, who performed a variety of experiments to show that a given amount of mechanical energy (measured, for example, in joules) is always transformed into the same amount of heat (measured, for example, in kilocalories).

Living systems require a supply of energy to maintain their vital processes and to do work on external objects. Plants obtain energy from sunlight and, by the process of photosynthesis, convert the energy into chemical energy stored in the molecules of the plant. An animal which eats plants, or which eats other animals which have eaten plants, releases the stored chemical energy in the process of oxidation, and uses the energy to run the "machinery" of its body and to do work on its surroundings. Different activities use food energy at different rates. A healthy college age person needs at least 1700 kilocalories of food energy a day merely to keep his body functioning, without doing any work on his surroundings. Yet there are countries where the average intake of food amounts to less than 1700 kilocalories a day.

In the early nineteenth century, developments in science, engineering, and philosophy contributed to the growing conviction that all forms of energy (including heat) could be transformed into one another and that the total amount of energy in the universe was conserved.

The newly developing science of electricity and magnetism revealed many relations between mechanical, chemical, electrical, magnetic, and heat phenomena, suggesting that the basic "forces" of nature were related.

Since steam engines were compared on the basis of how much work they could do for a given supply of fuel, the concept of work began to assume considerable importance. It began to be used in general as a measure of the amount of energy transformed from one form to another and made possible quantitative statements about energy transformations.

The German "nature-philosophers," concerned with discovering by intuition the inner meaning of nature, stimulated the belief that the various phenomena of nature were different manifestations of one basic entity—that which came to be called energy.

Of a large number of scientists and philosophers who proposed in some form a law of conservation of energy, it was von Helmholtz who most clearly asserted that any machine or engine that does work cannot provide more energy than it obtains from some source of energy. If the energy input to a system (in the form of work or heat) is different from the energy output, the difference is accounted for by a change in the internal energy of the system.

The law of conservation of energy (or the first law of thermodynamics) has become one of the very foundation stones of physics. It is practically a certainty that no exception to the law will ever be discovered.

Sec. 10.1: Work and Kinetic Energy

It may seem strange that we have brought our discussion of mechanics so far with hardly a mention of energy, which now holds such an important place among the concepts of physics. We might mention that the logical development of mechanics is quite possible without it. All the problems of classical mechanics can be solved without reference to energy. The "idea" of energy is historically much older than the name. It goes back to Galileo on his work with machines, where the concept of "work" was involved.

While there was confusion between the words force and work, due to Leibniz's interpretation, Leonardo da Vinci remarked that work implied a distance moved in the direction of the force (200 years before). In the literature of physics today, work means just this: force multiplied by the distance traversed in the direction of the force. The tack taken in the text is to present the concept of work in an easily digestible form first, and to qualify it later as necessary. Even using the text's approach, you might want to hedge the initial "definition" by pointing out to students that both force and displacement are vectors and that it isn't immediately obvious what to do when the force and displacement aren't along the same line. As long as they are in a line, it is quite correct to take work as Fd.

Many physicists use the concept of negative work because it allows one to treat these problems from a more general point of view with one equation instead of two. We have avoided doing this in the text to avoid excessive abstraction and instead stay closer to familiar concepts. Thus we say that B does work on A, rather than that A does negative work on B.

As we saw in Sec 9.6, Huygens made prominent the concept of vis viva or living force a quantity varying as the mass multiplied by the square of the velocity. The attribution of the term energy to the vis viva concept did not come about until the nineteenth century. The product Fd is called the "work" done by the force during the displacement, while the quantity $\frac{1}{2}mv^2$ is one-half that which used to be known as vis viva. We now call $\frac{1}{2}mv^2$ the kinetic energy.

If a body moves a distance d directly against a resisting net force F, it loses kinetic energy. The amount of kinetic energy lost can be shown as follows, starting from F = ma,

$$F = ma$$

$$Fd = ma\ d$$

$$= m\frac{(v_f - v_i)}{t} \cdot \frac{(v_i + v_f)}{2}t$$

$$= \tfrac{1}{2}m(v_f^2 - v_i^2)$$

$$= \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

Sec. 10.2: Potential Energy

Experiment E24 should be done before (or closely in conjunction with) the beginning of this section. The "dip" in the kinetic energy graph for the slow collision is a nice way of making the introduction ("discovery") of potential energy; some of the kinetic energy disappears and then reappears—where was it? The "explosion" from the tied steel loop makes a good follow-up demonstration; there is no kinetic energy in the system before and much afterwards—but there was something related to energy in the tied spring. If the lab can't be done until after the gravitational force of E_p has been considered, the dip can be predicted instead of discovered.

If we accept that a falling body is continually gaining kinetic energy due to the previous work we did in raising it, we must then accept that the "raised" body, before it started to fall, must possess the energy which is appearing in kinetic form. Energy due to motion is fairly obvious, which is why kinetic energy (vis viva) came relatively early upon the scene. But energy due to position is less obvious. In fact, it eluded both Leibniz and Descartes. The first mention of "position" energy was in a book by Carnot in 1803 in which he says

> Vis viva can figure as the product of a mass and the square of its velocity, or as the product of a moving power and a length or a height. In the first case it is a vis viva properly called; in the second it is a latent vis viva.

It was not until 1853 that the name potential energy was first used by Rankine and has been used ever since.

See Resource section for article "Energy Reference Level." Note that it is not used in text development.

Sec. 10.3: Conservation of Mechanical Energy

We see from the nature of energy two fundamental concepts: kinetic and potential. Kinetic is energy possessed by reason of motion while potential is possessed by reason of position or condition, as in a raised weight or stretched spring. It should be clear that either kind may come into being in consequence of the performance of work: in the first case work devoted to producing speed, in the second case, work involving some reversible process which subsequently can be made to yield up the energy thus stored.

One caution about using the momentum and energy conservation principles for collisions: prediction is possible only for rectilinear collisions of two balls. As many students may remember, a set of equations all of which are true can be solved for as many unknowns as there are independent equations. Energy and momentum conservation principles constitute a pair of such independent "simultaneous" equations and so can be solved for two unknowns only, e.g., the two velocities after collision of two balls. If the collision is in two dimensions, there are two unknowns for each ball after collision (x and y velocity components or speed and angle) giving four unknowns in all—two other values would be needed in order to be able to solve for the remaining two velocities. The situation of a single ball striking a row of balls in contact has, after the collision, as many parameters as there are balls—one velocity value for each ball. Energy and momentum conservation are not sufficient to find a unique set of "after" values (and, as a matter of fact, all the balls will have finite velocities after the collision, even if the balls are perfectly elastic). If the balls are not in contact, but separated enough so that the collision of the first and the second is over by the time the second touches the third, then the traditional all-zero-except-the-last-ball result is predictable, but it is no more than a series of two-ball collisions.

Historically, the energy conservation generalization was employed by Leonardo da Vinci near the end of the fifteenth century; Simon Stevin used it in 1605 as the basis for his development of the laws of statics, while Galileo employed a similar argument in his analysis of a frictionless inclined plane.

Sec. 10.4: Forces That Do No Work

The ideas of zero work being done when heavy bodies are held or transported horizontally are troublesome to students and should probably be qualified as follows. When someone stands and holds a heavy body, no work is being done on the body, but this is not to say that there is not a great deal of chemical transfer of energy—(work, in the technical sense, on the cellular or molecular scale) required to maintain the muscle contractions that result in the body being held motionless. Work can be going on inside you, transferring chemical energy ultimately into heat, without any work being done on anything outside the system "you." In this sense it is quite reasonable to say that you can get tired and hot without doing any work (on the outside).

It does take a force to move something horizontally, even without friction. If it is an "immeasurably small force," as in the text, then it will take an immeasurably long time for the body to make the change in position. A more reasonable treatment would be to admit that a finite force is required, and that therefore work must be done on the body in getting it moving; but point out that when the body reaches the new position it will still be moving. The work that went into getting it moving went into kinetic energy which it still has after reaching the new position, no energy was "used up" in changing position. (If a small spring gun were used to start the body, the sprung gun could be moved around and used to stop the body as it arrived at the new position, and the gun would be cocked in the process. Everything then would be the same as before except that the body had changed position.)

"No-work" forces, then, are those which are perpendicular to the direction of a body's motion and so do not contribute to kinetic energy changes. A motion which is restricted to a prescribed path, like a track or wire, or to a prescribed plane, is called a constrained motion. In such motion no-work forces are present but knowledge of them is not necessary for the calculation of changes in kinetic energy.

Sec. 10.5: Heat Energy and the Steam Engine

One must be aware that the development of heat and mechanics progressed along quite different paths. The study of motion and the mechanical interaction of bodies on the one hand and the study of temperature and the thermal interaction of bodies on the other. It was the work of Joule which joined these two almost entirely independent disciplines, and it was this joining of heat and energy along with early developments in chemistry which brought about the kinetic theory. This will be developed further in Sec. 10.9. We cannot here undertake to describe the long controversy over the nature of heat that extended from the time of the early Greek philosophers until the middle of the nineteenth century. In the long run, however, the multitude of physical processes in which heat seems to be generated by the expenditure of mechanical work must be reckoned with.

The detailed operation of steam engines is not an important part of the story line of Unit 3. Accordingly, it is necessary only that students learn enough about engine operation to make sense of the extended treatment of steam engine improvement and their technological and social effects. If students understand well the principle of Savery's first engine, no further detail is important, subsequent improvements can be adequately understood in terms of the operation of this fairly simple device. The Newcomen engine, for example, was different from the Savery only in that it had a piston which was moved instead of water directly and there was mechanical linkage that sprayed the water—inside—at just the right time. Watt's principal improvement was to have the steam condensed in a separate container, so the main cylinder did not need to be reheated by the steam for each cycle. The development of steam engines was primarily a commercial rather than scientific enterprise. The early stages of development, it should be noted, were the work of amateurs—there were as yet no professionals of engine design. Section 10.6 develops the steam engine's impact on the Industrial Revolution.

Sec. 10.6: James Watt and the Industrial Revolution

The Industrial Revolution was a vast and complex set of phenomena—only the barest account is given here, and can be treated as a reading assignment. Cottrell's Energy and Society (listed in the bibliography) is a good source for expanding class discussion. One point that should be made: distaste for mechanistic science could now be based on objections to its practical consequences as well as on its pholosophical implications.

Note on the Industrial Revolution

"The Steam Engine," from Technics and Civilization, by Lewis Mumford. Harcourt, Brace and Company, New York, 1934.

This excerpt provides background material that may be useful for a class discussion on the social and cultural effects of the development of the steam engine. The following definitions may be useful to the reader:

eotechnic civilization: the pre-machine age

paleotechnic civilization: the period beginning about 1750 which followed the Industrial Revolution.

The Watt engine:

For this type of engine the use of a separate condenser provides a great gain in efficiency. However, it is even more efficient to use high pressure steam. Early attempts to do this were frustrated because the materials available for constructing the boiler could not withstand high pressures and temperatures. But advances in metallurgy later in the nineteenth century made it possible to construct practical high pressure engines. Eventually the separate condenser became unnecessary—but by this time it had already served its purpose in getting steam engines established as the major source of energy in industry.

Watt's working model of Newcomen's engine had a problem of scale—a tangential topic of considerable interest in itself. If you have time, the PSSC chapter 4, "Functions and Scaling," and the PSSC film Change of Scale, could be used for a pleasant diversion.

An additional historical sidelight—the first U. S. Government research grant was made in the nineteenth century to the Franklin Institute of Philadelphia, to investigate the causes of explosions in steam engine boilers.

The section "The Steam Engine after Watt" (pp. 42-55), in I. T. Sandfort's Heat Engines, is a nice account of some of the first steps towards modern engines. Internal combustion, steam turbines and gas turbines, turbojets and rockets are taken up in Chapter 7 of Sandfort's book. The discussion includes some thermodynamics which are developed in the book, but it is not hard to skip over.

Sec. 10.7:

A single experiment cannot, of course, establish that there is a constant conversion factor between, say, work and heat. Only when the same proportionality constant is found in a number of different experiments with different substances can it be called *the* conversion factor. Student activity on the "mechanical equivalent of heat" is, accordingly, a way of finding a proportionality constant for internal energy and gross mechanical energy of lead shot. That this is *the* conversion factor for work and heat *is* an unsupported generalization.

The success of the caloric theory with a limited range of phenomena is a worthwhile story. If lab time is available, you may wish to pursue the calorie in Experiment E27, "Calorimetry."

The identification of calorie with kilocalorie should be emphasized to avoid confusion later.

Sec. 10.8: Energy in Biological Systems

The main point of this section is that a meaningful connection can be made between principles developed in the abstract world of physics—where a "body" is a "body" whether it is a molecule of hydrogen, Caesar's corpse, or a red giant star —and the familiar world of widely differentiated living organisms. The physical principles are no less true for bacteria and high school students than for asteroids and roller carts. The details of the section are not important to the story line of Unit 3.

It should be pointed out that the "energy available in food" must be measured—e.g., a known mass of the food substance is oxidized in a closed container of a calorimeter.

Throughout the section, however, there is mention of inefficiency and energy loss. The dissipation of energy is taken up in Sec. 11.6. If Secs. 10.7 and 11.6 are brought together as a reading assignment, students can be asked in the subsequent class discussion to explain what is meant by the "loss" of energy in the food chain. The point, of course, is that only a small a way as to be available for growth and movement. Most of it is either dissipated internally as heat or excreted without being digested.

Many students will have had previous exposure to concepts of energy in biology or chemistry courses. They should now be capable of achieving a deeper understanding of the role of energy in biological systems than previously. Those students who used the BSCS text can be encouraged to reread Chapters 6 and 26 of the BSCS "blue" text, *Biological Science—Molecules to Man*, Houghton Mifflin Co., Boston. Two *Scientific American* articles, "The Contraction of Muscle" (H. E. Huxley, November, 1958) and "Energy Transformation in the Cell" (A. L. Lehninger, May, 1960) contain interesting and appropriate discussions of how chemical energy is converted to mechanical work. The first of these articles provides an answer to the question of why muscles become fatigued even when *no* mechanical work is done.

See Resource section for notes on "Food Energy" and "Temperature of Outer Space."

Sec. 10.9: The Law of Conservation of Energy

See Resource section for "A note on classification of energy."

The philosophical material in this section should be treated as elsewhere; the details of the philosophy or their correctness are not important—what is important is the extent to which the philosophical concern influenced the development of physics. (Goethe was, by the way, not an unworldly man—if students translate his ideas on male and female principles, which are expressed in the abstract in the text, into less abstract forms, they may not be entirely unjustified.)

The majority of men who first comprehended the full import of the generality of energy conservation were all young men and were all outside the field of physics at the time they made their contributions. Mayer, a German physician, aged 28; Carnot, a French engineer who preceded all the rest in the discovery and who will be discussed in Sec. 11.6, aged 34, Helmholtz, a German physiologist, aged 32; Joule, an English industrialist, aged 25; and Colding, (not mentioned here) a Danish engineer who made the same discovery independently of the others and almost simultaneously, aged 27.

Summary of Chapter 11

The development of the kinetic theory of gases in the nineteenth century led to the last major triumph of Newtonian mechanics. A simple theoretical model of a gas was adopted: a large number of very small particles in rapid disordered motion. Since the size of the particles themselves is assumed to be very small compared to the space occupied by the gas, each particle moves most of the time at constant velocity, occasionally colliding with other particles or with the sides of the container. By applying Newtonian mechanics to this model, scientists could deduce equations which related observable properties of gases, such as pressure, density and temperature, to the sizes and speeds of molecules. With these equations, kinetic theorists could: (1) explain the known relations between observable properties of gases, such as Boyle's law; (2) predict new relations, such as the fact that the viscosity of a gas increases with temperature but is independent of density; (3) infer the sizes and speeds of the molecules.

The chapter first discusses the model and its consequences qualitatively; details and derivations are collected in an optional section (11.5) which can be omitted by most students. The rest of the chapter is devoted to exploring the relation between the second law of thermodynamics, or principle of dissipation of energy, and the kinetic theory. It would appear that if Newtonian mechanics were strictly applicable on the atomic level, then all molecular processes would be time-reversible, and any initial arrangement would eventually recur. Boltzmann's attempts to reconcile these reversibility and recurrence paradoxes with experience involved statistical reasoning, and introduced interesting speculations about the direction of time.

Sec. 11.1: Heat and Mechanical Energy

This section should be treated primarily as a reading assignment. Some treatment for discussion could be the following. The kinetic-hypothesis-of-heat idea is of much longer standing than is ordinarily realized. Disregarding Greek speculations, one can find that in the scientific era, 1706, John Locke said

> Heat is a very brisk agitation of the insensible parts of the object, which produces in us the sensation from whence we denominate the object hot; so what in our sensation is heat, in the object is nothing but motion.

In 1738 Daniel Bernoulli had remarked that

> it is admitted that heat may be considered as an increasing internal motion of particles.

In 1780, Lavoisier and Laplace were even more explicit. They said in their "Memoire sur La Chaleur"

> ...heat is the vis viva resulting from the insensible movements of the molecules of a body. It is the sum of the products of the mass of each molecule by the square of its velocity.

In 1798, Rumford, drawing conclusions from an experiment, stated that it appeared to him to be

> extremely difficult if not quite impossible to form any distinct idea of anything, capable of being excited and communicated, in the manner the Heat was excited and communicated in these Experiments, except it be MOTION."

All these pronouncements, except Rumford's went considerably beyond any really justifiable evidence from experimental data.

Again, most of the central ideas here will be developed later. Treat as introduction material but always be prepared to add to a discussion. The "equal-sharing" principle (equipartition) and the problem of "irreversibility" are mentioned only to show contradiction between Newtonian mechanics and observable properties of matter.

Sec. 11.2: A Model for the Gaseous State

Historically, suggestions along the lines of a particle theory of gases had been made in 1738 by Bernoulli. Detailed theory was developed during the latter half of the nineteenth century by such men as Clausius (German physicist), Maxwell (English physicist), Boltzmann (Austrian physicist), Kronig, Joule (English industrialist), etc. Joule is better known for his experimental confirmations of the principle of conservation of energy than he is for his work in developing the kinetic theory of gases. The theory stands very much as it was left left by them, but is refined primarily to include knowledge of intermolecular forces.

The theory can be looked at systematically in historical perspective in the following manner. (1) 1738, suggestions by Hooke and Bernoulli; (2) 1848, fresh attack by Joule in light of his mechanical equivalent of heat experiments; (3) 1857, chief difficulties encountered by Joule straightened out by Clausius; (4) 1859, application of statistical mechanics by Maxwell.

What do we mean when we say model? This idea should be discussed so that it can be applied fully to the kinetic theory of gases.

Sec. 11.3: The Speed of Molecules

The Newtonian mechanics of collisions can be used to derive the relation between P and v for a collection of perfectly elastic, vanishingly small particles. It turns out that the product of pressure and volume is proportional to the total kinetic energy of the particles. This idealized model does not appear to match real gases, however, because Newtonian mechanics shows also that the kinetic energy, and therefore the product Pv, changes when the volume is changed. The model might still be correct, however, if something happens during volume changes that continually acts to return the total kinetic energy of the particles to the initial value. This something will later be shown to be heat exchange with the surroundings; keeping the kinetic energy constant is the same thing as keeping the temperature constant.

Although Newton's static 1/r repulsion model does account for the empirical relation of P and v, and Bernoulli's kinetic model apparently does not, we have dismissed the former because it isn't necessarily correct and pursued the latter because it is "reasonable." Students may need an optimistically sly "now just wait and see" to get them over this hump with good humor.

The second part of this section touches on the very rich field of statistical description. The essential point is that normal distributions are likely to result when a very large number of independent small effects produce a measurable result.

Sec. 11.4: The Sizes of Molecules

There was no real need to consider the collisions between molecules in developing the gas laws and the specific heat of a gas. However, many effects and properties of gases and molecules cannot be understood on a molecular basis without a quantitative study of molecular collisions and primarily the mean free path. With a molecular radius of about 10^{-8} cm, we find that the mean free path is 10^{-5} cm in a gas at atmospheric pressure and at a temperature of about 300°K, it is 1000 times larger than the assumed molecular dimensions. To obtain a mean free path of 1 meter, the pressure must be only 1.5×10^{-5} mm H_g. Many effects and properties of gases and molecules, such as heat conduction and viscosity, cannot be understood on a molecular basis without a quantitative study of molecular collisions.

Sec. 11.5: Mathematical Derivation of the Ideal Gas Law from the Kinetic Theory Model (Special topic—optional)

The refusal of a liquid to fall out of a closed tube can be much more exciting if demonstrated before the text is assigned to be read. A 25-ft. length of transparent tubing suspended from an upper floor of the school makes the point much more strikingly than a meter tube of mercury, especially if students are led to expect a great rush of water—which fails to occur.

If you make a mercury barometer, be sure to take the usual care in handling mercury and in cleaning it up as it spills; the inhalation of mercury vapors has cumulatively poisonous effects.

The statement that "it is the balance of pressure, not force, that determines the height of the mercury column" may not be entirely clear. The point is that as the cross-section of the column changes, changing the weight of the column, so does the area over which the column is supported; thus the force up for different column sizes will change in proportion to the change in weight. The balance is not characterized by any particular pair of balanced forces, therefore, but is characterized by the pressure at the bottom of the column.

Before dealing quantitatively with atmospheric pressure, you should have students make rough calculations of the pressure exerted on the floor by an elephant's foot and a spike heel.

See reference section for a method of calculating the pressure of the atmosphere.

Sec. 11.6: The Second Law of Thermodynamics and the Dissipation of Energy

Note that Carnot's work led to principles with sweeping generality, even though his theory treated heat as a fluid.

His conclusions went beyond the first law of thermodynamics (i.e., the conservation of energy).

The first law states that the total output of any heat engine must equal the total energy input. It follows directly that the useful work cannot exceed the total input energy. Carnot's theory placed a further restriction, stating that the useful work output can never exceed a certain fraction of the total input energy. The fraction is

$$\frac{T_i - T_o}{T_i},$$

where T_i is the absolute temperature in the working part of the engine and T_o is the absolute temperature of the exhaust. A heat engine can approach 100 per cent efficiency only as the temperature of its exhaust approaches absolute zero—or as its working temperature becomes infinitely great! Because of design limitations, heat engines always fall short of even this theoretical fraction of useful work output.

The principle of dissipation of energy can be applied to all processes—mechanical or organic. The principle can be stated in many different ways and each way can be called the second law of thermodynamics. Because it was first formulated with regard to engines, many of these statements of the second law refer to restrictions on the operation of heat engines. The statement "second law of thermodynamics" is not particularly better than any other. Indeed, it leaves unclear what the "maximum amount of work" is. (It is the maximum allowed by the first law of thermodynamics, i.e., an amount equal to the input energy.)

It may not be particularly pleasing to have a major principle stated so negatively. Physicists have invented the concept of entropy, an abstract quantity which can be calculated for any process of heat transfer. In any real process, the entropy total for all participating systems will increase. This is a pleasantly positive statement of the second law of thermodynamics even if it lacks much intuitive meaning. Some intuitive meaning can be attached to the "entropy" quantity by associating it with the degree of disorder in a system. (This association has even led the developer of modern "information theory" to refer to the "entropy of a message," a quantitative measure of the disorder—and therefore information—in a set of symbols.) So another statement of the second law of thermodynamics could be: in any real process, the total disorder for all participating systems will increase. The topic of entropy increase is taken up again in terms of kinetic molecular theory, and it will be seen then that these simple statements need to be qualified to allow for brief statistical "fluctuations" during which total entropy can decrease. This qualification has been attempted by the use of "tends to." It won't be clear yet to students what "tends to" means, but it should be pointed out that it is a hedge which will be discussed further later.

There is sometimes reference made to a third law of thermodynamics, one statement of which is: "it is impossible to reduce, in a finite number of steps, the temperatures of any system to absolute zero." By using a very large number of cooling steps, temperature only .001°K or less can be produced. (This restriction is not the same as the "zero point energy" of quantum theory; not even the zero point energy can be reached.)

The "entropy death," called also the "heat death," appears to be a "cold death." The principle of dissipation predicts only that all ordered motion will eventually disappear in any closed system and only random (completely disordered) thermal motion remain. Whether this is "hot" or "cold" depends on how much energy there is to go around in how much space. The statement about the possibility of avoiding the entropy death refers to the recurrence theorem. If we could wait long enough, we might find that entropy decreases again and we come back to where we started from.

The principle of dissipation of energy is not best characterized as a consequence of the second law of thermodynamics; a better expression of the relation is that the second law is a quantitative but somewhat restricted version of the dissipation principle. The statement at the end of the paragraph suggests that heat performs work while flowing, without being used up. A more satisfactory statement would be as follows: mechanical work can be derived from internal energy only when there is a temperature difference between two parts of a system. As a system approaches a uniform temperature, the possibility of producing mechanical work from the internal energy approaches zero.

Sec. 11.7: Maxwell's Demon and the Statistical View of the Second Law of Thermodynamics

The main point of this section is very important to the overall goals of the course; it is to suggest that laws of physics need not be absolute descriptions of what must or must not happen; they can just as well give probabilities for what might or might not happen. An aspect of this kind of law which is not mentioned explicitly in the text is that the accuracy of the statistical description increases with the number of particles, or, in other words, the expected per cent fluctuation from the most probable value decreases as the number of particles increases. (Even a dust speck comprises some billion billion atoms, so virtually all directly observable events are very, very close to completely determined by statistical laws.) Also, the longer the time interval over which an average value is taken, the less becomes the expected percentage fluctuation from the statistically most probable value. For relatively small numbers of particles over relatively short time intervals (e.g., a thousand particles over a billionth of a second) the statistical description predicts pronounced fluctuations from the most probable value. Note however, that the fluctuations do not constitute error in the statistical description; the statistical description includes the expectation of fluctuation. It is the expected fluctuation which decreases as the number of particles and the time interval increase. The Steinberg cartoon from The New Yorker might be labeled "a very improbable fluctuation." The statistical form of the second law of thermodynamics would give an exceedingly small—but not zero—probability to so great a fluctuation as the boulder rolling up the hill as it and the hill cooled off.

Maxwell's demon is important to the development. It is an example of how the second law could be violated, on the basis of Newtonian mechanics, if only one could get information about molecular arrangements and sort out the molecules. In this respect it illustrates the connection between entropy and information mentioned in Section 11.6 development. Feynman gives the example of a mechanical demon consisting of a tiny spring door of so small a mass and loose a spring that it would open under the impact of fast molecules only. The impacts would result in the door becoming increasingly hot, so that its own random thermal agitation would progressively disrupt its function.

Sec. 11.8: Time's Arrow and the Recurrence Paradox

Your students will probably be feeling anxiety about the statistical turn that the course has taken. Statistics may be all right for describing the chaotic organic world, but the immutable laws of cold matter are something else. They may find some comfort in knowing that many scientists in the nineteenth century were not willing to accept as a basic law of physics one that gave only probabilities instead of certainties. They argued that if kinetic theory didn't lead to a completely deterministic second law of thermodynamics, then kinetic theory must be wrong.

Another way of stating this objection is the "reversibility paradox." The kinetic theory involved only reversible collisions of molecules so that events could just as easily happen backwards as forwards; e.g., the hill could cool slightly and propel the boulder up it as easily as the boulder could roll down and warm the hill, concentrating energy instead of dissipating it, increasing order instead of disorder. But all observed events in the universe go in just one direction, that of energy dissipation and increasing disorder. So it seems that the kinetic theory must be wrong. The sequence of photographs on page 106 is intended to suggest this very sticky question. The sequence of events in the very nearly perfectly elastic collision photographed stroboscopically in the top picture could be almost equally good physics whether beginning at the right or at the left. In the perfectly elastic molecular collisions of the kinetic model, there would be no way of choosing between a left to right and a right to left time sequence. In the other pictures there is no doubt about which way the process is going; if a motion picture of one of them were reversed, it would easily be spotted as "backwards."

Summary of Chapter 12

Chapter 12, perhaps more than any other chapter in the unit, depends upon experiments and/or demonstrations to give substance to the material discussed in the text. The purpose of the demonstrations and experiments is to provide a general understanding of waves.

Our previous work has been concerned on the one hand with gross motion—the motion of macroscopic particles and of rigid bodies—and, on the other, with heat and the kinetic molecular theory, which concerns itself with the internal properties and constituents of matter. Throughout our study we have assumed that various parts of a body remained at fixed distances apart. That is, we ignored the compressible nature of matter. We are now in a position to drop this approximation and ask how a compressible body responds to externally applied forces and see that this leads directly to the study of waves.

Here we are concerned with wave phenomena in a mechanical framework which sets the stage for the study of light and electromagnetic waves in Unit 4. It is important that students become familiar with wave phenomena, in particular the interference idea, so that what constitutes a wave vs particle model will make sense to them. This will be useful also in Unit 5 when they study the atom.

Sec. 12.1: Introduction

Our previous work has been concerned, on one hand, with gross motion—the motion of particles and of rigid bodies—and on the other hand, with heat and the kinetic molecular picture—some of the internal properties and constituents of matter. Throughout our study of rigid bodies, we have assumed that the various parts of a body remained fixed distances apart. That is, we ignored the compressible nature of matter. We shall now ask how a compressible body responds to externally applied forces and see that this leads directly to the study of waves. We shall see in chapters to come that these phenomena (waves) are particularly descriptive of many events in nature such as light, sound, radio and water-surface phenomena. We shall see in Units 5 and 6 that atoms, electrons, nuclear and subnuclear particles have wave-like, as well as particle-like, properties.

Sec. 12.2: Properties of Waves

Wave motion implies the transmission of a state. If we stand dominoes on end in a long line and then knock over the first one, we start a train of events which leads eventually to all the dominoes being knocked down. There was no net mass transport along the line of dominoes, rather, it was their state of falling that traveled, and in this simple case, the speed at which the state of falling traveled is called wavespeed.

All wave pulses possess momentum and energy which can be determined without much difficulty. This could be assigned to "better" students with some guidelines.

The first serious suggestion that polarization might be due to the vibrations of light being transverse to the direction of propagation came from Thomas Young in 1817. The very name "polarization" was first used in this connection in 1808 by a French investigator named Malus.

Sec. 12.3: Waves and Propagation

Energy can be transmitted over considerable distances by wave motion. The energy in the waves is the kinetic and potential energy of the matter, but the transmission of the energy comes about by its being passed along from one part of the matter to the next, not by any long-range motion of the matter itself. The properties of a medium that determine the speed of a wave through that medium are its inertia and its elasticity. Of course, this refers to mechanical rather than electromagnetic waves. Given the characteristics of the medium, it is possible to calculate the wave speed from the basic principles of Newtonian mechanics.

Sec. 12.4: Periodic Waves

It is most important for students to have a thorough understanding of the important definitions in wave motion. The wavelength "idea" is an obvious, imperative and prerequisite concept for an understanding of phase. Since the period T is the time required to travel one wavelength λ, it follows that $\lambda = vT$. This velocity is called the phase velocity and is the only velocity involved in a simple harmonic wave. In contrast to this is a group velocity, which is important when several wavelengths and phase velocities travel through a medium. We shall limit our discussion to phase velocity and simple harmonic waves. The relationship, $v = \lambda f$, is important and applies to all waves impartially—water waves, waves on slinkies or ropes, sound waves and electromagnetic waves.

Sec. 12.5: When Waves Meet: the Superposition Principle

The superposition principle appears so obvious that it might be worthwhile to mention when it does not apply. When the equations of the wave motion are not linear, superposition fails. This happens physically when we have relatively large wave disturbances where the laws of mechanics are no longer linear. An example is the linear relation in Hooke's law. Beyond the elastic limit, $F = -kx$ no longer applies. For this reason, shock waves in sound behave differently than ordinary sound waves and hence superposition does not hold. A quadratic equation governs their behavior. Very loud tones will not add linearly, thus giving rise to a distortion in high-fidelity jargon known as "intermodulation distortion." In water waves, ripples can travel independently across gentle swells but not across breakers.

It would be instructive to stress the fact that two waves acting independently of one another means that the displacement of any particle at a given time is simply the sum of the displacements that the individual waves alone would give it.

Sec. 12.6: A Two-Source Interference Pattern

It is interesting to note that Thomas Young (1773-1829) was a London physician who became interested in the study of light through his medical studies, particularly his discovery of the mechanism of accommodation of the eye (focussing). He had done extensive experiments with sound and was impressed with the study of beats. Two sounds can combine so as to produce silence and this is most easily explained on the basis of a wave picture. It isn't surprising then, that he presented reports to the Royal Society concerning experiments on light, namely, his now famous two-slit interference experiment.

The node and antinode lines are actually hyperbolas. The hyperbola being the curve for which the distance from two fixed points is constant, obviously fits the condition for a given fringe, namely, the constancy of the path difference. Although this deviation from linearity may become important with sound and other waves, it is usually negligible when the wavelengths are as short as those of light.

Sec. 12.7—Standing Waves

Standing waves are most commonly produced as a result of reflections. However, they can also be produced by two independent sources of disturbance, one of which produces a wave that travels to the right and the other, a wave that travels to the left. In terms of energy, the difference between a traveling and a standing wave is that in the traveling wave there is a net flow of energy; whereas in a standing wave there is none. The energy is "trapped" in the standing wave.

Each characteristic frequency of the one-dimensional spring, string, etc., or any other similar elastic system, corresponds to a certain characteristic mode of oscillation. Similar characteristic frequencies and modes of oscillation are found also in two-dimensional patterns. An important difference exists between the one- and two-dimensional cases. In the one-dimensional case, there is a one-to-one correspondence between the characteristic frequencies and the characteristic modes of vibration. In the two-dimensional case, however, there may be several modes of oscillation which have the same frequency.

Standing waves are of first importance in the study of sound, since every musical instrument without exception depends for its action on this principle. In 1807 Fourier showed that any periodic function can be expressed as the sum of a number of sine and cosine functions with appropriate amplitudes. This mode of analysis is of great value in almost every branch of physics.

Reference to Sec. 12.6 can now show that the pattern for two-slit interference between the sources is a standing wave pattern.

Sec. 12.8: Wave Fronts and Diffraction

Huygen's theory simply assumes that light is a wave rather than, say, a stream of particles. It says nothing about the nature of the wave, and gives no hint of the electromagnetic character of light. His theory is based on a geometrical construction, called Huygen's principle, which allows us to tell where a given wavefront will be at any time in the future if we know its present position.

This principle can today be stated somewhat succinctly as follows: every point on an advancing wavefront acts as a source from which secondary waves continually spread. The passage of a ripple through an aperture illustrates this effect. (Refer to ripple tank experiments.) It was a similar spreading of beams of light that gave rise to the phenomena which Grimaldi had classified under the name diffraction in his publication of 1665. Newton not only referred to Grimaldi's experiment but he repeated and improved upon them. But even with a man of Newton's caliber, the preoccupation with one theory blinded him to the significance of the evidence pointing toward another; and it was left to Thomas Young to find the most convincing evidence on the wave hypothesis.

Sec. 12.9: Reflection

The law of reflection was known to Euclid. It can be derived from Maxwell's equations which means it should hold for all regions of the electromagnetic spectrum.

The reflection of waves is familiar from such events as the echo of a sound wave or the reflection of a water ripple on a water surface or the reflection of waves on a rope. When waves are incident on a boundary between two media in which the velocity is appreciably different, the incident wave train is divided into reflection or refracted (or transmitted) trains. The reflected energy will be relatively greater the larger the change in velocity.

Sec. 12.10: Refraction

The observation of the bending of light waves goes back to antiquity. Cleomedes in the first century A.D. was one of the first to suggest that the sun remains visible for a time after it has actually set. Aristotle, in his Book of Problems, correctly described the appearance of an oar dipped in water. Ptolemy (about 150 A.D.) tabulated angles of refraction for air-water-glass mediums. He concluded that the angle of incidence and refraction were the same. Alhazen (965-1039), an Arabian investigator, pointed out the error of Ptolemy's generalization. Ptolemy's tables were extended (though not correctly) by Vitellio about 1270 and by Kircher (1601-1680). Kepler, in addition to his astronomy studies, made important contributions to optics and investigated reflection-refraction angles in media with somewhat more success than Ptolemy. It was not until 1621 that the legitimate relation was experimentally discovered by Willebrod Snell and deduced from early corpuscular theory of light by René Descartes. The law of refraction is known as Snell's law or (in France) Descartes' law. There is reason to believe that Descartes had seen Snell's manuscript, though he subsequently published the law of refraction as his own discovery. Descartes' plagiarism upon Snell points out, perhaps unfortunately, that unethical practices in the pursuit of personal ambition are not entirely unheard of in the history of science.

Sec. 12.11: Sound Waves

Sound waves are longitudinal mechanical waves. They can be propagated in solids, liquids and gases. The material particles transmitting such a wave oscillate in the direction of propagation of the wave itself. The distinction between the subjective and objective attributes of sound has not always been recognized, but John Locke, the seventeenth century philosopher, said

> That which is conveyed into the brain by the ear is called sound, though in truth until it comes to reach and affect the perceptive part, it is nothing but motion. The motion which produces in us the perception of sound is a vibration of the air caused by an exceedingly short but quick tremulous motion of the body from which it is propagated, and therefore we consider and denominate them as bodies sounding.

But in any case the origin of sound can be traced to motion of some kind. Again, historically speaking, Aristotle made this observation when he said

> All sounds arise either from bodies falling on bodies or from air falling on bodies. It is due to air... being moved by expansion, contraction and compression.

Robert Boyle in 1660 improved the air pump recently invented by von Guericke and with it performed a bell-in-vacuo experiment where it is reported he said

> We silently expected the time when the alarm should begin to ring... and were satisfied that we heard the watch not at all.

Transparencies

T19 One-dimensional collisions: Facsimiles of stroboscopic photographs of two events involving two-body collisions in one dimension are provided. Measurements may be made directly from the transparency to establish the principle of conservation of momentum. See Unit 3 Transparency Visu-Book.

T20 Equal mass two-dimensional collisions: A stroboscopic facsimile of an elastic collision between spheres of equal mass is shown. Overlays show accurately drawn momentum vectors before and after collision, illustrating conservation of momentum. (See Unit 3 Transparency Visu-Book.)

T21 Unequal mass two-dimensional collisions: A stroboscopic facsimile of an elastic collision between spheres of unequal mass, both of which are moving before collision. Overlays show accurately drawn momentum vectors before and after collision, illustrating conservation of momentum. (See Unit 3 Transparency Visu-Book.)

T22 Inelastic two-dimensional collisions

A stroboscopic facsimile of an inelastic collision between two plasticene-covered spheres of equal mass, both of which are moving before collision. Overlays show accurately drawn momentum vectors before and after collision, illustrating conservation of momentum.

See Unit 3 Transparency Visu-Book.

T23 Slow Collisions

A stroboscopic facsimile shows a collision between two dynamics carts equipped with spring bumpers and light sources. Analysis of momentum and kinetic energy before, during and after the collision may be made directly from the transparency. See Unit 3 Transparency Visu-Book.

T24 The Watt Engine

Overlays depict a schematic diagram of the Watt external condenser engine during the steam-expansion and condensation phases of operation. See Unit 3 Transparency Visu-Book.

T25 Superposition

Shows two pulses crossing at four instants of time, first with both pulses above the equilibrium line, then with one on either side of the line. The superposed wave for each case is also shown. See Unit 3 Visu-Book for teacher notes.

T26 Square wave analysis

Shows how first four Fourier terms add to begin to produce a square wave. May be used for variety of superposition problems. See Unit 3 Visu-Book for teacher notes.

T27 Standing waves

A set of sliding waves permits a detailed step by step analysis of a standing wave pattern. See Unit 3 Visu-Book for teacher notes.

T29 Interference pattern analysis

Overlays illustrate crests and troughs for two independent sources. Other overlays show nodal and antinodal lines and geometry for deriving the wavelength equation. See Unit 3 Visu-Book for teacher notes.

Loops

L4 A matter of relative motion

A collision between two equally massive carts is viewed from various stationary and moving frames of reference. Use qualitatively here. Student notes and quantitative treatment are found in Unit 3.

L18 One-dimensional collisions: Slow motion photography of elastic one-dimensional collisions. (R); (L)

L19 Further examples of one-dimensional collisions: A continuation of the preceding loop. (R); (L)

L20 Perfectly inelastic one-dimensional collisions: Slow motion photography of inelastic one-dimensional collisions. (R); (L)

L21 Two-dimensional collisions: Slow motion photography of elastic collisions in which components of momentum along each axis can be measured. (R); (L)

L22 Further examples of two-dimensional collisions: A continuation of the preceding loop. (R); (L)

L23 Perfectly inelastic two-dimensional collisions: A continuation of the preceding two loops; plasticene is wrapped around one ball. (R); (L)

L24 Scattering of a cluster of objects: In slow motion photography, a moving ball collides with a stationary cluster of six balls of various masses. Momentum is conserved. (R); (L)

L25 Explosion of a cluster of objects: A powder charge is exploded at the center of a cluster of five balls of various masses. One ball is temporarily hidden in the smoke; the position and velocity of its emergence can be predicted using the law of conservation of momentum. (R); (L)

L26 Finding the Speed of a Rifle Bullet—Method I

A bullet is fired into a block of wood suspended by strings. The speed of the block is measured directly by timing its motion in slow-motion photography. (R); (L)

L27 Finding the Speed of a Rifle Bullet—Method II

A bullet is fired into a block of wood suspended by strings. The speed of the block is found by measuring its vertical rise. (R); (L)

L28 Recoil: A bullet is fired from a model gun. Direct measurements can be made of the bullet's speed, and the speed of recoil of the gun. (R); (L)

L29 Colliding freight cars

The collision of two freight cars is photographed in slow motion during a railroad test of the strength of couplings. (E); (L)

L30 Dynamics of a billiard ball

Slow motion photography of a rolling ball striking a stationary ball. The target ball slides, then starts to roll. Linear momentum and angular momentum are conserved. (E); (L)

L31 A method of measuring energy—nails driven into wood

A nail is driven into wood by repeated identical blows of a falling weight. A graph of penetration depth versus number of blows can be made; the result is nearly a straight line. This loop establishes a criterion for energy measurement used in the next two loops. (E): (L)

L32 Gravitational Potential Energy

Dependence of gravitational potential energy on weight; dependence on height. (E); (L)

L33 Kinetic energy

Dependence of kinetic energy on speed; dependence on mass. Slow-motion photography allows direct measurement of speed. (E); (L)

L34 Conservation of Energy I—Pole Vault

The total energy of a pole vaulter can be measured at three times: just before takeoff, the energy is kinetic; during the rise it is partly kinetic, partly gravitational potential, and partly elastic energy of the distorted pole; at the top, it is gravitational potential energy. (E); (L)

L35 Conservation of Energy II—Aircraft Takeoff

Flying with constant power, an aircraft moves horizontally at ground level, rises and levels off. Kinetic and potential energy can be measured at three levels. (E); (L)

L36 Kinetic theory—gas model

Impacts of plastic beads on a piston can be measured to show the dependence of average force on molecular speed and on number of impacts per second. (E); (L)

L37 Reversibility of time

After some introductory shots of real-life actions that may or may not be reversible, the film shows events of increasing complexity: a two-ball collision on a billiard table; a four-ball event; finally a ball rolls to a stop while making some 10^{23} (invisible) collisions with the molecules of the table surface. (E)

L38 Superposition of waves

Amplitudes and wavelengths of two waves are varied; the resultant is shown. Display is in three colors on the face of a cathode-ray tube. (E)

L39 Standing waves on a string

Production of standing waves by interference of oppositely-moving equal waves is shown in animation. Then a tuning fork sets a string into vibration; several modes are shown as the tension is adjusted. The string's motion is also shown stroboscopically. (E)

L40 Standing waves in a gas

A loudspeaker excites standing waves in a glass tube containing air. Nodes and antinodes are made visible in two ways: by the motion of cork dust, and by the cooling of a hot wire inside the tube. (E)

L41 See Unit 5

L42 Vibrations of a rubber hose

A long vertical rubber hose is agitated by a variable-speed motor at one end. The frequency is adjusted to show a succession of modal patterns. (R)

L43 Vibrations of a wire

A horizontal stiff wire is set into vibration; the driving force is supplied by the interaction of alternating current through the wire and a fixed magnetic field. Several modes of vibration are shown, both for a straight wire (antinode at free end) and for a circular wire (nodes equally spaced around the circumference). The patterns are shown in real time, and also stroboscopically. (E)

L44 Vibrations of a drum

A loudspeaker is placed beneath a horizontal circular rubber drum head. Several symmetric and antisymmetric modes are shown stroboscopically, in apparent slow motion. (R)

L45 Vibrations of a metal plate

Vibration patterns are made visible by sprinkling white sand on a vibrating plate; sand collects at the nodal lines. (E)

16mm Films

F17 Elements, Compounds and Mixtures

33 minutes, color, Modern Learning Aids. A discussion of the differences between elements, compounds and mixtures, showing how a mixture can be separated by physical means. Demonstrates how a compound can be made and then taken apart by chemical methods, with identification of components by means of their physical properties such as melting point, boiling point, solubility, color, etc.

F18 The Perfection of Matter

25 minutes, color, Nuffield Foundation. This film is mainly for atmosphere. A cameo treatment of medieval culture and science, principally alchemy. Some explicit discussion of closed systems and conservation of mass.

F19 Elastic Collisions and Stored Energy

27 minutes, B & W, Modern Learning Aids. Various collisions between two dry ice pucks are demonstrated. Cylindrical magnets are mounted on the pucks producing a repelling force. Careful measurements of the kinetic energy of the pucks during an interaction lead to the concept of stored or potential energy.

F20 Energy and Work

28 minutes, B & W, Modern Learning Aids. Shows that work, measured as the area under the force-distance curve, does measure the transfer of kinetic energy to a body, calculated from its mass and speed. A large-scale falling ball experiment, a non-linear spring arrangement and a "Rube Goldberg" graphically establish work as a useful measure of energy transfer in various interactions.

F21 Conservation of Energy

27 minutes, B & W, Modern Learning Aids. Energy traced from coal to electrical output in a large power plant, quantitative data is taken in the plant; conservation law demonstrated for random and orderly motion.

F22 Mechanical Energy and Thermal Energy

22 minutes, B & W, Modern Learning Aids. This film shows several models to help students visualize both bulk motion and the random motion of molecules. It shows their interconnection as the energy of bulk motion. Demonstrates that random motion can average out to a smooth effect. Shows model of thermal conduction. Demonstrates a model using dry ice disc and small steel balls, in which bulk mechanical energy of the disc is converted to "thermal" energy of random motion of the balls. Develops a temperature scale by immersing canisters of two gases in baths of various temperatures, reading the resulting pressure; through this, explains the origin of the absolute temperature scale.

F23 Demonstrating the Gas Laws

21 minutes, B & W and color, Coronet. A discussion and demonstration of Boyle's law showing that the volume of a gas varies inversely with the pressure, provided the temperature remains constant; and of Charles' law, which states that the volume of a gas varies directly with the absolute temperature, if the pressure on the gas remains constant. The combined gas law is then derived and demonstrated.

F24 Gas Laws and Their Application (Boyle, Charles and Gay-Lussac)

13 minutes, B & W, EBF. Demonstrates some of the early research which led to the discovery of the relationships between the temperature, volume and pressure of gases. Applications of the gas laws are shown in compression, refrigeration, heat engines and low temperature research.

F25 The Molecular Theory of Matter

11 minutes, B & W and color, EBF. Through stop-motion photography and animation, this film presents the basic assumption of the kinetic molecular theory: matter in its three phases may be considered to be composed of small particles in motion. Experiments show the passage of one substance through another, how pressure of a gas can be measured, change of phase and the diffusion in liquids.

F26 Progressive Waves: Transverse and Longitudinal

9 minutes, B & W, McGraw-Hill. To introduce the subject of transverse progressive waves, a pulse is sent down a stretched rope. Its velocity of travel is shown to be dependent upon elastic and inertial factors. Steady wave-train propagation along a stretched string is then illustrated. Wavelength and frequency are identified and related. Similar treatment is followed for longitudinal wave disturbance moving down a row of beads connected by light springs.

F27 Stationary Longitudinal Waves

9 minutes, B & W, McGraw-Hill. The treatment in this film follows closely that used in Stationary Transverse Waves. A string of beads connected by light springs constitutes the medium. Both a pulse and a steady train of waves are shown. The situation at a fixed end and at a free end is discussed. At the conclusion the film touches on pipe resonance.

F28 Stationary Transverse Waves

9 minutes, B & W, McGraw-Hill. The nature of stationary waves is approached from the standpoint of kinematics. Pulse reflection at the fixed end of a stretched string is studied, including phase change. Conclusions are applied to the reflection of a steady wave-train at a string boundary. Resulting stationary waves are compounded of two oppositely traveling wave-trains. Nodes and antinodes are identified and their positions related to the wavelength. The film concludes with a consideration of the resonance patterns for a stretched string of limited length.

F29 Sound Waves in Air

35 minutes, B & W, Modern Learning Aids. The wave characteristics of sound transmission are investigated with large scale equipment using frequencies up to 5000 cycles. Experiments in reflection, diffraction, interference and refraction are supplemented with ripple-tank analogies. Interference is shown in both the pattern of standing waves and in a pattern reflected from a grating. A gas-filled lens is used to refract sound.

Film Sources

Films in this unit can be obtained from the following sources:

Modern Learning Aids
1212 Avenue of the Americas
New York, New York 10036

Nuffield Foundation
Color Film Services, Ltd.
London, England

Encyclopaedia Britannica Films
1150 Wilmette Avenue
Wilmette, Illinois 60091

Coronet Films
Coronet Building
Chicago, Illinois 60601

McGraw-Hill Book Company
Text-Film Division
330 West 42nd Street
New York, New York 10036

A more complete film source reference is given in the Unit 2 Teacher Guide, pages 48 - 51.

Reader

R1 "Silence, Please"
By Arthur C. Clarke

A fictional scientist tells of an apparatus for producing silence. Although the proposed scheme is improbable, the story has a charming plausibility.

R2 The Steam Engine Comes of Age
R. J. Forbes and E. J. Dijksterhuis
1963

The invention of the steam engine was a major factor in the early stages of the Industrial Revolution.

R3 Energy
William Thomson and Peter G. Tait
1862

The principle of conservation of energy was proposed simultaneously by many physicists, including von Mayer, Joule, von Helmholtz, and Thomson. This popularization appeared soon after the discovery. William Thomson is better known as Lord Kelvin.

R4 The Barometer Story
Alexander Calandra
1964

A physicist and educator tells a parable to illustrate the inadequacies he sees in the present system of teaching.

R5 The Great Molecular Theory of Gases
Eric M. Rogers
1960

The kinetic theory of gases is a marvelous structure of interconnecting assumption, prediction and experiment. This chapter supplements and reinforces the discussion of kinetic theory in the text of Unit 3.

R6 On the Kinetic Theory of Gases
James Clerk Maxwell
1872

Abandoning a mechanical view of studying the behavior of each individual gas molecule, Maxwell adopts a statistical view and considers the average and distribution for velocity and energy.

R7 The Law of Disorder
George Gamow
1947

Completely random motion, such as the thermal motion of molecules, might seem to be out of the realm of lawfulness. But on the contrary, just because the motion is completely disorderly, it is subject to statistical laws.

R8 The Law
Robert M. Coates
1947

The "law of averages" applies to all randomly moving objects whether in kinetic theory or in city traffic. This story from The New Yorker raises in fictional form the question of the meaning of a statistical law.

R9 The Arrow of Time
Jacob Bronowski
1964

How can a viewer distinguish whether a film is being run forward or backward? The direction of increasing disorder helps to fix the direction of the arrow of time.

R10 James Clerk Maxwell
James R. Newman
1955

The biography of this great Scottish physicist, renowned both for kinetic theory and for his mathematical formulation of the laws of electricity and magnetism, is presented in two parts. The second half of this selection is in Reader 4.

R11 Maxwell's Demon
George Gamow
1965

Throughout history, there have been men who endeavored to design machines that produce energy from nothing. All their efforts have been thwarted by the law of conservation of energy. But why not a machine that extracts unlimited energy by cooling its surroundings? Unless Maxwell's demon intervenes, this machine is highly improbable.

R12 Randomness and the Twentieth Century
Alfred M. Bork
1967

The use of random elements is common today not only in science, but also in music, art and literature. One influence was the success of kinetic theory in the nineteenth century.

R13 Introduction to Waves
Physical Sciences Study Committee
1965

Everyday observation of waves includes ripples in water or vibrations set up by a slammed door. A wave is a traveling pattern, not the mass movement of matter.

R14 What is a Wave?
Albert Einstein and Leopold Infeld
1961

Two masters of physics introduce the wave concept in this section from a well-known popular book.

R15 Wave Motion and Acoustics
Robert Bruce Lindsay
1940

The basic equation which summarizes the properties of waves is developed as an example of the application of mathematics to physics. Lindsay's detailed discussion will be rewarding for the student who has some knowledge of calculus coupled with persistence to work on the more advanced passages.

R16 Musical Instruments and Scales
Harvey E. White
1940

Many aspects of the music produced by instruments, such as tone, consonance, dissonance and scales, are closely related to physical laws.

R17 Founding a Family of Fiddles
Carleen M. Hutchins
1967

The four members of the violin family have changed very little in hundreds of years. Recently, a group of musicians and scientists have constructed a "new" string family.

R18 The Seven Images of Science
Gerald Holton
1963

Some nonscientists hold odd views of the nature of science. This article catalogs and analyzes the most common fallacies.

R19 Scientific Cranks
Martin Gardner
1957

Science's greatest men met with opposition, isolation and even condemnation for their novel or "heretic" ideas. But we should distinguish between the heretical innovator and the naive crank.

Demonstrations

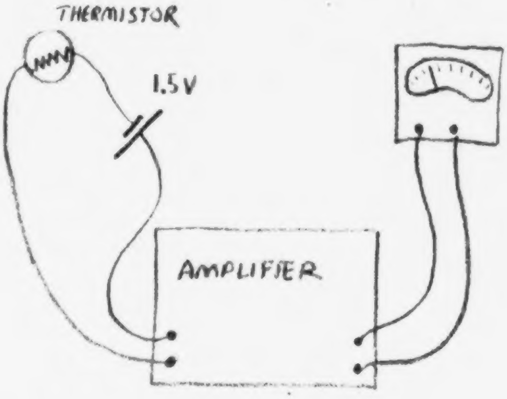
D33 An inelastic collision

A perfectly elastic collision is one in which the total amount of kinetic energy present is the same before and after the collision. An inelastic collision, then, must be one in which kinetic energy is lost. What happens to it? You can show qualitatively by a very simple demonstration that a loss in kinetic energy is associated with a rise in temperature of the interacting bodies: pound a nail into a piece of wood, and have students touch the nail. Remind students that when they analyzed a slow elastic collision they found that the total kinetic energy of the carts decreased temporarily, then went back nearly to its original value. Ask what would happen if the bumpers on the carts had instead been made from a soft metal such as lead. Then demonstrate such a collision with the apparatus described below.

A practical experiment to show the conversion of kinetic energy into heat in a collision requires a very sensitive thermometer: the rise in temperature will be only a few tenths of a degree.*

A sensitive thermometer with very low heat capacity is made from a thermistor (a pellet of semiconductor material whose resistance drops markedly with increase in temperature) and an amplifier. A thermistor, already embedded in a thin strip is supplied by Project Physics. An identical thermistor, not imbedded, is provided for other demonstrations and activities. Briefly, an increase in temperature of the thermistor increases the input current to the amplifier; which increases the output current many times more. Small changes in the large output current (and, therefore, small changes in temperature) can be detected by bucking out most of the output current with the OUTPUT OFFSET control and connecting the output to a sensitive meter.

Connect the thermistor between a 1.5 volt cell and the input terminals of the amplifier.* Turn the ac/dc switch to dc. Set the OUTPUT OFFSET control to its maximum (full counterclockwise) and the gain to about half-way up. Connect a meter across the output of the amplifier. It is most convenient to use a multimeter with several voltage ranges. Start on the least sensitive (highest voltage) dc voltage scale. The reading should be near zero; if it is not, adjust the gain control until it is. Change down to progressively more sensitive scales, using the gain control as necessary to keep the reading on scale.



Finally, with the meter on a scale of not more than 250 mV or 100 μA full scale deflection, adjust the gain or OUTPUT OFFSET so that the reading is approximately zero. Final adjustment is very sensitive, and it is not necessary to set to zero exactly. If you do not have such a sensitive scale on a multimeter, substitute an independent millivolt- or microammeter, but always use the coarser scales of the multimeter first, so that the sensitive meter will not be overloaded. Use a projection meter if you can.

With the meter reading approximately zero, increase the temperature of the thermistor by bringing your finger, a hot iron, or a match close to it (but not actually touching). There should be an appreciable increase in meter reading. Even blowing on the thermistor may send the reading off scale. Use one of these techniques to show that the system responds to temperature changes and that an increase in temperature increases the meter reading.

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*Although students are not ready for the quantitative treatment given here, the calculations are shown so that you will appreciate the difficulties involved in making the experiment quantitative.

If two carts, each loaded so that its mass is 2 kg, approach each other with speeds of 1 m/sec, their total kinetic energy is:

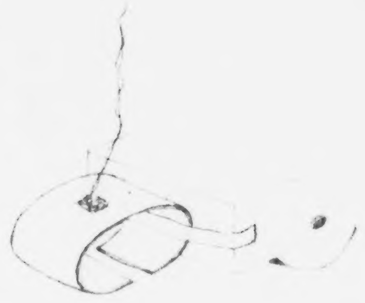
$$2 \quad \tfrac{1}{2} \times 2\text{kg} \times (1\text{ m/sec})^2 = 2\text{ joules}$$

If this energy goes entirely into heat, it amounts to about 0.5 calorie. The temperature rise in a 50-gram strip of lead with a heat capacity of 0.03 cal/g C° will be

$$\Delta t = \frac{H}{mc} = \frac{0.5\text{ cal}}{50\text{g} \times 0.03} = 0.3\ \text{C}^\circ.$$

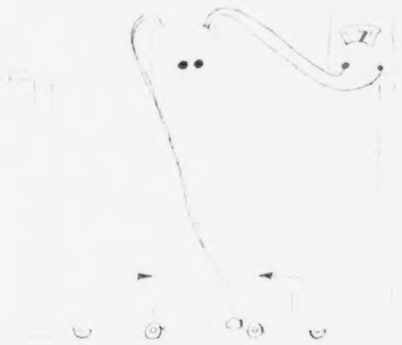
*For detailed notes on the amplifier/power-supply unit see the Equipment Note at the end of this Teacher Guide.

Bend the lead strip into a ring and tape it to the end of a dynamics cart:



Do this several minutes before you want to perform the demonstration so that the lead will have time to recover from the heating caused by handling and bending it.

Wait for the meter reading to reach a steady value before doing the collision demonstration.



Arrange the apparatus so that the wires from the amplifier-power supply unit to the cart are hanging freely. Push the carts together so that the lead ring absorbs the kinetic energy impact. The meter deflection shows the increase in temperature of the lead. If you plan to calibrate the thermistor later for quantitative work, record the meter readings from this demonstration for future reference.

In a trial we found that when a cart moving at about 1 meter/second collided with a stationary object (the wall), the temperature rise in the lead caused a change of 50 mV in meter reading on a 250 mV meter, or 30 μA on a 100 μA meter.

Suggestion for Quiz or Class Discussion

In order to show that the meter deflection is not simply due to mechanical shock (as one might suspect with such a sensitive instrument) you can try putting the lead strip on a brass block and hitting the block with a hammer. In this case there will be no temperature rise.

D34 Predicting the range of a slingshot

In this experiment, the impact point of a slingshot projectile is predicted from the drawing force and distance. The objectives are to provide (1) an exercise in energy conservation that both can (and will) engage the students' intuition, and (2) and experience of successfully predicting from the dry machinery of theoretical mechanics an event which is interesting to the students. The derivation of the expression for range requires some analysis not treated in the text: resolution of vectors, and the work done by a varying force.

Procedure

A satisfactory sling can be made from a large rubber band. To insure a reasonably small error in measuring the length of draw, the rubber band should allow a draw of at least 20 cm without overstraining. The support can be almost anything that is sufficiently rigid; a pair of ring stands clamped to the table will suffice. The rubber band should be attached to the supports in a manner that will allow a minimum of friction during draw and release. (Alternatively, we have found that a toy shop model firing a 3/4" steel ball gives very satisfactory results.)

An excellent projectile can be made by twice folding a 1" × 5" piece of 1/16" lead sheet; its mass should be great enough so that only a negligible fraction of the kinetic energy will appear in the rubber band upon release, but small enough to give an impressively long range—about 50 gm is good. Be careful to fold the lead in such a way that it will not catch on the rubber band when it is released. The margin of error is adequately represented (and the drama is increased) by placing the mouth of a wastebasket at the expected impact point. (See Fig. 1.)

Fig. 1

The draw and release can be made satisfactorily with the thumb and forefinger on the edges of the projectile, but some skill is required to avoid frictional losses on release. A thread-burning technique such as is suggested in Fig. 2 below may prove to be better. The drawing force and distance are measured with a spring balance and meter stick, as indicated in the diagram. (The meter stick should be removed for launching.)

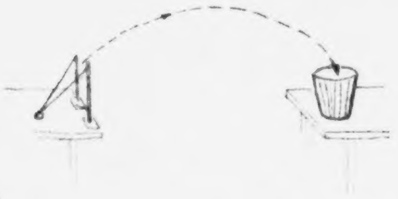


Fig. 1

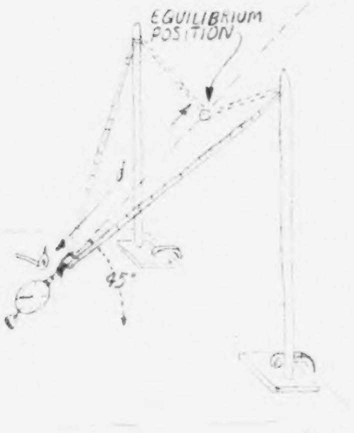


Fig. 2

Derivation of range R

(1) $R = v_{\rightarrow} t$

(2) $t = 2t_{\uparrow}$

$R = 2v_{\rightarrow} t_{\uparrow}$

(3) $t_{\uparrow} = \dfrac{v_{\uparrow}}{a_g}$

$R = \dfrac{2v_{\rightarrow} v_{\uparrow}}{a_g}$

(4) $v_{\rightarrow} = v_{\uparrow} = \dfrac{v}{\sqrt{2}}$

(for 45° launch angle*)

$R = \dfrac{v^2}{a_g}$

(5) $E_{K_{max}} = \frac{1}{2}mv^2$

$R = \dfrac{2E_{K_{max}}}{ma_g}$

(6) $E_{K_{max}} = E_{P_{max}}$

$= \bar{F}d_{max}$

$R = \dfrac{2\bar{F}d_{max}}{ma_g}$

(7) $\bar{F} = \frac{1}{2}F_{max}$

(for linear force)

$R = \dfrac{F_{max}d_{max}}{ma_g}.$

*Vector resolution is not treated in the text, but should be familiar from programmed instruction and problems.

The derivation is lengthy, but that is part of the point. If you know that each input principle is correct and if your math is proper, you should have confidence in your prediction even if the derivation is long.

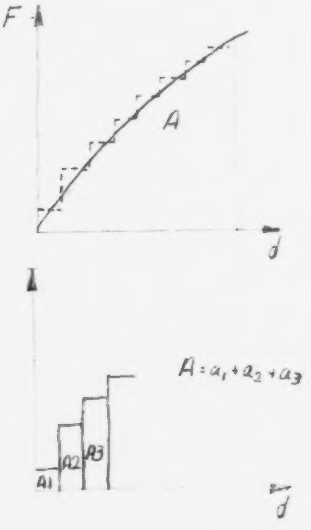


Fig. 3

With the exception of (4) and (5), these relations are all idealization. As it happens. all the actual deviations from the idealizations are in a direction that reduces the actual range. Only the step represented by (7) allows the deviation from the idealization to be accounted for—students can plot F against d and find $E_{p_{max}}$ (or $\overline{F}$) from the area under the somewhat non-linear curve. But remember that the work done by a varying force is not considered in the text. Probably the best way to treat a non-linear force-extension curve is to replace it by an equivalent bar graph. (See Fig. 3.)

The most appropriate approach to the derivation for a particular student must be determined by the teacher. Below are listed some approaches to suggest the broad range of possibilities. Any one approach would not have to be used for a whole class. For example, approach "C" might be planned for most of the class while several faster students could be begun as in the "A" previously. In any case, the rationale for bothering with prediction instead of just trying it should be easy to concoct in this era of probing space with extremely expensive machinery.

Possible Approaches

A. Students are requested to derive an expression for the range of a projectile as a homework assignment, perhaps over a week or longer, the teacher being available to discuss problems.

B. As in A, but the problem is given better initial direction by a class discussion of "what would you have to know in order to figure out how far it will go?"

C. A discussion is begun as in B, but it is pursued through the entire derivation, suggestions for steps coming almost always from the students. ("You say you want to know the initial speed—how could you find it?")

D. As in C, but with the teacher providing the structure by means of concrete leading question. ("We can find the speed if we know the kinetic energy; what is the relation of v and KE?")

E. Lecture presentation of a polished derivation.

The more that students are able to come up with on their own, the more valuable the experiment is likely to be for them; on the other hand, painfully heavy demands on the students will spoil the effect of the demonstration and the time involved must always be weighed. In any approach, it should eventually be pointe out that the procedure is one very common to science (see Fig. 4).

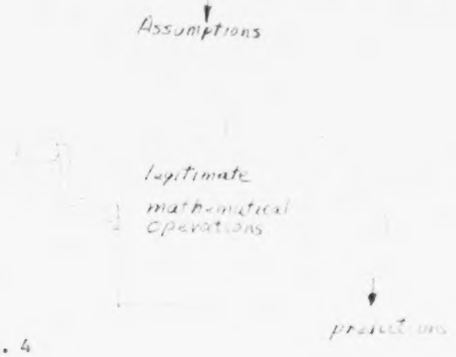


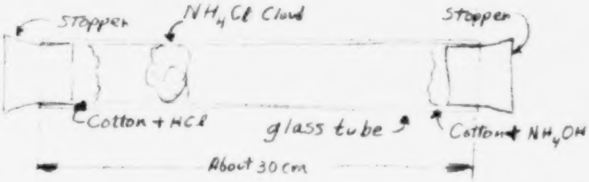
Fig. 4

An interesting extra discussion might center on the question, "What do you do when the actual event doesn't match the prediction?" Any discussion of the uncertainty in the predicted location of the impact point should precede the actual launching. In addition to uncertainties in measurement, the effects of the idealizations should be considered.

The amount of derivation required of the students may vary a great deal; whatever treatment of the experiment is used, the central intent is that the experiment be predictive and interesting. For these ends it is important that the actual launching be the very last step and that the anticipated range be as long as is practicable.

(Courageous teachers who are using approach D or E might want to try a dramatic tack wherein an erroneous step is surreptitiously included in the derivation or calculation, causing the projectile to overshoot the target by a factor of two or more, perhaps proceeding out a door or window. The immediate drop in the stock of science must, of course, be quickly recouped—ideally by the students discovering the error themselves.)

D35 Diffusion of gases



The formulation of a cloud of NH_4Cl vividly demonstrates the diffusion of HCl and NH_3 molecules through the air.

The demonstration also shows:

a) The lighter NH_3 molecules diffuse more quickly than the heavier HCl ones—the cloud of NH_4Cl is formed further from the NH_3 source.

b) Diffusion is comparatively slow at atmospheric pressure.

D36 Brownian motion

The molecular-kinetic theory of matter developed in Chapter 11 as a model for a gas is consistent with many experimental observations (gas laws, specific heat, etc.). The Brownian motion is the most direct evidence we have been able to present so far for the molecular-kinetic theory of matter. The phenomenon is called Brownian because the botanist Robert Brown, although not the first to observe it, showed (circa 1827) that it was found for a wide variety of particles, both organic and inorganic.

Historical Significance of Brownian Motion

Brown himself had no theory to account for the motion. He found that it existed for all kinds of inorganic particles, and various suggestions including the irregular molecular bombardment of the particle were made to account for it. (In fact, Brown used the persistence of the phenomena in isolated drops to "prove" that it could not be due to molecular activity.) But until work by Einstein and Smoluchowski and by Perrin early in the twentieth century, several eminent scientists still disputed the existence of atoms.

Einstein and Smoluchowski, using the mathematics of probability, were able to make quantitative predictions about the observed motion of the particles which were closely confirmed by Perrin's measurement. One of the achievements of the theory was the first accurate determination of Avogadro's number.

A Note Concerning Demonstrations and Experiments

Chapter 12, perhaps more than any other chapter in the unit, depends upon demonstrations and experiments to give substance to the material discussed in the text. If you examine the chart on page 73, you will notice that the wave properties are carried entirely by either the experiments or the demonstrations. If you have the time and wish to do both experiments, it is recommended that you only do demonstration D41. If only E30 is done by the students then D38, D41 and D42 are recommended. If only E31 is carried out, D37, D39, D40 and D41 should be considered. If no experiments are planned, the demonstrations can carry the burden. The list below summarizes some possible approaches:

E30, E31, D41

E30, D38, D41, D42

E31, D37, D39, D40, D41

All Demonstrations

In all cases, D46 should be included.

Wave Property	E 30*	E 31*	D 37	D 38	D 39	D 40	D 41	D 42	D 43	D 44	D 45	D 46
Pulse amplitude and length	S,R		S,B									
Pulse velocity	R		S,B,R									
Traveling pulse as energy transport				S,B,R								
Absorption at a barrier		P		B,R,P								
Superposition	S				S,B							T
Reflection from a free end	S					S,B						
Reflection from a fixed end (phase inversion)	S					S,B						
Partial reflection at an interface where $v_1 \neq v_2$	S,R					S,B,R						
Two dimensional reflection from barrier	R	P				R,P						
Transmission (impedance, or index match)		P				S						
Wave trains (reflection, refraction, etc.)							B,R,P					
Refraction as a function of angle	R							R,P				
Refraction as a function of velocity	R							R,P				
Refraction through wave shaping elements (lenses)	R	P						R,P				
Interference patterns	R								R,P			
Young's experiment		R							R			
Diffraction around obstacles (frequency dependence)	R	P								R,P		
Single, double and multiple slit diffraction patterns	R									R,P		
Standing waves	R	P									S,B,R,P	
Longitudinal — Transverse	S		S									

KEY:
S — Slinky
B — Bell Wave Machine
R — Ripple Tank
P — Project Equipment
T — Turntable Oscillator

D37 Wave propagation

	coil slinky	bell wave machine	ripple tank	Project equipment
Pulse amplitude and length	X	X		
Pulse velocity	X	X	X	

S - Slinky

The slinky is employed because pulses propagate slowly enough along it to be easily observed. It can be used either with a partner or suspended horizontally with strings attached to a horizontal wire above it. If used with a partner, pull the slinky out to a length of about 30 feet on a smooth floor. Be sure never to let go of an end of the spring when it is stretched because the resulting snarl is almost impossible to untangle. This problem is eliminated when it is suspended properly. The strings holding the slinky suspended should be at least two feet long, and spaced a few inches apart along the entire length of the spring. One end of the spring may be tied to some support. Stretch the slinky slightly by fastening a light, 5-foot string to the other end.

Pulse amplitude and length can be demonstrated by sending different size pulses along the spring. The amplitude of a pulse sent may be defined as the maximum displacement of any point on the spring. Ask questions such as: In what direction do the actual spring coils move as the wave motion travels along the spring? Does the shape of the wave change as it travels along the spring? What determines the length of the pulse? Is there a relation between the length of the pulse and its amplitude? Pulse velocity can be determined by measuring the time it takes the pulse to travel back and forth several times over a measured distance. How is the velocity affected by changing the amplitude? the pulse duration? the tension? What determines the pulse velocity?

Transverse waves can be demonstrated by grasping one end of the spring and snapping it rapidly at right angles to the length of the spring while longitudinal waves are brought about by displacing the end of the spring in a direction parallel to its length. A logical question to pose is: As the wave travels along the spring, in what direction do the actual spring coils move?

B - Bell wave machine

See Exp. 1, Getting Acquainted with Waves and Exp. 4, Wave Speed in the book Similarities in Wave Behavior. This book accompanies the Wave Machine, whether rented or bought.

R - Ripple Tank

Straight waves can be generated by placing a three-quarter inch dowel or section of broomstick handle along one edge of the tank and rolling it backward a fraction of an inch and then stopping, or employing an electric rippler as supplied by most scientific apparatus houses. By using a hand (or electronic) stroboscope, determine the speed of the waves and verify the relation $v = f\lambda$. Stress the fact that this relationship is applicable to waves in the ripple tank and on a coil spring (slinky). Define the amplitude as the maximum displacement of any point on the surface. Make several measurements of frequency and wavelength when you determine the wave speed. How is the frequency of your strobe related to the frequency of the waves? What can be said about the accuracy of your determination of the wave speed?

D38 Energy transport

	coil slinky	bell wave machine	ripple tank	Project equipment
1. Traveling pulse as energy transport	X	X	X	
2. Absorption at a barrier (receiver, damping)		X	X	X

S - Slinky

Energy transport concerns itself with the size or amplitude of a traveling wave, and what happens to the amplitude as the wave propagates. Under the assumption that it takes "work" to deform a medium we associate the energy a wave possesses with its amplitude and ask such questions as: Does the "shape" of the wave change as it travels along the spring? Does the size, or amplitude of the wave change? Why does it change? What determines the amplitude of the pulse? The progressive loss in amplitude as the pulse travels along the spring is called damping. What happens to the energy lost?

B - Bell Wave Machine

See Exp. 2, Wave Damping, page 10, and Exp. 3, Waves as Carriers of Energy, page 11, in the book Similarities in Wave Behavior that accompanies the Wave Machine. Absorption at a barrier can be demonstrated in Exp. 10 on page 26 where a dash-pot-and-piston arrangement is employed for a mechanical load.

R - Ripple Tank

Generate straight-fronted and circular-fronted waves. Does the amplitude of the straight-fronted wave change as the wave travels? If so, how? Why? Is any energy lost? Energy absorption at a barrier can be shown by allowing waves to strike the gauze fences and observing what happens to the amplitude. A discussion of reflection is not warranted at this time.

P - Project Equipment

The absorption of audible sound, ultrasound and microwave "waves" can be demonstrated with an assortment of materials such as pieces of metal, wood, glass, styrofoam, paraffin, masonite, etc., while the fact that energy is transmitted can be associated with the effect these waves have on various receivers.

D39 Superposition

	coil slinky	bell wave machine	ripple tank	Project equipment
Superposition	X	X		

S - Slinky

The demonstration set-up should be the same as it was for D37.

Superposition can be demonstrated by generating two simultaneous pulses, one from each end of the slinky.

Ask such questions as: What happens to the pulses as they collide? When the pulses meet, how does the resulting amplitude compare with the amplitude of each individual pulse? When the pulses are on the same side of the slinky? On opposite sides?

B - Bell Wave Machine

See Exp. 5, Criss-crossing of Waves in Similarities in Wave Behavior.

D40 Reflection

	coil slinky	bell wave machine	ripple tank	Project equipment
Reflection from a free end	X	X		
Reflection from a fixed end (phase inversion)	X	X		
Partial reflection at an interface where $v_1 \neq v_2$	X	X	X	

D40 Reflection (continued)

	coil slinky	bell wave machine	ripple tank	Project equipment
Two-dimensional reflection from barrier			X	X
Transmission (impedance, or index match)	X			

S - Slinky

Reflection from a fixed end (phase inversion) is demonstrated by observing the reflected pulse when one end of the coil (slinky) is held rigidly in place (infinite impedance). The other case, reflection from a free end (zero impedance) can be observed by having the end of the spring unconnected to anything but a long thin thread. Observe these two cases to see whether the reflected pulse has its displacement on the same side or on the opposite side of the spring from the incoming pulse. Partial reflection from an interface where $v_1 \neq v_2$ is then investigated by tying together two coil springs on which waves travel with different speeds. Send a pulse first in one direction and then in the other, asking what happens when the pulses reach the junction between the two springs. Employing different springs, which give rise to different media, and observing the amplitude of the transmitted and reflected waves, one can qualitatively demonstrate impedance, or index-match, in terms of the media velocities.

B - Bell Wave Machine

Reflection from free or fixed ends is demonstrated in Exp. 6, page 18, while partial reflection at an interface is contained in Exp. 11, page 31, of Similarities in Wave Behavior which accompanies the Wave Machine.

R - Ripple Tank

The speed of water waves depends on the depth of the water. Two different depths of water therefore constitute two different media in which waves can be propagated. With a glass plate supported in the ripple tank, this situation can be brought about. Ask what will happen if straight waves, generated in deep water, cross the boundary between the two media involved? Here we are primarily interested in the "reflected" wave. The transmitted wave and refraction is presented in D42. Two-dimensional reflection from an "opaque" barrier is always shown using paraffin blocks. The angle of incidence and angle of reflection can be measured.

P - Project Equipment

The reflection of sound, ultrasound and microwaves can be demonstrated by reflecting them against an assortment of materials. From D38 you will have an idea of what materials will do well for optimal reflection.

D41 Wave trains

	coil slinky	bell wave machine	ripple tank	Project equipment
Wave trains (reflection, refraction, etc.)		X	X	X

B - Bell Wave Machine
R - Ripple Tank
P - Project equipment

Demonstrations of reflection, refraction, etc., are presented employing a constant frequency source so as to distinguish between a pulse and a wave train.

D42 Refraction

	coil slinky	bell wave machine	ripple tank	Project equipment
Refraction as a function of angle			X	X
Refraction as a function of velocity			X	X
Refraction through wave shaping elements (lenses)			X	X

R - Ripple Tank

Refraction in the ripple tank can be observed by laying a sheet of glass in the center of the tank to make a shallow area. To make the refraction quite obvious, the frequency of the wave should be low (less than 10 cycles/sec), and the water over the glass should be as shallow as possible; use just enough water to cover the glass. The waves refract at the edge of this area because they travel more slowly where it is shallow. The wave that passes over the plate is the refracted wave; the acute angle between its front and the boundary of the new medium is the angle of refraction, r. Try varying the angle at which the pulse strikes the boundary between deep and shallow. Measure the angles of incidence and the corresponding angles of refraction over a wide range of values and determine how the angle of refraction varies as a function of the angle of incidence. Determine the velocities of the waves in the deep and shallow parts of the tank. What is the ratio of their velocities? Of their wavelengths? Compare with the data from angle measurements. Paraffin lenses can be cut or plastic ripple-tank lenses can be bought to demonstrate the focussing of waves.

P - Project Equipment

Sound—Fill a "lab gas" balloon with carbon dioxide (less dense gases do not work well). The resulting spherical "lens" will focus sound a few centimeters beyond the balloon. Explore the area near the balloon on the opposite side from the source. Try two or more frequencies.

Ultrasound—at the higher frequency of ultrasound, the gas lens may be too large. Experiment with various materials which are transparent to ultrasound and can be formed into a sphere, or hemisphere. Try the gas-filled balloon with this higher frequency.

Microwaves—cast a hemisphere or hemicylinder of paraffin wax about 3 cm radius, perhaps in a frozen-fruit juice can. It will act as a short focal-length lens. Observe the area behind the lens with the lens in position and while removed.

D43 Interference patterns

	coil slinky	bell wave machine	ripple tank	Project equipment
Interference patterns			X	X

R - Ripple Tank

The interference of waves from two point sources are demonstrated best with the ripple tank, since the student can see the development of the nodal lines as waves progress from the point sources. It is enlightening to observe first a single circular pulse, then two simultaneously generated pulses. Follow the path of the intersection of the two pulses. Next observe two pulses that originated at different points and at slightly different times. The locus of the intersection points is seen to curve away from the sources. Next produce two or three successive pairs of simultaneous pulses, observing their intersections, and finally the intersections of continuous waves. By marking the positions of the nodal lines as projected, it is possible to establish quantitatively the wavelength from the double slit equation. For detailed suggestions for demonstrations consult the PSSC Laboratory Guide, Experiments II-8 through II-13, or Lehrman and Swartz, Laboratory Experiments, Nos. 36, 37, 39, 40 and 43, or Brinckerhoff and Taft, Modern Laboratory Experiments in Physics, Nos. 31 and 35.

P - Project Equipment

The double sources used should be placed to minimize reflections from hard surfaces, otherwise spurious nodal points will be present. Set the source transducers at the edge of the table top and directed away from nearby walls.

Mount the two sources so that the distance from center to center can be measured, as well as the distance along the perpendicular bisector of the line connecting the sources.

Sound—Connect the two loudspeakers in series to the oscillator and mount them at the edge of the table about 25 cm apart. Observe the signal strength as the ear is moved along a horizontal line in front of the sources. Move further away from the sources and change the source separation to see what happens. Note the effect of changing the frequency.

Two Source Interference (quantitative). Again the ripple tank and film loops are best for showing how the interference pattern is produced by two waves. The double sources used in the other experiments (except light) should be placed to minimize reflections from hard surfaces, otherwise spurious nodal points will be present. Set the source transducers at the edge of the table top and directed away from nearby walls.

Mount the two sources so that the distance from center to center can be measured, as well as the distance along the perpendicular bisector of the line connecting the sources.

Sound—Connect the two loudspeakers in series to the oscillator and mount them at the edge of the table about 25 cm apart. Observe the signal strength as the ear is moved along a horizontal line in front of the sources. Move further away from the sources and change the source separation to see what happens. Note the effect of changing the frequency.

This demonstration can be made quantitative by mounting a meter stick parallel to the line of the speakers so that the nodes can be located and their positions noted from the stick. Plot the positions (x) of maxima and minima; also record D and d.

Ultrasound—Plug the second source into the amplifier (the plugs "stack") and arrange the two sources about 5 cm apart. Explore the field with the detector about 25 cm in front of the sources, and plot the maxima and minima positions (x). Also record D and d.

Microwaves—A two-source extension horn is supplied with the generator. Fit the two-source horn into the horn of the generator. It should fit snugly, but if necessary support it with a block of wood or a rubber stopper. Explore the field about 25 cm in front of the two sources and plot the positions of maxima and minima (x). At least 3 maxima should be picked up on either side of the central antinode.

D44 Diffraction

	coil slinky	bell wave machine	ripple tank	Project equipment
Diffraction around obstacles (frequency dependence)			X	X
Single, double and multiple slit diffraction patterns			X	X

R - Ripple Tank

Demonstrate the behavior of pulses and waves at openings in barriers, around obstacles, and edges of barriers (diffraction).

P - Project Equipment

Diffraction—Around Obstacles and Edges. The obstacle must be at least a few wavelengths in size, and yet not too large.

Sound.

a) Use a long piece of thick plywood or celotex about 25 cm wide, placed about 25 cm in front of the source. Explore slowly the area about 75 cm beyond the obstacle, along the x, y and z axes. Try other obstacles—ear separation distances. Try other frequencies.

b) Use a very large piece of wood placed about 25 cm in front of the source, and with its edge lined with the center of the source. Explore the area in the "shadow zone" and immediately out of the shadow zone. How many "fringes" can be counted?

Ultrasound.

a) Use an obstacle of about 3 cm width, placed about 10 cm in front of the source. The detector should be placed 5 to 10 cm beyond the obstacle. Probe on the x, y and z axes.

b) Use a large screen to explore the edge diffraction pattern.

Microwaves.

a) Use an obstacle about 4 or 5 cm wide (a narrow aluminum screen is provided) placed about 10 or 12 cm in front of the source. Explore the field at 5 cm behind the screen and at greater distances. Observe the "maximum" in the center of the shadow.

b) Mask one-half the source with a large screen placed about 12 cm in front of the source. Explore the intensity of the field as the detector is moved parallel to the screen and about 5 cm behind it. You might use the meter to record and plot intensity as a function of x. You should be able to resolve at least two maxima. If the output is weak, use an amplifier to drive an ac (decibel) meter. Note that at the first maximum intensity is greater than when there is no screen.

D45 Standing Waves

	coil slinky	bell wave machine	ripple tank	Project equip-ment
Standing waves	X	X	X	X

S - Slinky

The slinky, pulled out and held rigid at one end, can show standing waves with one to several nodes.

R - Ripple Tank

Place a straight barrier across the center of the tank parallel to straight-fronted advancing waves. When the generator speed and barrier position are properly related, "standing waves" will be formed. How does the length of a standing wave appear to compare with the length of a moving wave? Can you measure the wavelength from the standing wave pattern? Change the depth of the water and ask if a change in speed can be detected.

B - Bell Wave Machine

See Exp. 8, Interference and Standing Waves, page 22, in Similarities in Wave Behavior.

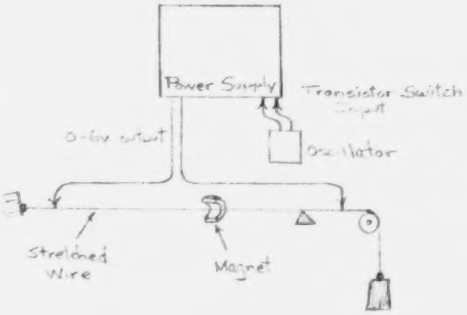
P - Project Equipment

These demonstrations, which are modified versions of "Melde's Experiment," show that a wire or spring of given length, mass-length ratio and tension, can be made to oscillate at only certain predictable frequencies that depend upon mass, length and tension. These frequencies are related to each other by integers. This may be used as an introduction to the concept of characteristic (eigen) frequencies, normal modes and eigenvalues. By analogy, this prepares the student for the concept of quantum numbers.

The demonstration may also be used to show how currents and magnetic fields interact. A current-carrying wire placed in a steady magnetic field has a force acting on it proportional to the current. This concept will be covered in detail in Chapter 14 and should not be mentioned at this time. An alternating current in the wire forces the wire to oscillate. Standing waves can be set up in the wire or spring if the frequency is adjusted to one of the resonant frequencies of the wire or spring.

Transverse Waves

Standing transverse waves in a wire under tension are produced in the arrangement shown schematically in Fig. 1. Clamp one end of a copper wire (#18 or #20) an inch or two above the tabletop and stretch the wire over a pulley with a weight suspended on the other end.



P - Project Equipment

A variable frequency alternating current of about 3 amperes is required for this demonstration. This current is provided by the Project Physics TRANSISTOR SWITCH which pulses the current from a 6-volt power supply. The switch is driven by an audio oscillator set for square wave output. On the new DAMON equipment, connect the oscillator to

TRANSISTOR SWITCH INPUT, move the slide switch to the TRANSISTOR SWITCH position; the switched current output is available at the "0→6V" terminals.

Alternatively a power amplifier, rated at 20 watts or more, can be used. Add a foot or two of #30 nichrome wire in series with the output to provide a load of about five ohms.

The magnet should provide as large a field as possible, and a surplus magnetron magnet serves very well. Remember that the plane of oscillation is perpendicular to the field, so the magnetic field must be horizontal if the standing waves are to be in the vertical plane.

Hold the length and tension constant, and vary the frequency. The wire is fixed at both ends, and may vibrate in the fundamental mode and in many harmonic modes. Many of these modes are seen as the frequency is swept through several multiples of the fundamental frequency. The amplitude is smaller at the higher frequencies. Harmonics are easy to obtain if you position the magnet so that it is at an antinode. The frequency f_n of the n^{th} harmonic is given by:

$$f_n = nf_1 \qquad (1)$$

where f_1 is the fundamental frequency. The frequency of the fundamental can be calculated from:

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\sigma}} \qquad (2)$$

where L is the length of the wire in meters, T, the tension in newtons, and σ the mass per unit length in kg/m.

You can do the same experiment without an audio oscillator. Use a current source with fixed frequency—the output of a 6-volt 3-ampere filament transformer—and adjust either length of or tension on the wire for maximum amplitude standing waves. The tension is adjusted simply by changing the weights, and the length is easily changed by inserting a hardwood wedge under the string between the fixed edge and the pulley.

A typical experiment yields the following results: a 2-meter length of #24 nichrome wire has a mass of about 3.42×10^{-3} kg, so σ is about 1.71×10^{-3} kg/m. The wire is stretched between its supports, and L is measured as 64.3 cm. The tension is varied until the wire resonates at 60 cps with a mass of 1.020 kg attached to its free end. Substitute these values into Eq. (2)

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\sigma}}$$

and you obtain a frequency $f_1 = 59.5$/sec. This is within 1 per cent of the expected 60.00/sec value.

Longitudinal Waves

You may also produce longitudinal standing waves in a stretched spring. The spring is mounted between two fixed supports with electrical connections made from each end to a power amplifier, a transformer, or audio switch. A surplus magnetron magnet (U-shape, not C-shape) has a cylindrical iron slug about one inch long held magnetically to one pole, and the slug is inserted a short distance into the end of the coil.

As the frequency of the current is varied, the spring responds at each of its resonant modes. Handwound coil "springs" are adequate for qualitative demonstrations. For quantitative work a brass SHM spring (Cenco #75490 or equivalent) is desirable. The spring may be mounted horizontally on the stage of an overhead projector.

The harmonic series for a spring (or the series of frequencies at which standing waves appear) is given by Eq. (1). The fundamental frequency f_1 is given by the equation:

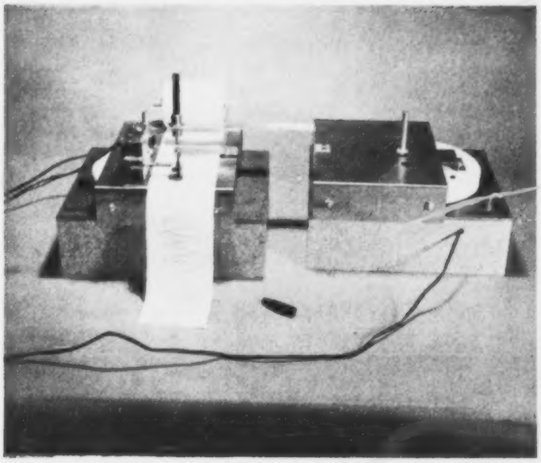
$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad (3)$$

where k is the spring constant, $(k = \frac{\Delta F}{\Delta \ell})$ found experimentally by Hooke's law; and where m is the total mass of the spring being subjected to oscillation. It is possible, therefore, to compare a set of predicted values with a set of experimental values.

You will find that if you use a fixed frequency, you cannot "tune" the spring to resonance by changing the tension, or, which amounts to the same thing, by changing the length of the spring by stretching. Once you have found the resonant condition, you can stretch the spring, and it continues oscillating in the same mode. You can only tune the spring by changing the mass of the portion of the spring that is oscillating. To do this, clamp the spring firmly at various points other than the end until resonance is found.

D46 Two turntable oscillators

Beats—Two oscillators are set up so that the motions of the two platforms are parallel. One oscillator has a pen attached to the platform; the other carries a chart recorder positioned so that the paper moves perpendicularly to the platforms' oscillations. The pen writes on the moving paper.



If the first turntable only is switched on, the pen will draw a sine curve on the moving paper. If the second turntable only is switched on, the moving paper will be driven back and forth in SHM under the stationary pen, and a sine curve will be drawn. If the two turntables are set to the same speed, the two curves will have the same wavelength. Now, if both turntables are switched on, the trace will be the result of the superposition of two sine curves. Even if both oscillators have been set to nominally the same frequency, there will almost always, in practice, be a detectable difference, which means that the resulting pattern will show beats.

With the turntable set to a given frequency (say 78 rpm), small adjustments in frequency can be achieved by loading down the platform to increase friction between platform and its support. [Some phonograph motors can be adjusted over a small range by a Variac (Powerstat) in the supply line.] Thus, one can change the beat frequency by adjusting the frequency of one of the component oscillators.[1]

Approach to Harmonic Synthesis—Two oscillators are set up as before, so that a pen attached to one writes on a chart recorder mounted on the other. If the frequency of one is a multiple of the other, then the resulting trace illustrates in a simple manner, the elements of harmonic synthesis. One particularly interesting trace representing the addition of the first two terms in the Fourier synthesis of a square-wave ($a \sin\theta + 1/3a \sin 3\theta$) is shown in Fig. 1. The "course" tuning was done by setting the two turntable speeds to 16 and 45 rpm. See resource section for a description of the turntable oscillator.

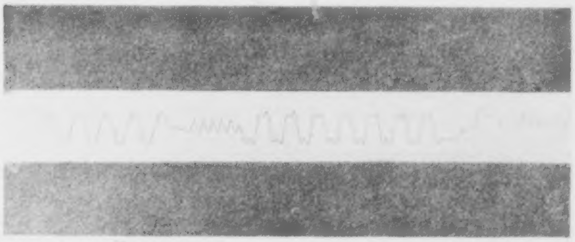


Fig. 1

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[1] Incidentally, the discussion and analysis of beats provides a good opportunity to point out to students the low precision of a result which is the difference of two large numbers. Measure the wavenumber (reciprocal wavelength) of the two component oscillations, together with an estimate of the uncertainty. Calculate the wavenumber of the resultant beat by taking the difference of the two, nearly equal, component wavenumbers. The percentage uncertainty of this result will be very large. On the other hand, the beat wavenumber can be measured directly with high precision, which demonstrates how sensitive the method of beats is in showing up small differences in frequency.

Introduction—Loops and Strobe Photos

TRANSPARENCY/FILM LOOP—STILL PHOTOGRAPH CORRELATIONS

Several one- and two-dimensional collisions are shown in more than one way in our materials. The table below shows these correlations.

STILL PHOTOGRAPHS OF ONE-DIMENSIONAL COLLISIONS

Event 1: Film Loop 18, first example
T19 One-Dimensional Collisions

Event 2: Film Loop 18, second example
T19 One-Dimensional Collisions

Event 3: Film Loop 19, first example

Event 4: Film Loop 19, second example

Event 5: Film Loop 20, first example

Event 6: Film Loop 20, second example

STILL PHOTOGRAPHS OF TWO-DIMENSIONAL COLLISIONS

Event 8: Film Loop 22, first example
T20 Equal Mass Two-Dimensional Collisions
(Note: This is referred to as Photo #69 in the transparency notes.)

Event 10: Film Loop 22, second example

Event 11: Film Loop 21

Event 12: Film Loop 23, first example

Event 13: Film Loop 23, second example

Event 14: Film Loop 24

Teacher notes for Loops 21, 22, 23, 24 and 25 are found in the stroboscopic activity notes. There are no teacher notes for Loops 25, 28, 32, 36 and 37.

LOOP	PAGE
4	114
18	110
19	112
20	113
21	102
22	102
23	102
24	104
25	no notes
26	114
27	114
28	no notes
29	116
30	116
31	117
32	no notes
33	117
34	117
35	118
36	no notes
37	no notes
38	118
39	118
40	118
41	118
42	119
43	119
44	119
45	119

Stroboscopic Photographs of One Dimensional Collisions

I. Materials Provided and their Uses.

A set of stroboscopic photographs has been produced which permits detailed quantitative study of seven two-body collisions in one dimension. In the text which follows, these will be called, simply, Events 1 to 7.

The student may be assigned one or more of these events as take-home problems, or as study-period (or laboratory-period) tasks. It may be advantageous to assign a pair of students to a given problem.

Prints and student notes are provided and may be kept by the student.

Two overhead transparencies are also supplied to the teacher, showing the stroboscopic photos of Events 1 and 2. With these the teacher can, in a few minutes, describe the problems qualitatively to the class before he distributes the assignment to students.

Or, the transparencies of the two collision events may be used by the teacher to work out these examples in detail with the whole class, taking measurements directly from the wall (or blackboard).

The better students may profit from working through more than one event. In fact, the series of events was so chosen that there are certain relations between events. In one pair (Events 1 and 2) the events are mutually inverted; in another (Events 3 and 4) the same balls collide strongly and weakly, respectively; and so on.

In each pair interesting discussion questions can be raised from which the student may learn something. These paired groupings and their points of interest are listed below.

The problems fit into Project Physics, Unit 3, Chapters 9 and 10. They are, primarily, illustrations of the principle of conservation of momentum.

A calculation of the kinetic energy in the system before and after collision is also of interest. It reflects on the elasticity of the colliding balls. This energy is conserved if the collision is "perfectly elastic."

None of the examples is a perfectly elastic collision, although kinetic energy is 98% conserved in Event 4.

II. Apparatus Used to Obtain the Photographs

The two colliding balls were, in each case, hung in bifilar suspension from thin piano wires. See Fig. 1. They were confined to move on circular paths in the same vertical plane. The radii of these paths were the same for both balls, and roughly equal to 32 feet. Thus the balls acted as pendula of equal periods (about 2π seconds) to excellent approximation. They were released

PLEASE NOTE: To keep the Student Handbook to a manageable size, Events 3, 5, 6, and 7 have been omitted. For additional work on one-dimensional collisions, you may wish to assign some students to use the appropriate pages in the Teacher Guide for these Events.

simultaneously by relays from chosen initial positions and therefore collided at the bottom of their swing (point B) a quarter period (or about 1.6 seconds) later.

A camera was placed in front of B for a field of view as indicated. The (circular) path of the balls within this frame was illuminated by four synchronized General Radio Company Stroboscopes (not shown). The flash rate was always a simple integer submultiple of 60 cycles per second. This permitted very accurate calibration of this rate against the power company's time frequency by observing beats in a small neon bulb in the stroboscope circuits.

Two vertical rods were placed into the field of view. The center points of the tops of these rods were, in a all photos taken, 1 meter apart (as precisely as possible). Therefore, the student can, by scaling, calculate actual distances from his measurements on the photographic prints.

Typical pictures can be found among the figures below. Clearly, the paths are not exactly straight. Nor should the velocities found be exactly the same for a given ball as it moves toward point B of collision, or away from B afterwards. But, if the amplitude of swing exceeds the portion within camera frame (about 1½ feet on each side of B) by a sufficient factor, the velocity variation will be small.

The speed of the ball near the frame's edge is smaller than near the collision point B. (By the way, it is smaller by less than 1% if the full swing of the pendulum is 12 feet; by 3% if it is 6 feet; by 7% if it is 3.75 feet; and so on.) But that need not be of great concern because the stroboscopic record allows one to check by how much the ball is slower near the frame's edge.

If our ruler detects this variation from the various exposures of ball position, then the best positions from which to calculate velocities are those nearest the center. If this rule if followed, the errors introduced by this aspect of the experimental setup are well within the estimated error of a reading of distance on the photograph by use of a good metric ruler with millimeter rulings. In none of our photographs is this "built-in" error greater than 2%.

III. Discussion of the Events

The seven collision events present similar analytical problems. For any given event, the following are the steps of this analysis:

A.

A schematic diagram is provided for each event. It specifies qualitatively what the conditions were before and after the collision. It also gives the masses of the colliding balls.

B.

The student can then proceed to a qualitative study of the stroboscopic photographs provided for the event. He can actually tell the order, in time, in which the stroboscope flashes occurred, and number them.

C.

He then must find values for the speeds of the balls before and after collision, by measurement.

Here the student may discover more than one time interval from which measurement of displacement might be made. He must make a "best choice."

Also, in some cases, conditions rule out an interval because he can tell from a study of the picture that the collision occurred during that interval. The displacement of a ball in that interval occurred at different speeds; the speed before, during, and after collision.

A measurement of displacement is best made by measuring the distance between successive left (or right) edges of the ball in question.

Each photograph shows two vertical rods. The centers of these rods are 1 meter apart, with precision of 2mm, a fraction of 1%. Thus, measurements of displacement taken on the print, can—and should—be converted to the actual values (in, say, meters or centimeters). This can be done by simple scaling. The student then finds the "real speeds" of the balls before and after collision, giving the exercise the flavor of the real experiment.

We recommend that the student be provided a good quality, but simple, see-through plastic ruler marked in millimeters; that he take care in positioning this ruler for every measurement; and that he estimate the tenth of the millimeter. But he should be aware that this smallest significant digit may be in doubt.

D

He can now proceed to the calculations required by the problem.

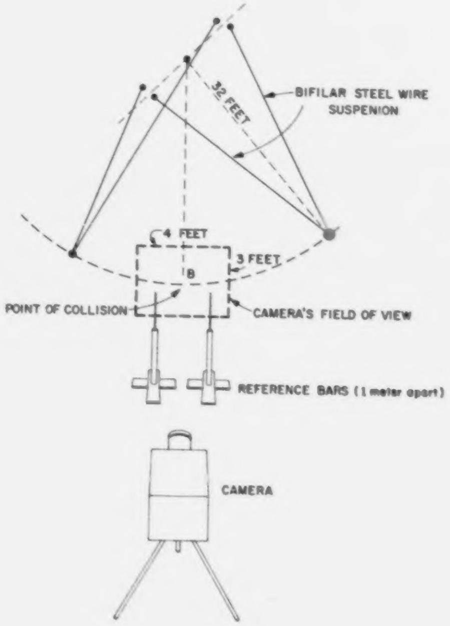


Figure 1. Schematic diagram of the experiment. Not to scale. The pendulums are very long. The height of the room in which the pictures were taken is 45 feet.

The teacher is provided an overhead transparency for Events 1 and 2. It is most strongly recommended that he discuss their qualitative aspects as prototypes before assigning events as problems for the student. As regards Events 2 to 7, only their special aspects are discussed here.

Event 1

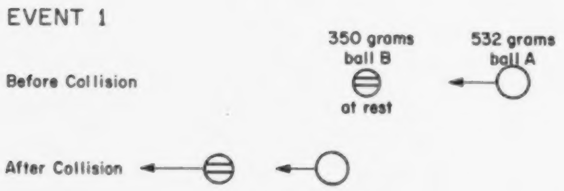


Figure 2. Schematic diagram of Event 1.

Figure 2 shows, schematically, the conditions before and after colli sion. Proceeding to the stroboscopic photograph (Fig. 3), we see two balls of different size. To reduce confusion, two dark stripes have been painted on the smaller one (ball B). large ball (ball A) comes in from the right. The time and order of the flashes can therefore be analyzed.

Ball A clearly was at rest at center frame during flashes 1, 2, and 3. The collision must have occurred between flashes 3 and 4. Therefore, the displacements experienced by either ball in interval "3 to 4" must be ruled out from the measurements useful for calculating speeds before or after collision. Ball A was considerably slowed down by the collision. At the time of flash 7, ball B was already out of frame. The camera shutter was closed before flash 8 occurred.

The student has two choices (intervals "1 to 2" and "2 to 3") to calculate ball A's incoming speed, and three choices ("4 to 5," "5 to 6," "6 to 7") to measure its speed after collision.

When using a ruler marked in millimeters on an 8 × 10 inch print of Fig. 3, the careful student will find that the displacement of ball A was about ½ millimeter larger in time interval "1 to 2" than in interval "2 to 3." The difference amounts to about 1%.

The student should be encouraged to decide himself whether there is a "best choice" in finding the speed before collision in view of the experimental setup illustrated in Fig. 1. He may in this particular case even decide, with reasonable arguments, that the average of both displacements is adequate.

As a rule of thumb, of course, the interval closest to the instant of collision is the "best choice." By this reasoning, "4 to 5" is also "best choice" to find both balls' speeds after collision.

In any case, the student should convert each measurement of a displacement, taken off the print, to the actual displacement of the ball. This he may readily do by measuring the distance between the two vertical reference bars which are 1 meter (± 2 mm) apart.

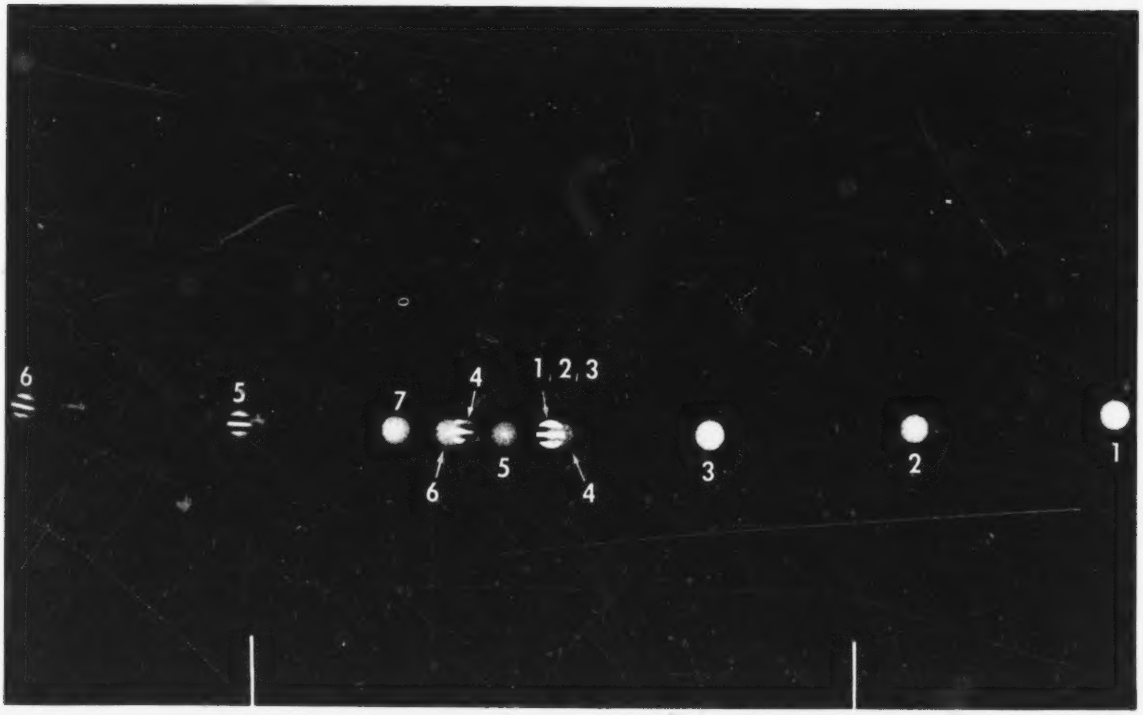


Figure 3. Stroboscopic photograph of Event 1. 10 flashes per second. The numbers shown do not appear in the photograph given to the student. They correspond to the order in which the successive flashes occurred at 1/10 second intervals. Each position of each ball can be associated with one of these numbers. The centers of the tops of the 2 rods appearing near the bottom of the picture are 1 meter apart and serve as a scale reference. (Available as a transparency.)

Table I shows the results obtained from a photographic 8 × 10 inch print by careful measurement. The "Best Value" is based on measuring displacement with a good ruler, of 1 millimeter least count (estimating the tenth of a millimeter), during time intervals "2 to 3" (before collision) and "4 to 5"(after).

Momentum of the system of two balls, before and after collision, comes out the same (to the number of significant digits available): 1.79 kg-m/sec (directed to the left). On the other hand, the kinetic energy of the system goes from 3.02 joules before collision to 2.64 joules after. It is only 87.4% conserved due to imperfect elasticity.

Table I

EVENT I

Scale: 12.65 cm to 1 meter on an 8 × 10 inch print

Ball A: 0.532 kg

Ball B: 0.350 kg

Flashrate: 10 per second

Item	Ball	Time	"Best Value"	Direction
velocity	A	before	3.37 m/sec	left
	A	after	.917	left
	B	before	0 m/sec	
	B	after	3.72	left
momentum	A	before	1.79 kg-m/sec	left
	A	after	.488 "	left
	B	before	0 "	
	B	after	1.30 "	left
kinetic energy	A	before	3.02 joules	
	A	after	.223 "	
	B	before	0 "	
	B	after	2.42 "	

Event 2

See Fig. 4. Notice that the balls have the same masses as in Event 1 and that the collision is the inverse of Event 1. This was intentional. In Event 2 ball B is coming in (from the left), striking ball A at rest. The incoming speed of ball B is, in fact, roughly equal to that of ball A in Event 1.

Moreover, note that in this case the incoming ball is reflected by the collision. This is so because the ball it strikes has the greater mass.

There are two stroboscopic photographs recording this event. See Figs. 5 and 6. The first shows the event before collision, the second after collision. The pictures were taken by allowing the event to control the camera shutter—using electric relays. A relay closed the shutter before collision in Fig. 5 and opened it just after collision in Fig. 6. If some such division of this event had not been made, it would be difficult to tell which exposure of ball B occurred when because this ball

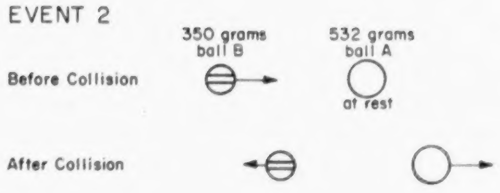

EVENT 2
350 grams
ball B
532 grams
ball A
Before Collision
at rest
After Collision

Figure 4.

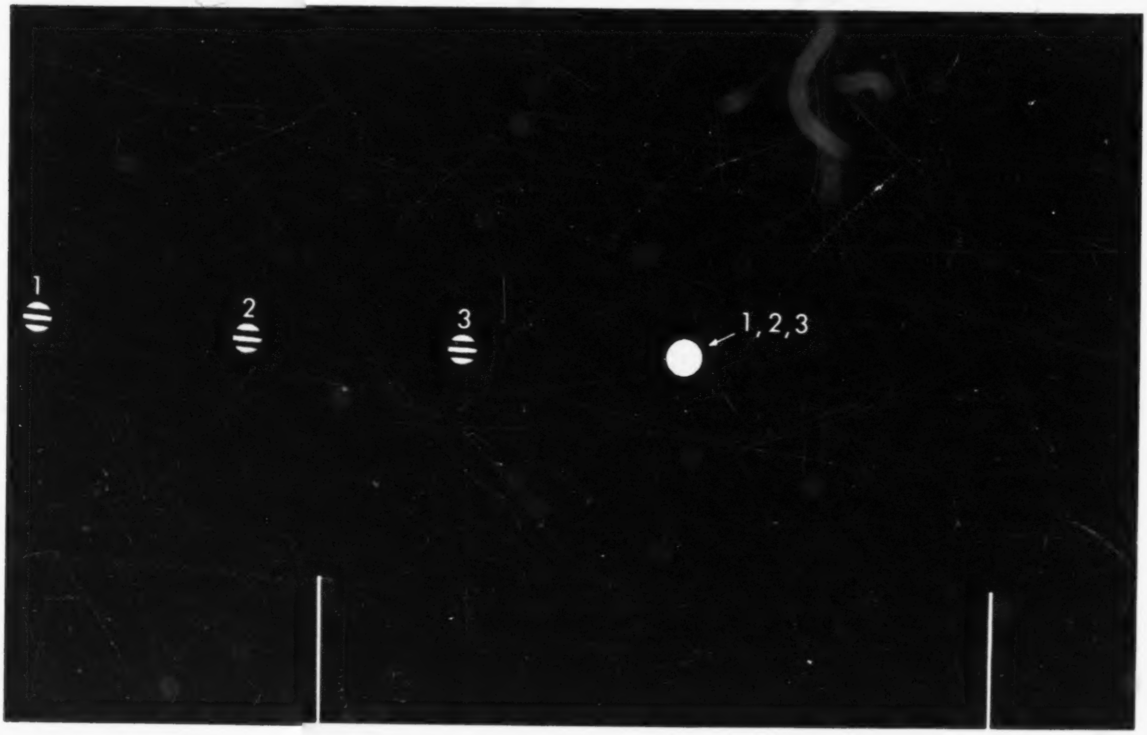


Figure 5. First half of Event 2. 10 flashes/second. Figures 5 and 6 are also available as an overhead transparency.

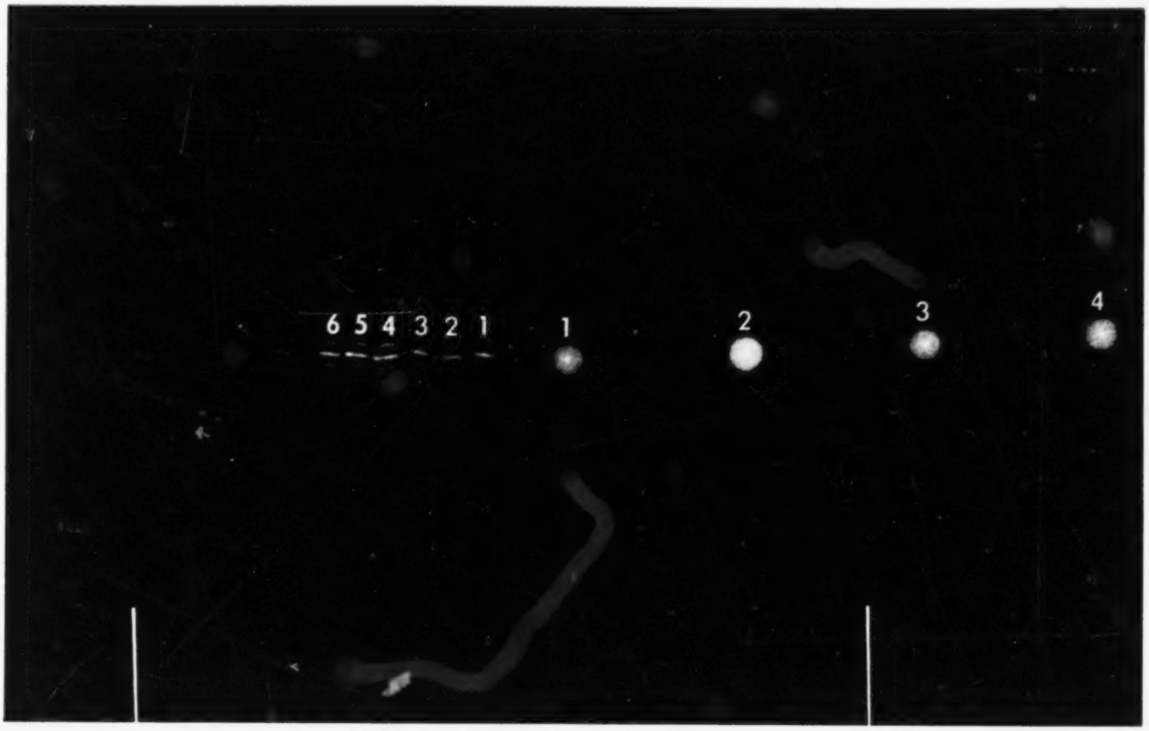


Figure 6. Second half of Event 2. 10 flashes/second.

retraces its path. The transparency overlay of Figs. 5 and 6 for overhead projectors provided to the teacher can be used to show this to the student.

The measurement of displacement for ball B after collision is in this case difficult. See Fig. 6. It emerges from collision with small speed, which —as is clear from the picture—is decreasing visibly. Interval "1 to 2" in Fig. 6 is clearly "best choice" for both balls. As regards ball B, care is required to find its displacement in that interval.

Our own values of total momentum before and after collision were 1.14 and 1.16 kg-m/sec, respectively (directed to the right). The difficulty in measuring speed of ball B after collision, and the large relative error arising from this difficulty, do not greatly affect absolute errors. The reason is due to the fact that the momentum of ball B after collision is so small. Discovery, by students, of these aspects of measurement are in themselves worthwhile goals of our laboratory instruction.

Our values for total kinetic energy showed it to be 91% conserved.

Event 3

As Figs. 7, 8, and 9 illustrate, a massive ball A enters from the left, and ball B of considerably less mass comes from the right. The speed of ball B before collision, however, is so large (compared to the speed of ball A before collision) that the net

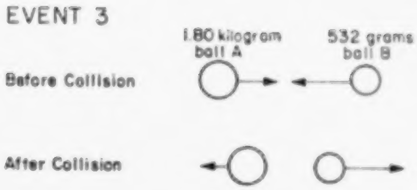

EVENT 3
1.80 kilogram ball A
532 grams ball B
Before Collision
After Collision

Figure 7.

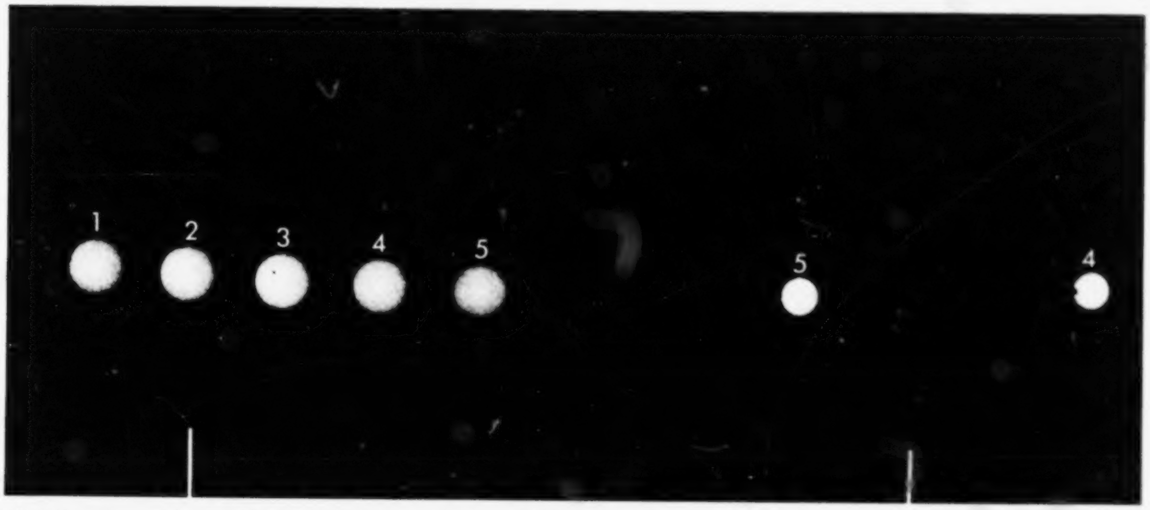


Figure 8. First half of Event 3. 10 flashes/second.

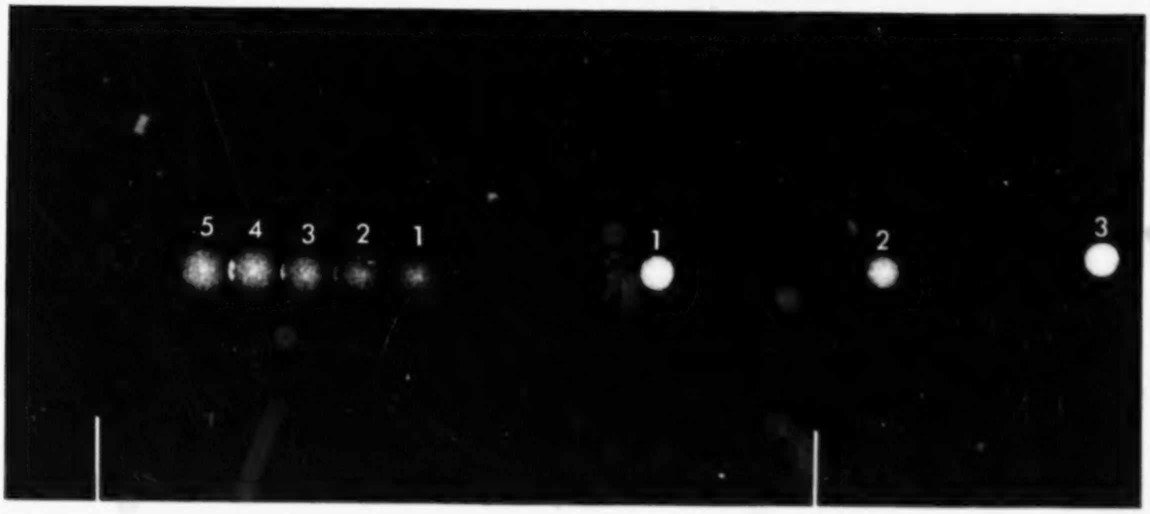


Figure 9. Second half of Event 3. 10 flashes/second.

momentum vector of the system actually points to the left.

This has two consequences which we feel to be of pedagogical value. One of these concerns the aspects of measurement: the net momentum of the system is here a small difference of two large numbers. The two "large numbers" are the momenta of the balls (before or after collision), and these the student can find—if he is careful—with a precision of 1 or 2%. However, in their small difference this becomes a large relative error. And matters are made worse because of the rule: in additions or subtractions, the error in the result is the algebraic sum of the absolute errors of the two numbers involved.

Our own results were: .30 kg-m/sec and .27 kg-m/sec net momentum (directed towards the left) before and after. This is a 10% difference. We have only two significant digits in the result, although each ball's momentum was, itself, known to 3 digits.

The second consequence concerns a physical process. In this collision the impact of the steel balls was a violent one. A real "whack"! The student can study this with his own data. Each ball's change in momentum, produced by the collision, was of the order of 2.8 kg-m/sec. [Change in momentum = (average force during collision) × (time of collision). Estimating, conservatively, 1/100 sec for this time, the average force was 280 newtons, or the weight of 29 kg.]

Obviously, the hard and elastic steel case of these case-hardened balls was strongly deformed in this collision; this deformed the soft, and far less elastic, inner core. The result was a great deal of internal friction (and heating). This explains why the initial kinetic energy of the system is wasted more here than in other events. We found it to be only 51% conserved.

Event 4 involves the same balls in a weak collision, resulting in 98% conservation of kinetic energy.

Event 4

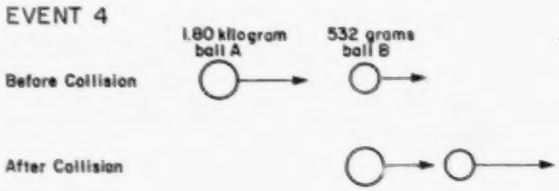


Figure 10.

Figure 10 specifies this event. The massive ball A comes in from the left at high speed and overtakes the lighter ball B which is going in the same direction. After collision both balls are still going in the same direction, but ball B is much faster than before whereas ball A is slowed down relatively little.

Since each ball crosses the entire field of view from left to right, a stroboscopic view of the whole event leads to superposition of images created by flashing of the bulb at different times. See Fig. 11. As a consequence, such a picture is difficult to analyze.

We got around this difficulty by photographing Event 4 in its entirety twice. Once (Fig. 12) ball B was painted black while ball A was painted white. The other time (Fig. 13) the colors were reversed.

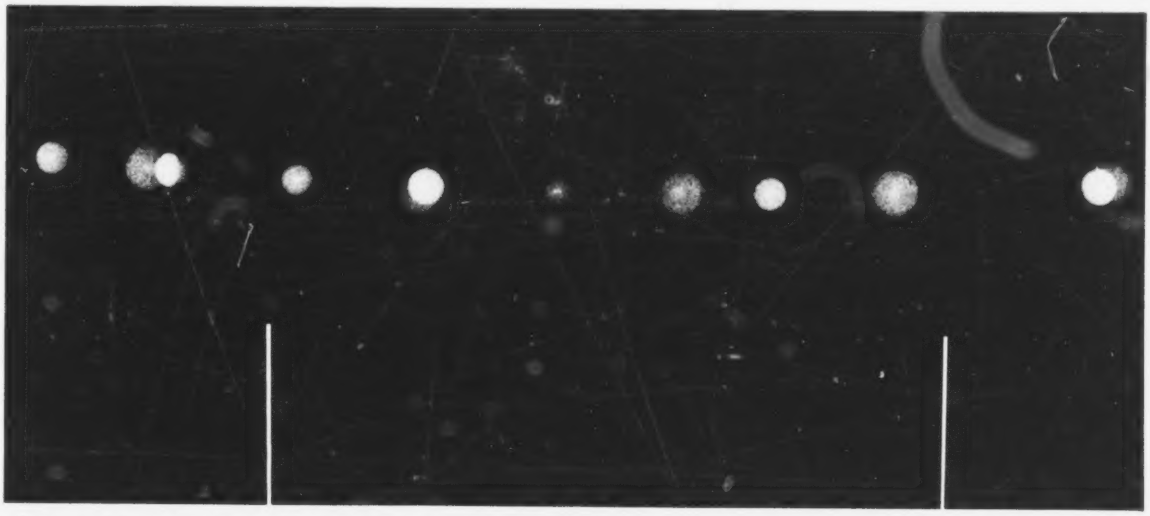


Figure 11. Stroboscopic photograph of the whole of Event 4. 10 flashes/second. Both balls cross the field of view in the same direction (from left to right) as time goes by. It is not easy to analyze this picture. Figures 12 and 13 show how this difficulty was resolved.

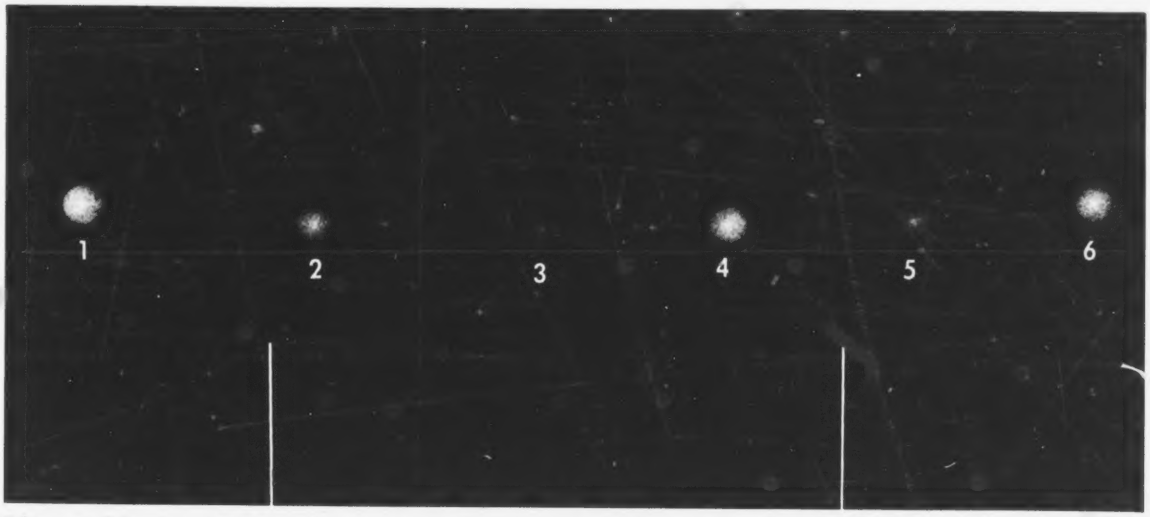


Figure 12. Event 4. 10 flashes/second. Whole event. Ball B (small mass) black.

Consider Fig. 12. It can be used to determine the speed of ball A before and after collision. However, it turns out that position 3 is difficult to identify. It could, at first inspection, have been before or after collision. A good student will be able to show, by making measurements, that the flash which exposed position 3 occurred after collision. [Distance "2 to 3" is slightly shorter than distance "1 to 2," while at the same time "3 to 4' is slightly longer than "4 to 5." If this is clear to the student, then the "best choices" for ball A are "1 to 2" (before collision) and "3 to 4" (after).]

However, this is a fine point of the analysis. It can be ignored. One can call position 3 doubtful. This rules out intervals "2 to 3" and "3 to 4." Equally precise results are obtained from Fig. 12 by choosing "1 to 2" and "4 to 5."

Analogous difficulties arise in analyzing Fig. 13. The intervals before and after collision corresponding to safe choices are "3 to 4" and "6 to 7."

We find, from the safest choices, that the total momenta before and after are 8.35 and 8.38 kg-m/sec (to the right), respectively. The total kinetic energies before and after are 15.9 and 15.5 joules, which correspond to 98% conservation.

This collision is much more elastic than another collision between these balls, namely, Event 3. This was already discussed above. Event 4 forms an instructive companion to Event 3. The student notes contain questions about the surprising differences in apparent elasticity.

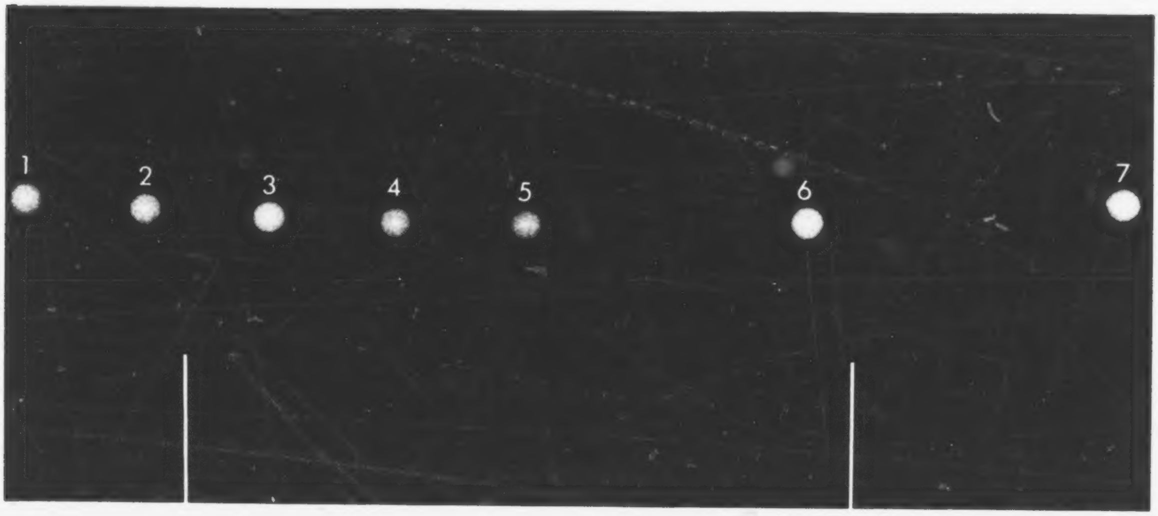


Figure 13. Event 4. 10 flashes/second. Whole event. Ball A (large mass) black.

PERFECTLY INELASTIC COLLISIONS
(Events 5, 6, and 7)

In these examples the colliding objects are steel balls covered by a thick layer of plasticene. They remain lodged together after collision.

Event 5

See Figs. 14 and 15. Ball A, coming in from the right, strikes ball B which is at rest. Our measurements yield momenta for the system of balls of 2.66 kg-m/sec before and 2.67 kg-m/sec after collision (pointing to the right). The kinetic energy after collision was 61% of that before collision.

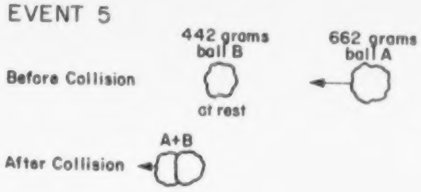

EVENT 5
442 grams
ball B
662 grams
ball A
Before Collision
at rest
A+B
After Collision

Figure 14.

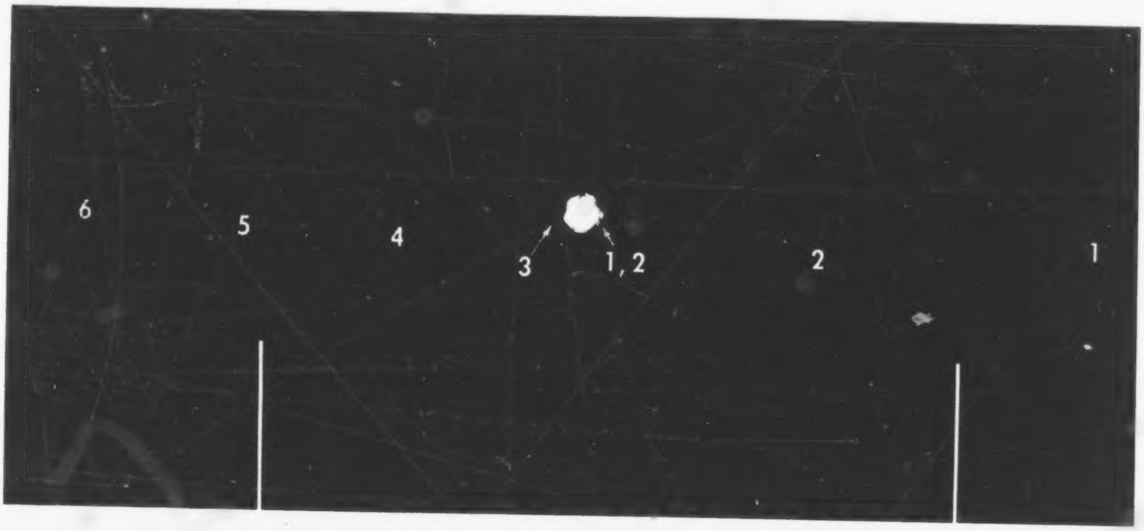


Figure 15. Event 5. 10 flashes/second.

Event 6

Figures 16, 17, and 18 tell the story. This case involves the same balls (i.e. masses) as Event 5. Momentum of the system before and after is 2.06 and 2.08 kg-m/sec (pointing to the left) by our measurements.

Event 6 is an instructive example, when considered together with Event 5, in which over 60% of the kinetic energy was conserved. Here, only 24% is conserved. The collision was more violent in Event 6.

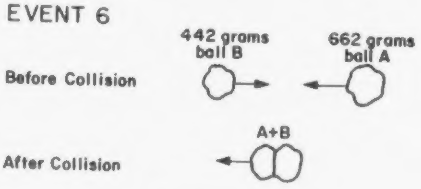

EVENT 6
442 grams
ball B
662 grams
ball A
Before Collision
A+B
After Collision

Figure 16.

Figure 17. First half of Event 6. 10 flashes/second.

Figure 18. Second half of Event 6. 10 flashes/second.

Event 7

A very massive ball A is coming in from the left (see Fig. 19) at a speed which is small when compared with the speed of ball B which enters from the right. But the mass of ball B is small. When the balls become lodged together, they move off to the right, but the speed of ball A after collision is not very greatly reduced from what it was before. This is the case of a head-on collision of a truck with a small car.

Figures 20 and 21 are the stroboscopic records before and after collision.

Momenta before and after are 6.58 and 6.71 kg-m/sec (directed to the right) respectively. Kinetic energy after collision is about 27% of that before.

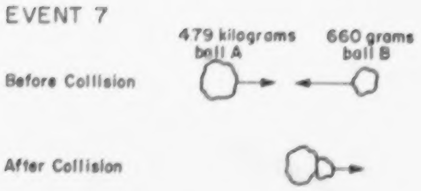

EVENT 7
479 kilograms
ball A
660 grams
ball B
Before Collision
After Collision

Figure 19.

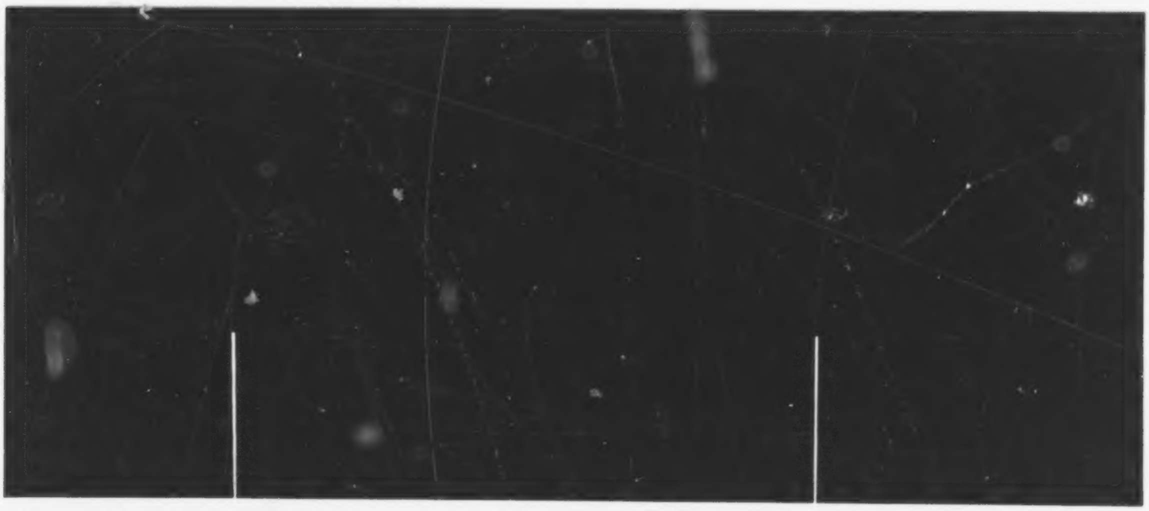


Figure 20. First half of Event 7. 10 flashes/second.

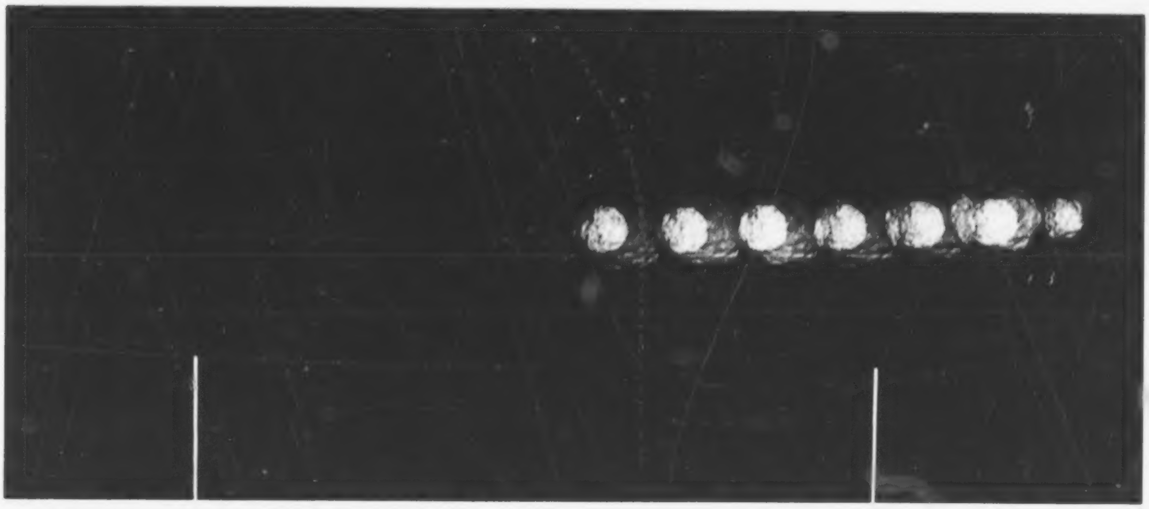


Figure 21. Second half of Event 7. 10 flashes/second.

Recommended Pairs of Events

Events 1 and 2 form a pair because they are inverse events involving the same balls.

Event 3 is a good exercise as long as it is not assigned alone but as a second exercise with one of Events 1, 2, 4, and 5. This is due to the instructive character of error propagation in Event 3. A large relative error arises when subtracting two nearly equal quantities which are themselves known with small relative errors.

Events 3 and 4 involve the same balls. Yet, in the former only ½ the kinetic energy is conserved, and the latter is almost perfectly elastic. Good discussion questions can be raised.

Events 1 and 5 proceed similarly until the collision takes place. The collision in 1 is elastic (not perfectly elastic) whereas in 5 is perfectly inelastic.

Stroboscopic Still Photographs of Two-Dimensional Collisions: Events 8 to 14

I. The materials.

Measurements in Events 8 to 14 are essentially of the same nature as in the one-dimensional problems. Analysis, on the other hand, requires construction of vector diagrams and is for this reason a bit more complicated.

The student may be assigned one or more of these events as take-home problems, or as study-period (or laboratory-period) tasks. It may be advantageous to assign a pair of students to a given problem.

Better students may profit from working through more than one example. Events 8 and 9 both involve collisions in which one ball is initially at rest. In Event 8, moreover, the balls have equal masses. Events 12 and 13 are perfectly inelastic collisions between balls of equal mass. The initial speeds are roughly the same in these two examples, but in one the angle between incoming paths is acute, whereas in the other it is obtuse. Event 14 is the most ambitious problem. Here, a rapidly moving ball scatters a cluster of six balls initially at rest. It should not be assigned unless the student has worked one of the simpler events (8 to 13).

The problems fit into Project Physics, Unit 3, Chapters 9 and 10. They are, primarily, illustrations of the principle of conservation of momentum.

A calculation of the kinetic energy in the system before and after collision is also of interest. It reflects on the degree of violence of the collision and on the imperfect elasticity of the steel balls. (In Events 12 and 13, however, the colliding balls are plasticene.)

II. Apparatus used to obtain the photographs.

Each ball was suspended from a thin piano wire. The lengths of the pendula were identical and roughly equal to 32 feet. Thus, the periods were likewise equal (about 2 seconds). The balls were released simultaneously by relays from chosen initial positions, and collided at the bottom of their swing (point C), a quarter period (or about 1.6 seconds) later.

The speed of a given ball near the frame's edge was smaller than near point C. The speed was smaller by less than 1 per cent if the full swing of the pendulum was 12 feet or more; by 3 per cent if it was 6 feet; by 7 per cent if it was 3.75 feet; and so on. This need not be of great concern because—in most cases—the stroboscopic photograph allows one to check by how much the ball was slower near the edge of the frame.

If our ruler detects such variation from the several exposures of a given ball's positions (before or after collision), than the best positions from which to calculate velocities are those nearest the center. If this rule is followed, the errors introduced by this aspect of our experimental set-up are well within the estimated error of reading distances on the photograph by use of the ruler. In none of the photographs is this "built-in" error significant.

III. Discussion of the events.

The vector diagram.

With the print securely taped to a corner of the large paper sheet, the student next uses the large triangles to draw a line parallel to the incident direction of motion of one of the balls.

Choosing a convenient scale factor, he measures off the momentum of this ball before collision. To the tip of this vector he now adds the momentum of the next ball, before collision, by the same procedure of drawing a parallel line and using the same scale factor. Thus adding vectors, the total momentum before collision can be drawn—and its magnitude measured.

The procedure is repeated to find the total vector momentum of the system after the collision.

It is recommended that the initial point of the procedure of adding vectors tip to tip be the same point for both the "before" and "after" phases of the collision event. It will then be relatively easy to find a value for the angle between the two vector sums. This angle is one of the two measures of error. The other is the per cent difference in magnitudes.

Kinetic energy.

Total kinetic energy of the system before and after collision can be calculated from the given masses and from the speeds determined by the measurements.

In none of these events were the collisions perfectly elastic. Even though the hardened steel cases approach ideal elasticity, the soft steel inner cores were frequently permanently deformed by the collision. The loss in kinetic energy goes into the work of deformation and into heat.

Clearly, the per cent loss of kinetic energy should be larger for more violent collisions. If a student were working through more than one Event (especially if the different events involved the same balls, such as in Events 8 and 10, and again in 9 and 11, or in 12 and 13) a comparison of the percentage of kinetic energy conserved in the collision can lead to interesting discussions.

PLEASE NOTE: To keep the Student Handbook to a manageable size, Events 8, 9, 11, and 12 have been omitted. For additional work on two-dimensional collisions, you may wish to assign some students to use the appropriate pages in the Teacher Guide for these Events.

Let us forget about rotation!

In Events 8 to 14 each ball was suspended by a single wire. It is not likely that the balls were spinning appreciably before collision. However, they may have been spinning considerably after collision. No provision was made (by marks on the balls, or other means) to permit measurement of these rates of spin. Only translational momentum and translational kinetic energy is accessible to measurement.

In the perfectly inelastic collisions (Events 12 and 13) care was taken to use photographs in which these spins are essentially zero. See Fig. 15 and 16. Hence the possible complications pertaining to angular momentum (and rotational kinetic energy) do not enter in Events 12 and 13.

Event 8

See Fig. 5. The balls have equal mass. Ball A comes in from the upper left-hand corner while ball B is initially at rest. During the stroboscope flashes numbered 1, 2, 3 on the figure, ball B is strongly exposed, photographically, in its rest position. The collision occurred in the interval between flashes 3 and 4. This interval is therefore inappropriate for determining ball speeds since we are interested in speeds before and after collision.

Following the idea that the intervals nearest the collision points constitute "best choices," it is clear that interval 2 to 3 is "best" before, and that interval 4 to 5 is "best" after, collision.

Table I summarizes results, as regards magnitudes, for a typical measurement. Figure 6 reproduces the vector diagram and gives the values for total momenta "before" and "after" collision. These two vectors subtend an angle of about one-half degree and differ by one per cent in magnitude.

The kinetic energies of the two balls are scalars. They add numerically. By our own measurements (Table I), the system possessed 1.39 joule of translational kinetic energy before collision and 1.23 joule afterwards (88.5 per cent conservation).

Table I

Event 8

Scale: 23.1 cm to 1 meter on an 8" x 10" print.
$m_A = m_B = 0.366$ kg
Flash-rate: 20 per second.

Item	Ball	Time	"Best Value"
speed	A	before	2.76 m/sec
	A	after	2.16
	B	before	0
	B	after	1.44
magnitude of momentum	A	before	1.01 kg-m/sec
	A	after	.774
	B	before	0
	B	after	.527
kinetic energy	A	before	1.39 joule
	A	after	.854
	B	before	0
	B	after	.379

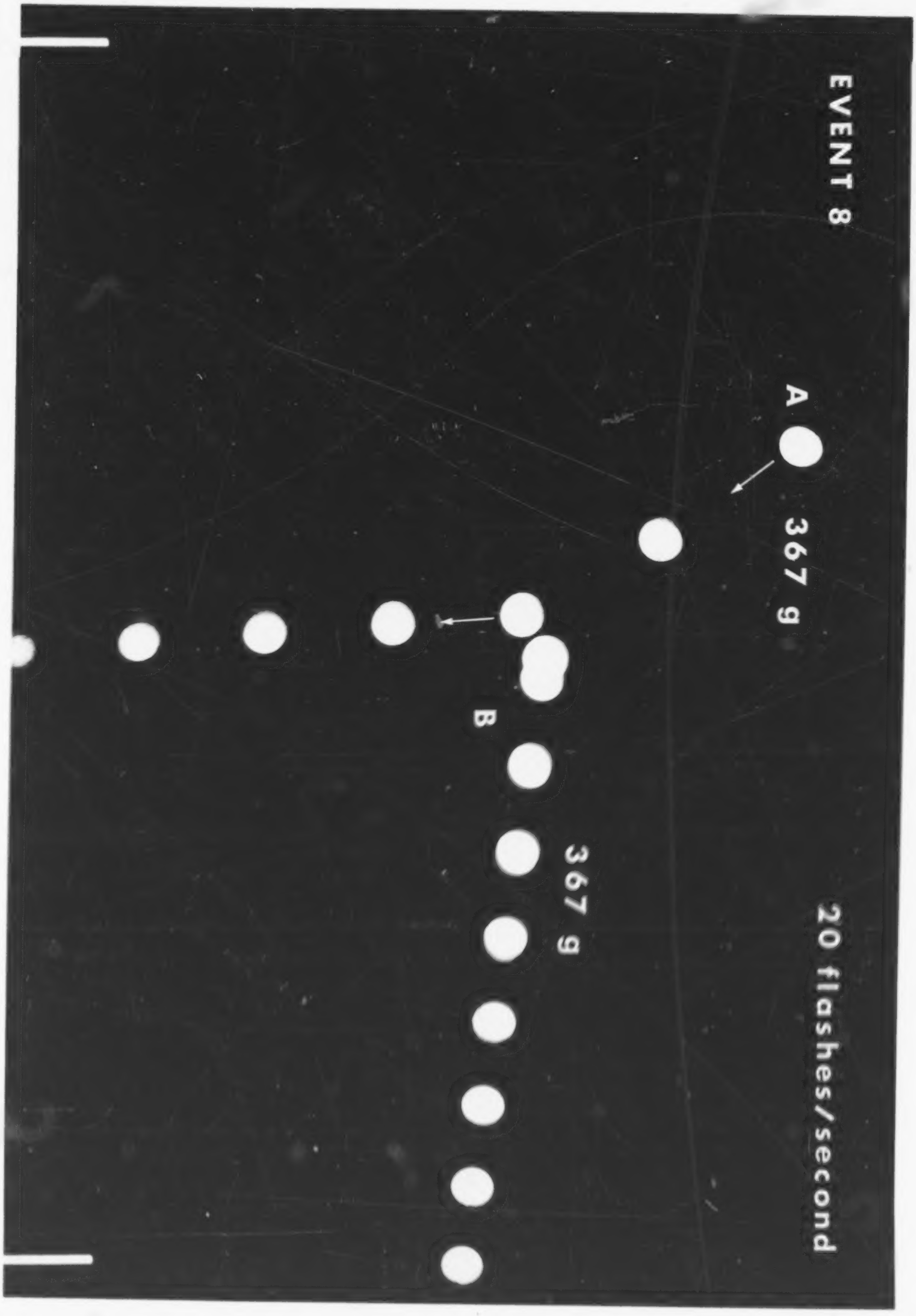
Center of mass in Event 8.

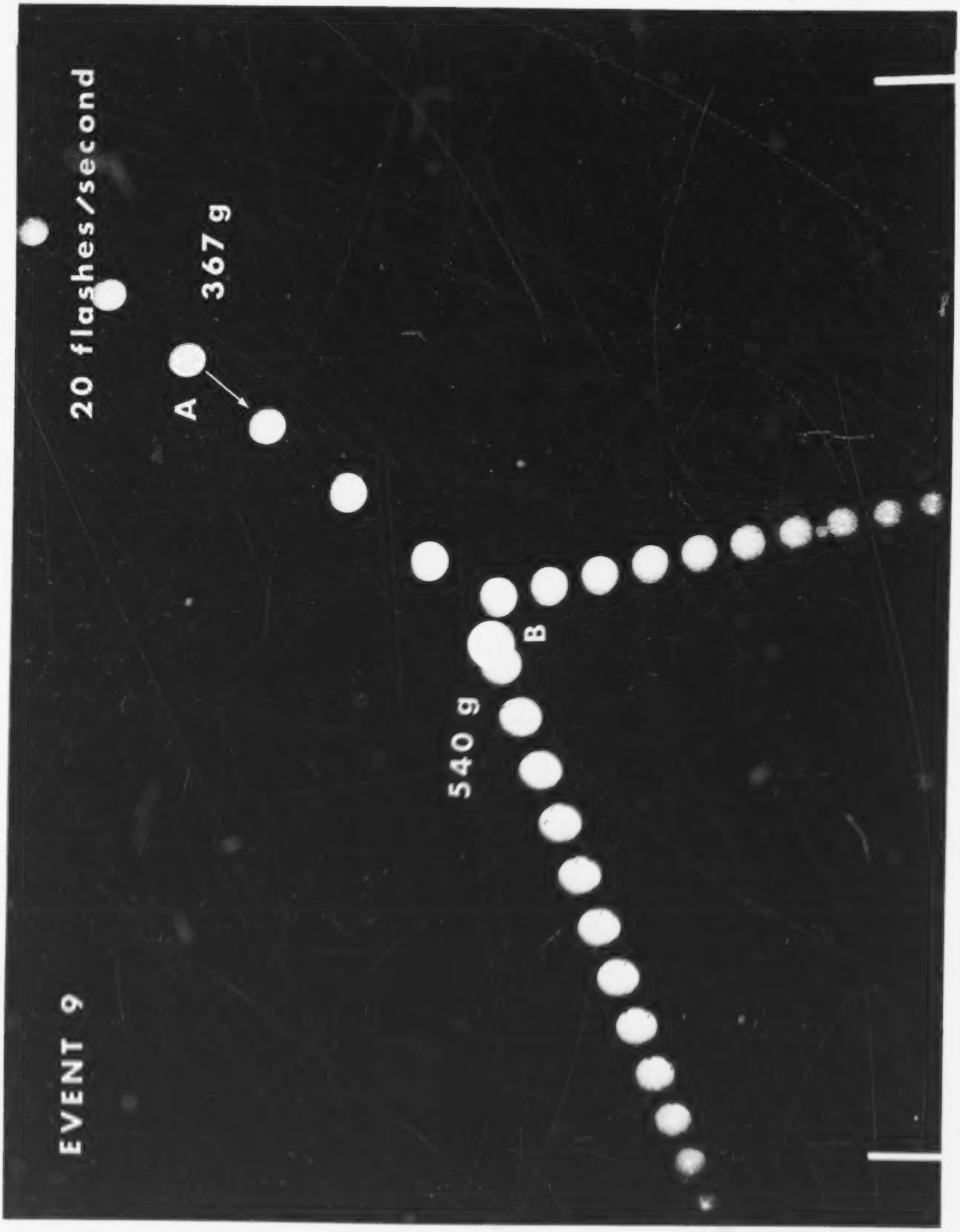
Because the masses of the balls are equal in this event, the photograph lends itself to an easy demonstration of the properties of the center of mass of the system. Figure 7 shows the construction. The legend provides the explanation.

The results illustrate the following theorem: "The center of mass of a system of particles, which is subject to zero net external force, travels uniformly along a straight line."

It is not recommended that the teacher present this theorem and the construction in Fig. 7 to all students. Rather, we mention it here because of its intrinsic interest and because it may be appropriate to expose it to your best students.

Strobe Photos
Event 8


EVENT 8
A
367 g
B
367 g
20 flashes/second


20 flashes/second
367 g
A
B
540 g
EVENT 9

Strobe Photos
Events 9, 10, 11, 12, 13
L21, L22, L23

Event 9

Our discussion of this and subsequent events will emphasize only special points of interest. It will be less detailed than for Event 8 because procedures are identical.

Event 9 (see p. 101) is similar to Event 8. Again, one ball is initially at rest. However, ball masses are different in Event 9.

Our resultant momenta before and after collision coincided in direction. They differed in magnitude by 0.004 kg-m/sec (about one-half per cent).

The kinetic energy of the system fell in value from 0.727 joule before, to 0.481 joule after, collision. Only 66.2 per cent of this energy is conserved. This should be compared to the 88 per cent conservation in Event 8.

Notice that the balls A have the same mass in these two events. Hence, why is the percentage loss of kinetic energy larger here? The answer is connected with the larger mass of ball B in Event 9. In Event 9, each ball experiences a greater change in momentum during collision than in Event 8. Hence, the interaction forces during collision were larger on the average. The collision was more "violent". The balls were permanently deformed to a greater degree.

Event 10 L22

This event is slightly more complicated than the two discussed so far because both balls are moving before collision. Note, however, that the balls again have identical masses as in Event 8.

In our measurements, we found an angular separation of 0.6° and 1.5 per cent difference in magnitude between the total momenta before and after collision.

Total kinetic energy was 2.17 joules before and 1.73 joules after collision (79 per cent conserved).

Center of mass in Event 10.

Since Event 10, like Event 8, uses balls of equal mass, it is again quite easy to locate the instantaneous images of balls A and B. It is clear that the path of the center of mass is a straight line and that its motion is uniform.

One merely measures the distance between two successive positions of the center of mass and converts to "real" distances, by scaling. [I.e., the reference rods at bottom frame are (1.000 ± .002) meters apart.] Successive positions are 1/20th second apart in time. We find:

speed of the center of mass = 1.89 m/sec.

The center of mass of a system of particles can be treated as if it were a particle whose mass equals the total mass of the system and whose momentum equals the total momentum of the system. This is a general result of theoretical mechanics.

Since we know the speed of the "center-of mass particle", the magnitude of its momentum is

$$\begin{aligned}(0.366 + 0.366)\ \text{kg} \times (1.89)\ \text{m/sec} &= (.732)(1.89)\ \text{kg-m/sec} \\ &= 1.38\ \text{kg-m/sec.}\end{aligned}$$

Notice that this comes close to the values found for the magnitude of the total momentum of the system by adding individual particle momenta vectorially.

Furthermore, one could easily verify that the direction of the straight-line path taken by the center of mass is parallel to the total momentum vector as found by the method of adding particle momenta.

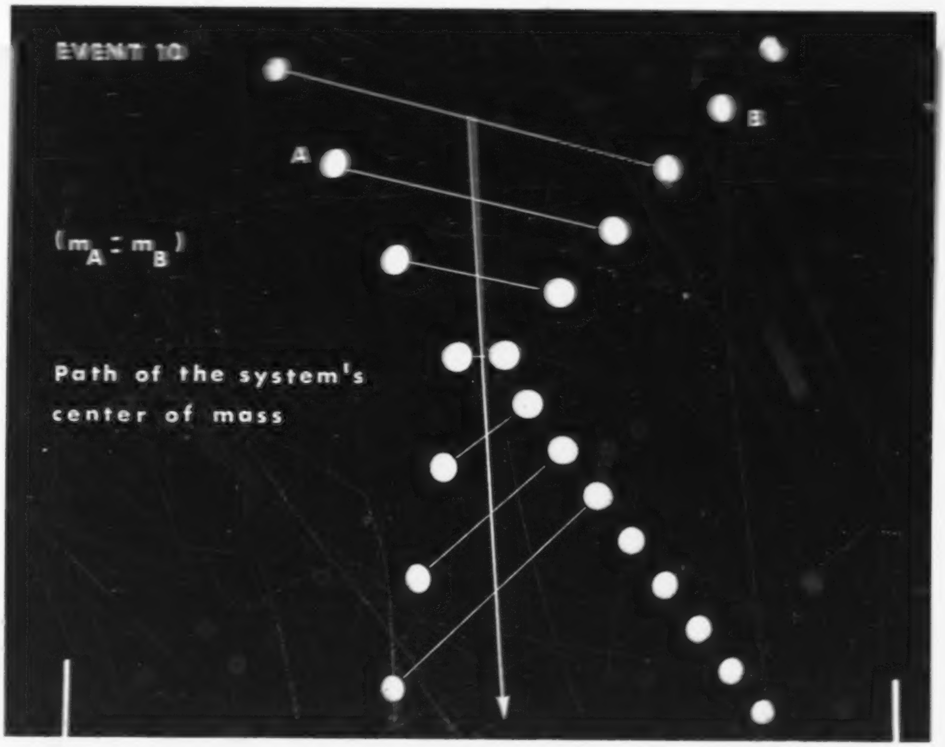
Event 11 L21

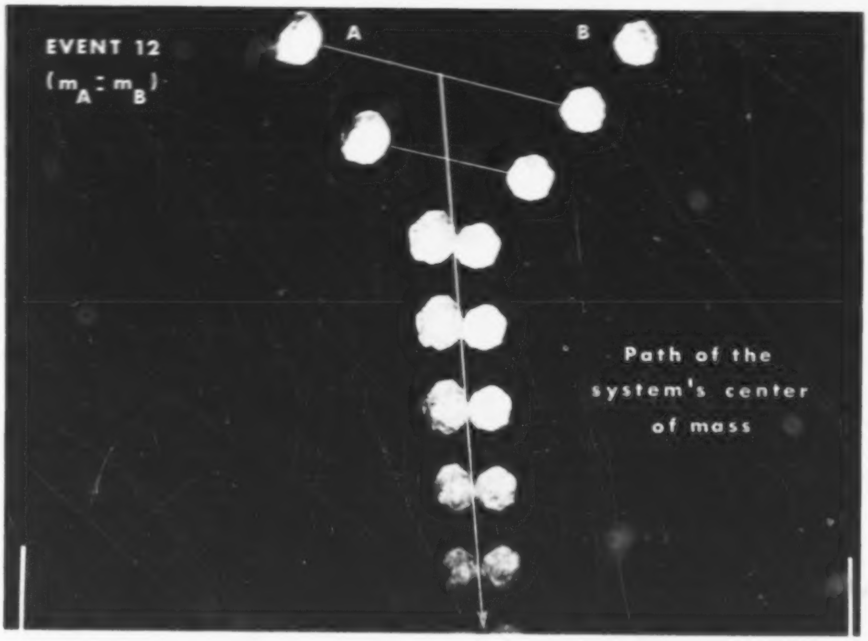
Event 11 is analogous to Event 10 except that the masses of the two balls are not equal. We find a difference of one per cent between the magnitudes and an angle of less than 1° between the directions of the vector sums. Total kinetic energy before and after collision is 3.10 and 2.64 joules, respectively (85 per cent conservation).

Events 12 and 13 L23

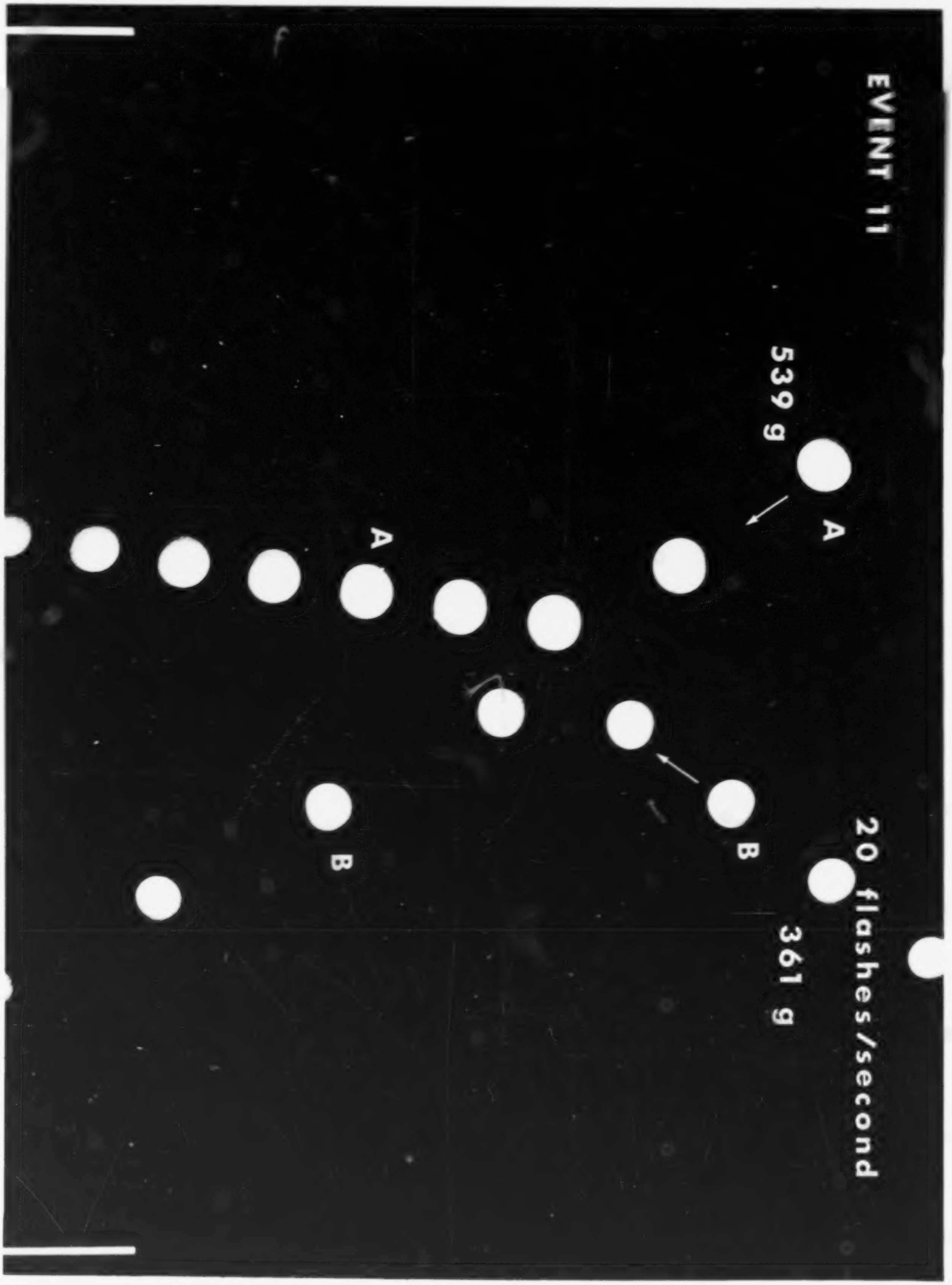
In these events, the steel balls were covered by a thick layer of plasticene. They remained lodged together after collision. The major difference between these two events was the fact that the angle between the initial directions of motion of the two balls was larger in Event 13 than in Event 12.

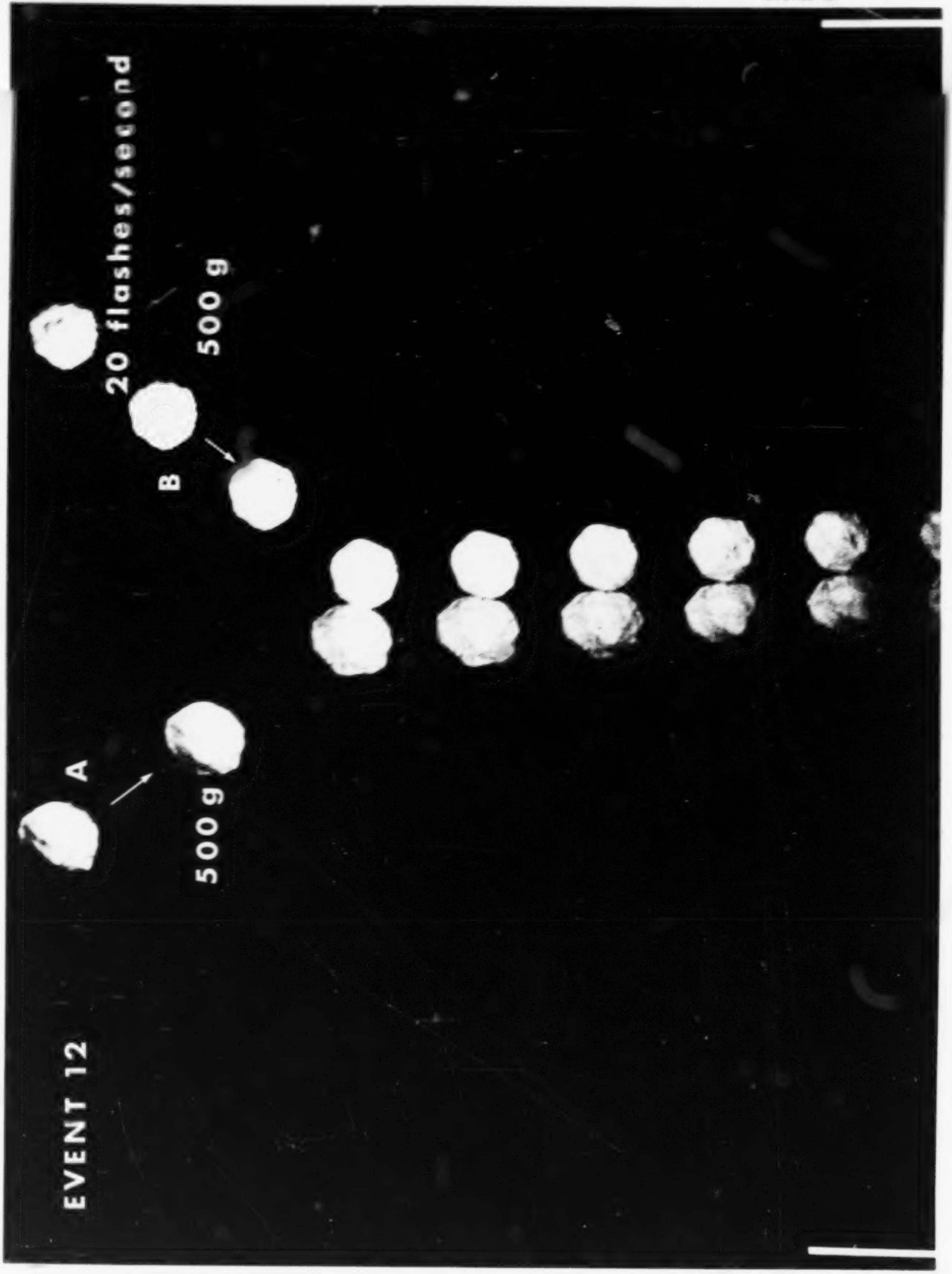
After collision, the "particle system" becomes a single, compound particle consisting of the coalescing balls. Its mass is the sum of the masses of the original balls. To find its speed and momentum, it is necessary to locate the center of mass. Even though the collision created noticeable deformation, it is possible to locate this point easily. Since the individual balls have identical masses, the "compound particle's" center of mass lies at its geometrical center. This can be judged by the naked eye and marked on the print by a pencil dot.


EVENT 10
A
B
($m_A = m_B$)
Path of the system's center of mass


EVENT 12
($m_A = m_B$)
A
B
Path of the system's center of mass

Strobe Photos
Event 11


EVENT 11
539 g
A
A
B
B
20 flashes/second
361 g


EVENT 12
20 flashes/second
A
500 g
B
500 g

In Event 12, the total momenta before and after collision differed by one-half per cent and subtended an angle of one-half according to our data. See Fig. 17. Kinetic energy fell from 3.10 joules during collision (85 per cent conservation).

As Fig. 18 shows, we found the directions of the total momenta for Event 13, before and after collision, to be the same. Their magnitudes differed by 3 per cent. Kinetic energy dropped from 2.81 joules to 0.33 joule. Only 12 per cent of this energy was conserved.

Why is the relative error significantly larger in Event 13 as compared to 12? The principal reason can be found by an inspection of the vector diagrams in Fig. 17 and 18. In Event 13, the total momentum of the system before collision is—compared to Event 12—a short vector sum of two vectors. On the other hand, the vectors being added together to produce these sums are, in fact, of nearly equal magnitudes in both events. They probably have nearly equal errors in them, too. When adding the vectors, the errors "pile up." And in the "shorter" sum-vector of Event 13, the absolute error is about the same. So it is a larger relative error.

Why is there so much greater loss of kinetic energy in Event 13? The answer is obvious from an inspection of the photographs (Fig. 15 and 16). The balls collide almost, but not quite, head-on in Event 13. Notice also, how much more flattening results at the points of contact in Event 13 compared to Event 12. The collision was more violent, more energy went into unrecoverable work of deformation.

Event 14 The scattering of a cluster. L24

A stationary cluster of six balls is struck by a rapidly moving ball coming in from the right (in Fig. 20). The initial position of the cluster is seen at the center of the figure.

The total mass of the cluster is more than 4 kilograms. After the impact, ball speeds are relatively low. This explains why a flash-rate of only 5 per second was used and, also, why the speed of some of the balls after impact decreases noticeably as they move toward the edge of the field of view.

On the other hand, ball A is so fast before impact that the stroboscope, flashing only every 1/5th second, did not capture within frame more than one of its positions before collision. For this reason, a second picture was taken. See Fig. 21. The cluster of balls B to G was removed and ball A released from exactly the same initial point. The flash rate was, moreover, greater (20 per second). The "impact speed" of ball A can be determined from measurements of displacement in the center of the photograph of Fig. 21.

The analysis of Events 14 is analogous to the analysis of the previous problems. It requires more measurements and comparison of the sum of seven momentum vectors (after collision) with the momentum of the incoming ball. We recommend that this problem is assigned only after one of Events 8 to 13 was worked out by the student, and then only if he or she is one of the better ones in your class.

Notice that balls A and C have identical radius (2 inches). The same is true about balls B and D (1.75 inches). To distinguish between them, a mark in the shape of a cross was taped to ball A, and a line-mark was taped to ball B.

In determining speeds, the appropriate displacements must not involve the initial cluster-positions of the balls. "Best choice" is the distance between the first and second "scattered" positions.

The reference laboratory distance of 1 meter between tip-centers of the reference rods in the laboratory may not appear to be in the same scale as the two prints provided to the student. Hence, the scaling factors will not be equal. If this difference in scaling factors is taken into account by the students, the calculated "actual distance" will reproduce the laboratory conditions.

After determining the speeds and the magnitudes of the individual momenta, the student must add the seven individual momentum vectors together by drawing parallel lines. The vector addition proceeds most simply if the head-to-tail method is used. The order of the vectors being added is immaterial because vector addition is "commutative." See Fig. 22.

In our own measurements and calculations, system-momentum, before and after, differed by 2.1 per cent in magnitude and by 1.5° in angle. Kinetic energy fell from 5.22 joules to 1.36 joule (26 per cent conserved), a very violent set of collisions, especially when we remember that we used case-hardened steel balls.

I. INTRODUCTION

Six different two-body collisions, occurring along one dimension, were filmed with a high-speed motion-picture camera.[1] Each loop contains two of these collision events. Loop 18, in addition, show establishing scenes, filmed at normal camera speeds, which orient the viewer around the experimental set-up used to produce these collisions. Loops 19 and 20 contain high-speed sequences only. The projector shows the high-speed footage in slow motion.

Student notes for each loop are also provided. One or more events can be assigned as study-period (or laboratory-period) problems. Students should work in pairs.

The apparatus used in these filmed experiments is described in Section II below, and in each set of student notes.

The films should be projected upon a (preferably white) sheet of paper taped to the wall. The 18" by 24" desk pads often found in offices are a good source of such sheets. Even better are the 17" by 22" Quadrille sheets, lightly ruled, 10 squares to the inch, available through drafting supply stores. The projected image should fill the paper sheet. Students also need a good plastic see-through ruler, marked in millimeters, and a stopwatch (or other timing device) precise to the tenth of a second. A pair of draftsman's triangles will be helpful for drawing accurately parallel lines. Precision up to three significant digits is possible if measurements are carefully made.

The experiments involve marking pairs of vertical lines down on the paper to serve as "timing bars." Their positions (and the distance between the lines) are to be chosen after initial viewings of the film and then accurately measured. The passage of the balls on their (horizontal) paths can now be timed. Care must be taken not to move the projector during the experiment.

Students calculate ball speeds before and after collision (in relative units, e.g., in terms of the apparent speed across the paper). Section III, which discusses the six events in detail, also describes further a typical series of such measurements and calculations. The balls' masses are given.

Each collision is governed by the principle of conservation of momentum.[2] But these loops may be used to teach students far more than mere verification of this principle, especially if two events are assigned as problems.

The events occurring in each loop do, in fact, constitute matched pairs which may be used to teach interesting additional lessons. In Loop 18 the two collisions are mutually inverse. In one collision one ball is reflected, whereas this is not the case in the other, a matter which often surprises the novice. In each loop the same balls collide strongly and weakly, respectively. Calculation of kinetic energy, before and after collision, can lead to interesting discussion on the imperfectly elastic nature of the balls involved. Moreover, study of the two events in Loop 19 contains a useful lesson about error propagation. These aspects are discussed further in Section III (below).

The films are integrated with Unit 3 of the Project Physics text and may be assigned when students have reached Chapter 9 and 10.

If only one event is assigned, this event should not be the first example in Loop 19. In many cases the assignment of one event may be enough. But better students may well find time to study with profit a pair of events.

II. APPARATUS USED TO PRODUCE THE FILMS

The two colliding balls were hung in bifilar suspension from thin piano wires. See Figure 1. They were confined to move along the same circular path in a vertical plane. The radius of the circle was large (32 feet). The balls acted as pendulums of equal periods (about 6.4 seconds) to excellent approximation. In some events one ball was initially at rest at point C. Whenever both balls were in motion before collision, they were released simultaneously by relays from chosen initial positions, and they therefore also collided at point C, the bottom of their swing (a quarter period, or about 1.6 seconds, after release). A close-up of the release mechanism in operation is shown in Loop 18.

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[1] An alternative set of materials on this subject has also been produced by Project Physics and provides a similar experience to the students: the Stroboscopic Still Photographs of One-Dimensional Collisions, found following the Experiments in Chapter 9 of the Student Handbook.

[2] Momentum is vector quantity. Since the collisions in this series of film loops are one-dimensional, the vectors in any one problem are all parallel or anti-parallel. This simplifies the calculations. Loops 21, 22, 23 and 24 (as well as a series of stroboscopic still photographs) involve two-dimensional collisions.

The high-speed camera was placed in front of C for a 4 x 3 foot field of view. The camera took pictures at 2000 frames per second. Two white crosses were placed into the field of view. They may be used to realign the projector if it should accidentally be moved while measurements are taken.

The balls' paths are circles. Compared to their arc lengths in the field of view, the radius of curvature is large. Nevertheless, the speed of a given ball is not exactly constant as it moves towards point C of collision or away from C afterwards. The speed of the ball near the frame's edge is smaller than near the collision point C. It is smaller by less than 1.5 per cent if the amplitude of the pendulum swing is 12 feet or more; by per cent if it is 8 feet; by 7 per cent if it is 5 feet; and so on.

As a consequence, students should be encouraged to place the "timing bars" on the projected image of the film as near to point C as possible when seeking values for the speed of a ball before and after collision.

The more slowly a ball is moving before or after collision, the shorter should be the distance between the "timing bars" used to find its speed. This may lead to a larger relative error for the value of this speed.

III. THE SIX EVENTS, AND HOW TO TAKE DATA

The collision events present similar analytical problems. Measurements consist of timing each ball's motion past "timing bars" whose position and separation is chosen by the students. In our discussion, we give details for the first example in Loop 18 only. As regards the other 5 events, only their special aspects are discussed.

The simplest problem out of the six is the first example of Loop 20. There we deal with a perfectly inelastic collision of two balls with one ball initially at rest. After collision, the balls move together as one "compound" particle. Only two timings are required.

A schematic diagram is provided for each event both in this article and in the student notes. It specifies the masses of the balls and qualitative conditions before and after collision.

L18 One-Dimensional Collisions

First example. See Figure 2. Ball B is initially at rest. After collision, it moves off to the left at roughly the same speed as the initial speed of ball A. Ball A comes in from the right and emerges from collision with unchanged direction of motion.

Three timings are required. Two of these (initial velocity of A, final velocity of B) are simple since the speeds are relatively large. The following describes a typical procedure. (Such detail will, however, not be found in the student notes.)

1. Align the projector with the paper sheet on the wall. This alignment must not be disturbed until measurements are completed. The loop is run at least once for orientation.

2. To find the initial velocity of A, two fine and vertical lines must be drawn on the sheet. Here the rulings of the graph paper sheets (recommended above in Section I) come to one's aid. The two lines must be placed to the right of the collision point as closely to that point as possible.

The separation between these parallel lines should be as large as possible in order that the distance between them can be measured with reasonable precision. Since a ruler marked in millimeters is used, it is possible to estimate the tenth of a millimeter, although this digit is a doubtful figure. Hence, a distance of at least 10 cm is preferable, for then it can be measured with three-significant-digit precision.

The separation between these lines, on the other hand, should be as small as possible because the ball, as it moves toward the collision point, gains slightly in speed. If the image from the projector fills the paper sheet as completely as possible, this source of error contributes a small fraction of one per cent for this case if we compromise on a separation of 10.0 cm exactly. The students could measure the separation several times, estimating the nearest tenth of a millimeter, and use an average.

Similarly, 10 cm "timing bars" can be used for ball B after collision. But ball A moves slowly after collision.

When the speed of a ball is small, the amplitude of its swing (as a pendulum, see Figure 1) is small compared to the field of view. Hence it loses considerably in speed while still in the field of view. To reduce this source of error, the "timing bars" must be placed as near as possible to the collision and must be separated by as little as possible. But this raises another source of error: now the separation cannot be measured with the same precision. We chose "timing bars" separated by 3 cm, and when this distance is measured by our ruler, we may have only 2 significant digits.

3. Three velocities must be determined, one for ball B and two (pre/post) for ball A. One value for each of the three times of passage across the corresponding pair of "timing bars" is needed. With a stopwatch, obtain three values for each time interval and calculate the average. This will require repeated running of the film loop through the projector.

Repetition of the measurements of distance with our ruler, in (2) above, may convert the tenths of the millimeter from doubtful to significant. A like conversion of digits from doubtful to significant may result from repeated time measurements.

4. Calculate velocities, momenta and kinetic energies from the data. Table I lists these for a typical run of the experiment. Note: Do not assume that the student will get the same numerical values as appear in the table. These values depend on how large the image is on the paper and on the frame rate delivered by a projector. The latter is not guaranteed equal by the manufacturers in all models. On the other hand, the conclusions reached (including those about errors) from Table I are roughly those the student should reach.

5. Results. In Table I we have intentionally omitted mean deviations to keep discussion simple. We have also for simplicity failed to take advantage of the gain in significant digits obtained through averaging repeated measurements. In this way, the results appear at their worst. A good student can get better precision than is demostrated here.

The small value of ball A's velocity after collision reduced the number of significant digits to two and affects the corresponding momentum in the same way. Total momentum after collision is 4.4×10^2 g cm/sec, and before collision it is 4.3×10^2 g cm/sec to the same number of significant digits. The difference is 2.3 per cent of the average.

Table I

First Example in Loop 18–One-Dimensional Collisions
Ball A: 532 grams
Ball B: 350 grams
A hook under a digit means "doubtful"

Item	Ball	Time	Average Values		Direction
velocity	A	before	.813	cm/sec	left
	A	after	.25$\underset{\llcorner}{2}$	"	left
	B	before	0	"	----
	B	after	.885	"	left
momentum	A	before	433.	g cm/sec	left
	A	after	13$\underset{\llcorner}{4}$.	"	left
	B	before	0	"	----
	B	after	310.	"	left
kinetic energy	A	before	176.	ergs	not a vector
	A	after	16.$\underset{\llcorner}{9}$	"	
	B	before	0	"	
	B	after	137.	"	

The average numerical values appearing in column 4 depend on the size of the projected image. Students will not get these values, nor will different groups get equal values.

On the other hand, the kinetic energy calculation does not suffer from a reduction from 3 to 2 significant digits. The kinetic energy of the system is 176 ergs before and 154 ergs after collision. Kinetic energy 87.5 per cent conserved. The collision is not perfectly elastic. The balls were case-hardened steel which means that they have an inner core of soft steel. The collision is sufficiently strong to permanently deform this core, even if only slightly, with subsequent loss of mechanical energy into internal energy (much of it heat).

When a student studies both examples in this loop, this percentage of energy conservation should be compared and would lead to interesting discussion. See below.

Loop 18, second example. See Figure 3. Again, one ball comes in and strikes another at rest. Again, three sets of timing measurements are required. But this time it is the more massive ball which is initially at rest. This reverses the direction of motion of the incoming ball.

This event is, initially, the exact reverse of the previous example. The initial speed of ball B is the same as it was for ball A in the first example (to three significant digits).

After collision, ball B moves to the left very slowly. The "timing bars" need to be brought even more closely together than in the first example. The number of significant digits again drops from 3 to 2.

But here there is no loss in significant digits in the calculation of total momentum. Our measurements gave:

after collision: $\vec{p}_B$ = 23. g cm/sec to the left

$\vec{p}_A$ = 317. g cm/sec to the right

$\Sigma\vec{p}$ = 294. g cm/sec to the right

(Numerical values given here are not representative of those in student data.)

The fact that there was no loss in significant digits does not, however, necessarily insure greater precision in the results. Our value for total momentum before collision is 285. g cm/sec to the right. The difference is 3.1 per cent.

This difference is larger than it was for the first example, but note that it is larger only in relative, not in absolute, value. The absolute difference is, in fact, 10 g cm/sec in the first, 9 in the second example. Since the measurements were equivalent in these two examples and equal in number, the fact that absolute differences are nearly equal should not surprise us.

Our data also revealed that only 82 per cent of the kinetic energy survives the collision. Why was this percentage higher (87.5 per cent) when the same balls collided in the opposite manner?

In our pair of examples, this question is answered in a surprising way. We should expect greater energy loss if two balls collide with each other "harder." The average collision-force of interaction is proportional to momentum change experienced by either ball. In the first example, these changes are (see Table I) 3.0 x 10^2 and 3.1 x 10^2 g cm/sec respectively; in the second example (see data above), 3.08 x 10^2 and 3.17 x 10^2. Thus, the second "whack" was slightly greater, and, therefore, a higher percentage of energy was lost.

But this conclusion must be discarded. For, if difference of momentum, which is in principle zero, was 3 per cent on basis of the data, this indicates an experimental error of at least that size. In calculating kinetic energy, we square the measured speeds and this doubles the relative errors! The apparent difference of 5 per cent in energy loss is explained by the errors of our observations and may be spurious.

L19 Further Examples of One-Dimensional Collisions

The two examples of this loop involve the same pair of balls in differenct collision. One ball is over 3 times more massive than the other.

In the first example the two balls come at each other from opposite directions at considerable relative velocity. In the second example both balls are moving across the field of view in the same direction, one ball catching up with the other, but the relative velocity is small.

In both cases four velocities must be found from the film footage, and four pairs of "timing bars" are required.

The first example should not be assigned alone. Together with the second, however, the first example forms a highly instructive problem.

1. First example. See Figure 4. Both balls reverse their direction of motion upon collision. Ball A is moving slowly before and after, and a separation of less than 10 cm may be needed between the "timing bars." Because ball A is massive, however, the magnitude of its momentum (before as well as after) is comparable to the corresponding values of momentum of ball B.

Since the momenta of the two balls are oppositely directed (both before and after the collision), the net momentum of the system can be calculated only as a small difference between large numbers. Hence, a large relative error may be expected.

When we performed these measurements, we found a difference of 8.7 per cent between the net result for the momentum before collision and for the net momentum after. The actual numerical momentum values (in g cm/sec) we are about to cite from our data will not be the same as those found by your students. We, nevertheless, quote them here: 133 net momentum before, 122 after, both directed to the left. But the individual momenta were much larger. (Before collision: ball A, 448 to right; ball B, 581 to left. After: ball A, 454 to left, 332 to right.)

The large relative error is completely accounted for by the circumstances of this collision. The large individual momenta are far more precisely known. No so the difference! Had it been the sum, the relative error would be small. This is the first part of the instructive content of the present example.

The second point of interest here relates to the strength of the collision and is of particular value when this example is studied with the other event in this film loop. The student will find that only about 43 per cent of the total initial kinetic energy survives the collision. In the other event, this energy is 97 per cent conserved, i.e., the collision is almost perfectly elastic. Why? Because the soft core of these case-hardened steel balls is sorely tested in a real "whack," as occurs in the first example, but it is hardly touched in a weak collision. A good deal of any deformation of the inner core is permanent with resulting loss of the "work of deformation" from the initial supply of kinetic energy. The loss goes into "internal" energy (mostly heat).

Another way to explain the idea of "strength" in a collision is the following: the average force of interaction, multiplied by the time of contact in collision, equals the change in momentum of one of the balls. This is the "impluse theorem," a derived form of Newton's second law. Hence, change in momentum is a measure of the deforming force which occurs during contact in the collision. Note from the above data that ball A's momentum changes from 448 to the right to 454 to the left, i.e., by 902 g cm/sec (by our figures, not the student's). In the second example it is only about 400 g cm/sec.

2. Second example. See Figure 5. All velocities and momenta have the same directions (to the right) in this event. Four sets of measurements must be made to find the four speeds. All the speeds are large enough to allow 10 cm separation between "timing bars." Hence, there is chance for 3-digit precision.

Net momentum (before or after) is here calculated by addition of individual magnitudes (and not by subtraction, as was the case in the previous event). Therefore, the relative difference between momenta before and after will be small. Our value for it is 0.48 per cent.

Kinetic energy is 97.2 per cent conserved. The collision is almost perfectly elastic. This should be compared with the 43 per cent conservation found when the same balls collided in the previous event.

L20 Perfectly Inelastic One-Dimensional Collisions

In these events, the colliding objects are steel balls covered by a thick layer of plasticene. They remain lodged together after collision. Both collisions involve the same balls. The second collision is far stronger than the first.

The first example is the simplest of all six problems in this set of three film loops. It requires only two sets of timing measurements. The other example requires three.

Precision is excellent in both events.

1. First Example. See Figure 6. Ball A is initially at rest. Our value for the relative difference between net momentum before collision, and after, is 0.4 per cent. Kinetic energy is 59.4 per cent conserved. Three-digit precision throughout.

2. Second Example. See Figure 7. Here the balls come in from opposite directions. They move off to the left after collision. Three-digit precision throughout. To three significant digits, we found identical values for net momentum before and after. But kinetic energy is only 32.7 per cent conserved because the collision is more violent here than in the first example.

L4 A Matter of Relative Motion

This is a qualitative demonstration loop for repeated classroom use by the teacher. The concepts used are: (a) relative velocity and Galilean relativity (Unit 1); (b) principles of conservation of momentum and conservation of energy in elastic collisions (Unit 3). It is suggested that the teacher stop the projector near the start of the loop when a message on the screen asks "How did these events differ?" Encourage the students to describe the events they have just seen, without attempting to speculate on the ways in which the events were photographed. Then project the rest of the loop and initiate a discussion of relative motion and frames of reference. Come back to the loop when the conservation laws are studied in Unit 3.

In a technical sense, the word "event" implies knowledge of both places and times. A student walks from his home to school between 8:00 and 8:20 on Monday, and again between 8:00 and 8:20 on Tuesday. These are two different events. They are similar events, one being a repetition of the other. In the loop, three events not only occur during different time intervals, but also appear to be physically different. The student should be encouraged to describe what he sees—and the events do seem to require different descriptions.

The principle of Galilean relativity is discussed on pp. 102-103 of the text of Unit 1: Any mechanical experiment will yield the same results when performed in a frame of reference moving with uniform velocity as in a stationary frame of reference. In other words, the form of any law of mechanics is independent of the uniform motion of the frame of reference of the observer. Einstein broadened the principle to include all laws of physics, not just the laws of mechanics. Thus the Einstein relativity includes the laws of electromagnetism, which describe the propagation of light, as well as the mechanical laws of conservation of momentum and conservation of energy which are sufficient for our study of colliding carts.

L21–L25

Teacher notes are contained in the still photograph notes. L21, L22 and L23 are on page 100, L24 on page 102. There are no notes for L25.

L26 and L27 Finding the Speed of a Rifle Bullet—Method I and Method II

A. INTRODUCTION

These films are not meant to provide a precise laboratory exercise. While it is possible to obtain representative values for the muzzle velocities, the main intention is to bring a few conservation principles into the context of a real experimental problem, e.g., the ballistic pendulum.

The bullet's speed is calculated from the log's speed after impact by use of conservation of momentum. In Method I the latter speed is determined directly.

In Method II, on the other hand, the log's speed after impact must also be calculated. The student can measure only the log's full height of rise during its swing away from impact. To relate the two quantities, we invoke conservation of the sum of the log's kinetic energy and its gravitational potential energy during the swing.

Finally, one can (in each film) compare the kinetic energy of the bullet before impact with that of the log (with the bullet embedded in it) after impact, and ask the student: how was energy conserved here?

The films are designed to integrate with Unit 3 of Project Physics.

GENERAL DESCRIPTION

In Method I, there is a slow-motion sequence which permits timing the motion of the log just after the bullet strikes. The circular path of the log has a very large radius (see Figure 1) and the film sequence is a close-up of the log for a field of view of about 12" x 16." Hence the motion of the log can be considered uniform along a horizontal straight line. The student must convert distance as well as time measurements taken from the projected image of the slow-motion scene to "actual distance" and "real time." The information necessary for this conversion appears in the film. It is listed in Table I. See also the discussion under D. Now the student can calculate the bullet's speed by invoking momentum conservation, as follows

$$mu = (M+m)\,v, \qquad (1)$$

m = mass of the bullet
M = mass of the log
v = speed of the log (plus bullet) after impact
u = speed of the bullet

The values of M and m are given in each film.

In Method II the measurements are simpler. They involve distances only. No measurements of time are required. The film contains a slow-motion sequence showing the log, close-up, as it goes through its full pendulum swing after impact. The student can measure the vertical height of rise, h, of the log. That is to say, h is the distance from the log's lowest initial position before impact to the highest point of its swing (readily identifiable because it reverses its direction of motion at that point.)

Since the log's kinetic energy just after impact (at the start of its swing) must equal the gravitational potential energy acquired by having risen to a standstill, vertically through a distance h (at the end of its swing), $(\frac{1}{2})mv^2 = ma_g h$. Therefore,

$$v = 2a_g h \qquad (2)$$

v = speed of log (plus bullet!) just after impact
h = height of vertical rise of log during swing
a_g = acceleration of gravity

Having calculated v from Eq. (2), u can be calculated by use of Eq. (1).

METHOD I

In the close-up slow-motion sequence which is intended for the taking of data, the student must make two horizontal marks on the paper sheet taped to the wall while the log is at rest in the image. These marks must span the vertical dimension of the rod (H in Table I) which is given in another portion of the film to correspond to 15 cm "actual distance." By later measuring the distance between these marks, conversion of distances measured on the paper to actual laboratory distances becomes possible by scaling.

A strip of white adhesive is taped to the log. Either of its vertical edges can be used as a reference line for timing the log's horizontal motion after impact. Two vertical lines are drawn on the paper as timing bars. The distance between them is measured, and converted by scaling. The motion is timed by stopwatch.

"Film time" can be converted to "real time" if the slow-motion factor is known. This factor is the ratio

$$\frac{\text{frames/second taken by the camera}}{\text{frames/second delivered by the projector}}$$

The quantity in the numerator is given in the film and in Table I.

The quantity in the denominator must be measured since the manufacturer of the projector does not guarantee this rate to within less than 10 per cent. The total number of frames is printed on the cartridge into which the film is looped. There is a single black frame with a large white circle, in the "black stretch" between the end and start of the film loop, which is visible on the screen as a brief flash. The student can time the length of the loop.

When we took the indicated measurements off this film loop, we found that our loop of 3849 frames ran 207.3 seconds—a projection rate of 18.57 frames/second. The slow motion factor was therefore

$$\frac{2,850}{18.57} = 153.4$$

Times measured on the film are converted to absolute time by dividing by the slow motion factor.

Furthermore we found

v = 106. cm/sec, speed of log plus bullet after impact,

u = 6.43×10^4 cm/sec ≑ 2,110 ft/sec, speed of bullet.

The kinetic energy of the bullet before impact was 2,890. joules. The kinetic energy of the log (plus bullet) after impact, on the other hand, was only 2.50 joules! Most of the initial kinetic energy supply is dissipated in tearing wood and producing heat in this perfectly inelastic collision.

METHOD II

In the close-up slow-motion sequence which is intended for measurement, the student again marks off the vertical dimension H of the log while it is still at rest to serve as his scaling reference. The actual value of H is given in the film (and Table I) as 9.0 cm.

There are two horizontal strips of adhesive taped to the log. Any horizontal edge of these can now serve to mark off the initial position of the log (before it is struck) and the final position at full swing to determine h.

Our measurements (on the film) yielded h = 5.33 cm, v = 102. cm/sec and u = 5.8×10^4 cm/sec = 1,900. ft/sec.

The kinetic energy of the bullet before impact was 1,200. joules. The kinetic energy of the log, with the bullet imbedded in it, was only 4.2 joules.

L29 Colliding Freight Cars

The test of coupling strength was made by the Uplands Railway Laboratory for Canadian Pacific Railroad. The test engineer's report for the trial shown in the film's slow-motion sequence gives peak coupling force as 1,085,000 lb; hammer car's velocity after impact, 3.0 mi/hr.

An alternative method of finding V_1, of less accuracy but easier to understand, is to measure the time for the hammer car to come to rest after the collision. Then the initial velocity is just twice the average velocity. Do any students suggest this procedure?

Measurements from the film (in arbitrary units) gave 286 units for total momentum before collision, 280 units after collision. Kinetic energy of the system decreased from 390 units to 167 units.

L30 Dynamics of a Billiard Ball

The film has value even if used only qualitatively to illustrate conservation of momentum in a "real-life" situation. Measurements can be made and interpreted at two levels of difficulty.

1. Students should have no difficulty with straightforward conservation of linear momentum, as outlined in the student notes. For best results the velocities after impact should be measured over short distances to avoid complications due to friction. The cue ball's forward linear velocity is negligible just after collision, but this ball does gain forward speed as its rotational speed decreases due to friction. In a typical measurement, balls were timed as their leading edges moved forward a distance equal to one radius. The measured speeds (hence also the measured momenta) agreed within one per cent.

2. The following analysis is given primarily for the teacher's background. The balls rotate as well as translate, so we have to consider both translational and rotational momentum. The force of friction between the ball and the table surface affects the motion of a ball whenever there is slipping (i.e., a relative motion between the ball's lower surface and the table). A basic assumption is that the coefficient of sliding friction (μ) is the same for each ball and is independent of the speed of slipping. We use Newton's second law for translation ($F = ma$) and for rotation ($\tau = I\alpha$) where τ = torque, I = moment of inertia ($= 2/5\ mr^2$ for a sphere rotating about an axis through its center) and α is the angular acceleration. When a ball is rolling without slipping, its linear velocity v and its angular velocity ω are related by the equation $v = r\omega$.

At the moment of impact, the only force on each ball is that due to the other ball, acting along the line of centers. Because these forces have no lever arms, they can cause no changes in angular velocity at the moment of impact. The cue ball, which was rolling, must continue to spin with the same ω, and the target ball, which had no initial ω must start to slide with no rotation. These conditions do not persist, however, because a frictional torque acts on each ball while its lower surface is slipping on the table.

(a) Time of spinning of cue ball A frictional force μma_g acting toward the right on the bottom surface of the cue ball (Fig. 2) does two things: it causes the ball's rotational velocity to decrease, and it causes the ball's translational velocity to increase in the forward direction. After a time t_1 the velocity of the ball's lower surface equals the forward velocity of the ball:

$$r\ (\omega_1 + \alpha t_1) = 0 + at_1$$

$$r\ \left(\frac{v}{r} - \frac{\mu ma_g r}{2/5\ mr^2}t_1\right) = 0 + \frac{\mu ma_g}{m}t_1$$

where

$$t_1 = \frac{2v}{7\ ma_g}$$

After this time has elapsed, the cue ball continues to roll without slipping.

(b) Time for target ball to slide While all this is going on, the target ball starts to slide with belocity V2. Friction acting to the left (Fig. 3) causes the ball's translational speed to decrease, and it causes the ball's rotational speed to increase from zero up to some value. The ball starts to roll without slipping after a time t_2 when the linear velocity of the ball's surface (due to rotation) becomes equal to the ball's forward translational velocity:

$$r\ (0 + \alpha t_2) = V_2 + at_2$$

$$r\ \left(0 + \frac{\mu ma_g r}{2/5\ mr^2}t_2\right) = V_2 - \frac{\mu ma_g}{m}t_2$$

where

$$t_2 = \frac{2V_2}{7\ a_g}$$

Now we can explain the strange behavior of the balls. From the law of conservation of linear momentum, $v = V_2$. Hence t_1 and t_2 are equal! The changeover to rolling without slipping occurs at the same time for both balls; the cue ball seems to "know" what the target ball is doing.

(c) Conservation of angular momentum Change in angular momentum equals (torque) x (time). While the cue ball slides, it loses angular momentum $(\mu ma_g r)\ (t_1)$. While the target ball slides, it gains angular momentum

($\mu m a_g r$) (t_2). We have seen that $t_1 = t_2$, hence there is no net change in angular momentum. Since the momentum of inertia is the same for each ball, this means that $\omega = \omega_1 + \omega_2$ where is the initial angular velocity of the cue ball, and ω_1 and ω_2 are the angular velocities measured after both balls are rolling without slipping. A typical measurement from the film confirm this to within about 6 per cent.

(d) Coefficient of friction The time for slipping was found to be $t_1 = 2V_2/7 \mu a_g$ from which μ can be found if t_1 and V_2 are measured in real-time and real-distance units. (The slow motion factor is 167 and each ball has a diameter 5.24 cm). Perhaps it is easier to work with distances than with times, as follows: The distance for the target ball to slide is

$$d = V_2\ t_2 = 1/2\quad t^2$$

$$d = V_2\ \left(\frac{2V_2}{7\ ma_g}\right) + 1/2\ \left(-\ \frac{\mu m a_g}{m}\right)\ \left(\frac{2V_2}{7\ ma_g}\right)^2$$

which simplifies to

$$d = \frac{12\ V_2^2}{49\ ma_g}$$

Thus μ can be found by measuring V_2 and d.

Calculated values of μ are about 0.33. This agrees with a direct measurement of about 0.3 for the coefficient of sliding friction (not shown in the film).

L31 A Method of Measuring Energy—Nails Driven Into Wood

As the nail penetrates deeply into the wood, the force of friction increases somewhat, so the penetration is less than would be expected on the basis of the first few blows. Therefore, the graph is curved downward, as shown. For many purposes this effect can be ignored, and the energy of the object striking the nail assumed to be directly proportional to the depth of penetration.

L32 No Teacher Notes Needed

L33 Kinetic Energy

The "clock" turns at 3000 rev/sec, so each revolution is 1/50 sec. This value is not actually needed by the student, since only the relative speeds are used in testing the dependence of kinetic energy on v_2.

L34 Conservation of Energy I; Pole Vault

The film can be used qualitatively. The intermittent stop-frame sequences are long enough so that the teacher can mention the different forms into which the total energy has been transformed while they are actually "happening." Quantitative measurements are good to about 10 per cent. It is best to concentrate on comparing energy at position 1 with that at position 3; leave the more difficult check at position 2 for students who enjoy this type of measurement. Even if no measurements are made at all, you can tell the class that the energy is divided approximately as follows:

1. initial kinetic energy	1300 joules
2a. kinetic energy	450 joules
2b. grav. potential energy	650 joules
2c. elastic potential energy	300 joules
	1400 joules
3. final grav. potential energy	1800 joules
less muscular work	-400 joules
	1400 joules

In position 2, each of the three forms of energy is a significant portion of the total.

The intermittent freeze-frame technique is used because even the slow-motion action is too fast for accurate timing of a one-meter displacement. By this new technique, a student can measure the speed to within a few per cent if he counts, say, 20 frames. (1 frame out of 20 is only a 5 per cent error).

Some fine points can be raised for class discussion: (1) The gravitational potential energy of the pole itself is only about 10 joules in position 3, and can be neglected. (2) To be precise, we should calculate the work done in bending the pole by finding force as outlined in the student notes amounts to using the shaded area in Fig. 2. This is sufficiently accurate for our purposes.

Film Loops
L35, L36, L37
L38, L39, L40
L41

L35 Conservation of Energy II: Aircraft Takeoff

Air resistance does depend on speed and therefore does decrease somewhat as the plane's speed decreases. Make clear to the student that we are using an approximation when we ignore this effect. The approximation is justified by the large mass (inertia) of the plane. For instance, a typical measurement shows that at the upper level the speed was only 2/3 the value at ground level. This means that the force of air resistance at the upper level is only 2/3 the value at ground level. If, as intended, the engine supplies a constant forward force, there builds up a small net forward force when the plane reaches the upper level. But the acceleration caused by this unbalanced force is small because of the large mass of the plane; $a = F/m$. It is this sluggishness of response to changes in air resistance that allows us to make the simple energy analysis outlined for student measurements. If this seems unreasonable, reflect on the fact that the plane will, if it flies long enough at the upper level, regain its original ground speed when it again reaches terminal velocity; but there is no sign of such an increase during the time the plane remains visible in the film. The plane's inertia is simply too much to allow a rapid change of speed due to such a small unbalanced force.

During preparation of the film, 5 trials were carefully analyzed. The trial selection for reproduction gave results approximately as follows (height in meters, energies in units of 10^4 joules):

h	E_k	E_p	E_{total}
0	43	0	43
24	29½	13	42½
46	20	24½	44½
		average	43.3

As the table shows, the total energy remained constant to within about 4 per cent.

L36 and L37 No Teacher Notes

L38 Superposition of Waves

The amplitudes of component waves are intentionally varied somewhat irregularly while "setting up" a superposition. This is to keep before the student the idea that a human operator is really causing these changes on the face of an oscilloscope and he may be "playing around" a little, just as you yourself might do, or the student might wish to do. Be sure that the student understands that these are not animations.

Among the less familiar aspects of superposition is the fact that when two sinusoidal waves, which have the same wavelength but different phases and amplitudes, are combined, the resultant is again a simple sinusoidal wave of the same wavelength but with an intermediate phase. The phenomenon of beats can also be seen to result from the superposition of two waves which have slightly different wavelengths.

L39 Standing Waves on a String

The three loops on standing waves (L39, L40 and L41) are designed to emphasize the underlying features common to all standing waves. The source is at the left (tuning fork, loudspeaker, or dipole antenna); a reflector is at the right (wooden rod, piston, or aluminum mirror). At the end of loop L41, the three types of standing waves are compared in one composite picture, in which the wavelengths are the same, and the distances between nodes (given by $\frac{1}{2}\lambda$) are the same. The wave speeds, and the frequencies, differ by as much as a factor of 10^6.

L40 Standing Waves in a Gas

In symbols, where L is the length of the tube,

$$L = (n + \tfrac{1}{2})(\tfrac{1}{2}\lambda)$$

$$\lambda = \frac{2L}{(n + \frac{1}{2})}$$

Since

$$f = \frac{v}{\lambda}, \text{ then } f = \frac{v}{2L}(n + \tfrac{1}{2})$$

and

$$\frac{f}{(n + \frac{1}{2})} = \frac{v}{2L} = \text{constant}$$

For this pipe, $f/(n + \frac{1}{2})$ is about 151 vib/sec, from which v can be found to be 348 m/sec if L is given as 1.15 m.

L41 Standing Electromagnetic Wave

See Unit 5 T.G. for L41 Notes

L43 Vibrations of a Wire

The wire was actually a standard brass welding rod 2.4 mm in diameter. A short horizontal right-angle bend near the clamped end of the rod was essential to allow that point to serve as a node without undue restraint of vertical vibrations at neighboring points.

A surplus radar magnet was placed near an antinode of the wire's vibration.

The magnetic force is perpendicular both to the current and to the magnetic field.

An audio frequency source of high current and low voltage was needed. An audio oscillator fed a 20-watt hi-fi amplifier whose output was matched to the wire by a surplus power transformer used in a reverse or "step-down" connection. The "high voltage" (plate) winding was connected to the amplifier's 16-ohm output, and the "filament" winding was connected to the vibrating wire. Audio currents of several amperes passed through the wire.

For the wire, the observed frequencies of the first four modes were 8, 24, 48, 78 vib/sec.

An example will make clear the camera techniques used. For the "time exposure" or blurred shot, the camera speed was 3 frames/sec and the shutter was set at a full opening of 200° (out of 360°). Each frame thus was exposed for $(200/360)(1/3) = 1/5.4$ sec; during this time the wire had a chance to make enough vibrations to give the desired blurred effect simulating what the eye sees. To obtain the "slow motion" sequence for, say, the 48 vib/sec mode, the camera speed was set at 45 frames/sec, and the shutter closed to only 20°. Each exposure was therefore $(20/360)(1/45) = 1/810$ sec, which was short enough to freeze the wire's motion. The strobe rate was $48 - 45 = 3$ per sec as photographed. This becomes about 1/sec when projected in the classroom at 18 frames/sec.

The circular wire was actually clamped at two points very close together, which served as binding posts for the current.

For the circular wire, the observed frequencies of the first four modes were 10, 24, 55 and 101 vib/sec.

In discussing the Bohr atom from the point of view of DeBroglie waves (Unit 5) a familiar argument is that in the nth energy state there are n wavelengths in a complete circle of radius r. Then, since $\lambda = h/mv$, we have $n(h/mv) = 2\pi r$, whence $mvr = nh/2\pi$. This is Bohr's quantum condition for angular momentum. But the analogy is not as powerful as it seems. The Heisenberg uncertainty principle prevents us from knowing simultaneously both the angular momentum and the direction of the normal to the plane of an orbit. Therefore the planetary model of an electron's plane orbit is not a valid one, although it is useful in many cases as a first step. The film of a vibrating circular wire can certainly be used to show the student how a simple mechanical system with circular symmetry has a discrete behavior. By analogy, this makes plausible the argument that a simple atom might behave in a similar discrete fashion.

L42 Vibrations in a Rubber Hose
L44 Vibrations of a Drum
L45 Vibrations of a Metal Plate

Introduction

This is a set of three qualitative demonstration films. No work notes for the students are provided. The subjects supplement the study of waves. They may be shown in class by the teacher, or viewed by students individually, after the concept of the standing wave has been covered. (Project Physics Unit 3, Chapter 12.)

The films make use of this concept and carry it further. Film 1 should be shown first. All of them demonstrate the following ideas.

1. The vibrations of bodies can be explained in terms of standing waves.

Suppose you have something (an elastic body) capable of a certain type of vibration. Then you will find that

2. the body can vibrate in more than one mode of this type of vibration. To each mode corresponds a fixed, but different, frequency of vibration.

Moreover, the films, especially Film 1, are so constructed that they suggest the following fact.

3. In principle, the number of possible modes of this type of vibration is infinite.

In each film we drive the body at continuously increasing frequency—with a motor in Film 1, with a loudspeaker in 2 and 3. When the driving frequency passes through one of the fixed frequencies of vibration of which the body is capable (see 2 above), something happens. This is shown in detail and illustrates the concept of

4. resonance.

II. VIBRATIONS IN A RUBBER HOSE

Unit 3 presents the concept "standing wave" in connection with one-dimensional transverse waves such as are found in stretched strings.

Whenever two identical transverse traveling sine waves pass over the string in opposite directions, the superposed wave pattern appears to be "standing." To put it in another way, the string is vibrating. The vibration occurs in loops. A loop is exactly one-half wavelength long. See Figure 1. If the string vibrates in more than one loop, neighboring loops vibrate in opposite phases. Loops are separated by points on the string which do not move at all, called nodes. Successive nodes are separated by one-half wavelengths.

The rubber hose is driven by a variable speed dc motor—see introductory photo—connected through an eccentric linkage to point A (Fig. 1) at the bottom of the hose. The motor shakes point A in a side-ways oscillatory manner, but the amplitude of this motion is so small that point A can be treated as a node when it comes to consideration of the waves in the hose. Motor speed is controlled by a Variac.

The film opens with a scene in which the hose is stretched to produce tension. The value T of this tension, together with the mass μ per unit length of the hose, determine the wave speed v.

$$(v = \quad T/\mu .)$$

It follows, from an inspection of Fig. 1, that the wavelength λ is restricted by the length of the hose and by the fact that end points A and B must be nodes. The wavelength λ can only take on the discrete values

$$\lambda = 2L,\ L,\ \frac{2L}{3},\ \frac{2L}{4},\ \frac{2L}{5},\ \ldots,$$

$$\lambda_n = \frac{2L}{n},\ n = 1,\ 2,\ 3,\ \ldots.$$

If f is the corresponding frequency, and since sine waves satisfy $\lambda f = v$,

$$f_n = \frac{v}{2L} n, = 1,\ 2,\ 3,\ \ldots.$$

The overtones are all integer multiples of the fundamental frequency $f_1 = v/2L$. In our hose $f_1 = 2$ cycles/sec.

The above numerical details need not—they probably should not—be presented by you in class. It would be better to keep the discussion on a qualitative level.

The main sequence of the film records what happens after the motor is turned on and its speed is continuously increased from zero. As the speed approaches 2 rps (not given in the film), the amplitude of pattern 1 in Fig. 1 steadily rises: resonance!

Motor speed continues to increase, and a transition to the second harmonic (pattern 2, Fig. 1) takes place, and so on.

The film shows the first 15 transverse modes of the rubber hose.

III. VIBRATIONS OF A DRUM

The vibrating body now is a circular rubber membrane under tension. (The wave speed is in this case the square root of the ratio of surface tension to mass per unit area.)

But here we are dealing with two-dimensional waves which pass radially inward or outward as well as "angularly" around the circle. The standing waves are now not the simple sinusoidal loops we saw in Film 1. The film was made primarily to show what two-dimensional standing waves might look like qualitatively.

The "drum" is also capable of transverse vibration in an infinity of modes. The characteristic frequencies are now not integer multiples of each other. In the model drum we used, they were 50, 152, 258 and 373 cycles/sec for the first four symmetric modes shown, and 100 and 205 cycles/sec for the two antisymmetric modes.

The drum head was photographed, in some of the sequences, with a variable-speed motion-picture camera. For each of the six modes, the speed of the camera was slowly varied from just below to just above the characteristic frequency of the mode in question, while keeping the drum in steady resonant vibration. The effect is stroboscopic. The vibration appears slowed down, revealing the shape of the membrane for each mode.

Fortunately for the viewer of this film, the film is silent. The loudspeaker driving the drum was running at 30 watts root-mean-square power.

IV. VIBRATIONS OF A METAL PLATE

A square aluminum plate is clamped tight at its center. A loudspeaker drives the plate from below at increasing frequency.

This system resonates for many different frequencies. The amplitudes of the two-dimensional standing-wave patterns in the plate are too small to make them visible.

Sand is sprinkled on the plate. It vibrates furiously. When the output frequency of the loudspeaker reaches resonance for one of the plate's modes, the sand collects along lines and curves in a symmetric pattern and is quiescent there. These lines and curves are the geometrical loci of the nodes of the two-dimensional standing waves in the plate (nodal lines). The patterns of nodal lines are called the Chladni Figures.

Fig. 1. A stretched string is capable of an infinity of modes of transverse vibration. Each mode constitutes a possible transverse standing wave satisfying the necessary condition that the fixed ends A and B be nodes.

E22* Collisions in One Dimension

Since the word momentum is never used and the concept of conservation of momentum is never assumed in the Student Handbook, the experiment may, if desired, be treated as a "discovery" lab.

The next experiment (Experiment 23, Collisions in Two Dimensions) may be done concurrently if apparatus and student background permit. The instructions for Experiment 23, however, assume a knowledge of conservation of momentum in one dimension.

Several different procedures are described in both experiments. They could be combined into one "circus" lab with as many different procedures being followed simultaneously as apparatus permits. After all the working groups have finished, they can bring their findings to a common class discussion.

Let students develop an intuitive feeling for "frictionless" collisions by "playing" with balloon pucks, disc magnets sliding on plastic (Dylite) beads, balls, etc., before beginning the quantitative work with airtrack, dynamics carts or film loops.

Some of the procedures require strobe photography. Remember that the room need not be completely dark (a dark background is important, though). It is possible to have two groups working on photography in one part of the room, without making it impossible for the rest of the class to work.

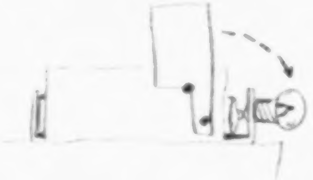
To save time, have the air track ready and the camera in position at the beginning of the period. Set about 4 feet from the track, the camera makes an image reduced in size about 10:1. Demonstrate the techniques of simultaneously opening the shutter and launching the glider and then closing the shutter after the interaction.

Some limits must be set on the kinds of interactions to be photographed if the experiment is to be done in one class period. The photographs are less confusing to analyze when all interactions start with one glider stationary in the center of the track and the other launched toward it from the left. Tape the lights to the glider with one light higher than the other (bend , one lamp socket) so that their images can be distinguished on the photograph.

As an alternative to the light sources mount a white or metallized drinking straw on each glider and use a xenon strobe. Make one straw taller than the other.

If the left-hand glider rebounds, the images will overlap and make measurements difficult. An inventive student could be encouraged to devise a way to distinguish the "before" pictures from the "after" ones. Otherwise, photograph only collisions in which the left-hand glider has a mass equal to or greater than the right-hand one so that the glider on the left does not rebound.

Here is one tested way to distinguish the images of the rebounding glider: pivot a small piece of colored transparent plastic or partly-exposed photographic negative on the glider so that, when the gliders collide, the plastic will fall in front of the light (see Fig. 1). The images formed after the collision will then be fainter.



Alternatively, on the dynamics cards but not on the air track gliders, the lamps themselves can be mounted vertically like the card in Fig. 1 so that the entire lamp tips to a horizontal position upon collision.

A third possibility is to have a student drop a filter in front of the camera lens at the instant of collision, thus dimming all subsequent images.

Although students must determine the mass of each cart with the light taped to it, the actual mass in kilograms is not needed, only the relative mass expressed as a multiple of the smallest glider's mass.

The notes on photography at the back of the Teacher Guide to Unit 1 suggest other ways of using the photographs.

The collision experiment between carts is possible as well, of course.

If bell timers and ticker tape are used to record the motion of the carts, notice that the tape attached to the left-hand cart in Fig. 1 passes under the right-hand cart to the timer which lies far enough to the right of the right-hand cart to allow room for the recoil motion of the right-hand cart. Conversely, tape to the right-hand cart passes under the left-hand cart. The frequencies of the two timers may not be the same. Warn students to check this.

When students tabulate their data, have them record speeds to the right as positive and to the left as negative, and remind them about the difference between speed and velocity.

Assemble data from all the photographs in a table to enable students to see the pattern which emerges.

With gliders of equal mass, students may conclude from a simple elastic collision that speed is a conserved quantity.

Use a linear explosion to show that the directions in which the carts travel must be considered if momentum is to be conserved.

The tabulated results make it clear to students that mv is the quantity which is conserved. Some students may notice that mv^2 is also conserved for elastic collisions; point out that it is not conserved for inelastic ones (or explosions since mv^2 is a scalar, not a vector, quantity), and defer discussion of this point until after Experiment 24 is done.

Summary of Results

Results of linear collision experiments lead to the general expression

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A' + m_B\vec{v}_B'$$

A number of special cases should be pointed out to the students:

1) When $m_A = m_B$ the expression simplifies to

$$\vec{v}_A + \vec{v}_B = \vec{v}_A' + \vec{v}_B'$$

2) When both carts are initially at rest ("explosion")

$$\vec{v}_A + \vec{v}_B = 0$$

so that

$$m_A\vec{v}_A' + m_V\vec{v}_B' = 0$$

or

$$m_A\vec{v}_A' = -m_B\vec{v}_B'$$

In this case

$$\frac{-v_A'}{v_B'} = \frac{m_B}{m_A}$$

which means that the carts go in opposite directions at speeds that are in inverse proportion to their masses.

3) When the carts stick together with one initially at rest, then

$$\vec{v}_A = 0$$

and

$$\vec{v}_A' = \vec{v}_B'$$

so that

$$m_B\vec{v}_B = (m_A + m_B)\vec{v}_B'$$

and

$$\vec{v}_B' = \vec{v}_B \cdot \frac{m_B}{m_A + m_B}$$

The canoe's speed will be two meters per second in the opposite direction. The results may be disastrous!

$$1 \times \frac{5 \times 10}{8 \times 10} = 6 \times 10^{-7} \text{ m/sec}$$

Ask students to estimate how far the ship would drift in an hour at this speed if there were no friction:

$$\Delta d = v\Delta t = 6 \times 10^{-7} \text{ m/sec} \times 3600 \text{ sec}$$

$$= 2 \times 10^{-3} \text{ m or 2 mm.}$$

To extend this idea ask what effect a boy jumping vertically upward would have on the earth, which has a mass of 6×10^{24} kg.

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A' + m_B\vec{v}_B'$$

$$(7 \times 2) + 0 = (7 \times 1) + 2\vec{v}_B'$$

$$\vec{v}_B' = 3\tfrac{1}{2} \text{ m/sec}$$

Thus, the pin will move off with a speed of about 3½ m/sec. (These numbers have been rounded off; the actual masses are closer to 7.3 kg and 1.5 kg. After an elastic collision the velocities would be 1.2 m/sec and 3.9 m/sec).

The effects of friction have been avoided thus far in the discussion. When a wooden block slides across the table, for example, the table and ultimately the earth itself become part of the interaction. As the block slows down, momentum is transferred from the block to the earth. This point is often missed by students; momentum is conserved, even when friction acts on the objects.

E23 Collisions in Two Dimensions

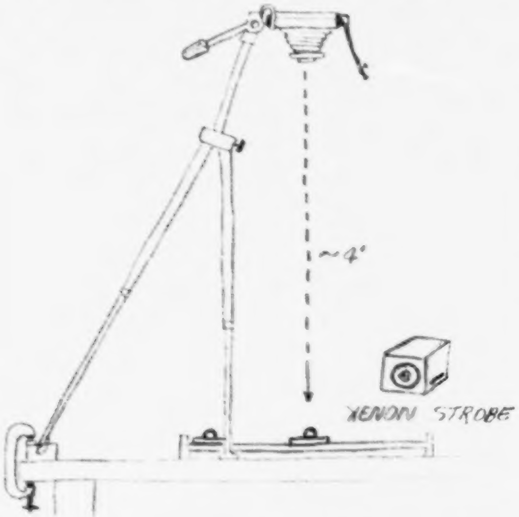
Even if your students do not do this experiment quantitatively, let them play with balloon pucks, pucks on beads and magnet pucks on beads to get an intuitive feel for two-dimensional collisions.

Two sizes of puck are supplied. The mass ratio is 2:1. They may be used as balloon pucks (remove the stopper with the balloon when inflating) on any flat and fairly clean surface. (You can improve sanitation—and avoid saliva on the pucks—by placing puck tightly up against the exhaust of a vacuum pump to inflate the balloon.)

The "new rule" is that momentum is a vector quantity.

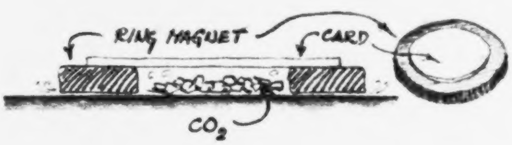
For strobe photographs remove the balloons and slide the pucks on a low friction surface made by sprinkling plastic (Dylite) spheres on a ripple tank. Use a small piece of clay or tape to fix a steel ball (e.g., from the trajectory apparatus) or white styrofoam hemisphere*, at the center of the puck. (Other reflectors, such as a white painted stopper, are also possible, but give less satisfactory photographs.)

The xenon strobe must be at the side of the tank, not above it. This is to reduce reflections from the beads themselves which may still be a nuisance: don't use too many beads!



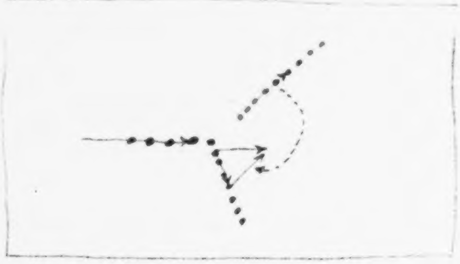
To get the camera directly above the working surface extend one leg of the tripod more than the other. The tripod will then be unstable so the long leg must be held down as shown in the sketch.

Alternative procedure with ring magnets (e.g., from Edmund Scientific): glue a piece of card over the top of the magnet. Put a piece of dry ice inside, or fill with "dry snow" from a CO_2 fire extinguisher. Place the magnet down on a flat surface (ripple tank): it will float on the film of CO_2 gas.



Another possibility here is an "explosion" between magnets. With only two must they move apart in opposite directions along a straight line? If the masses are equal will the speeds be equal? What about three or more magnets held close together and released simultaneously?

Or students can draw their vector diagrams by pricking holes and drawing on the back of the photograph:



The conclusion that momentum is conserved in an interaction is not sufficient to enable us to predict the final velocities of the interacting bodies, except in a few special cases. Given m_A, m_B, $\vec{v}_A$, $\vec{v}_B$, the equation

$$m\vec{v}_A + m\vec{v}_B = m\vec{v}_A{}' + m\vec{v}_B{}'$$

has two unknown quantities, $\vec{v}_A$ and $\vec{v}_B$. Students realize from their knowledge of algebra that two equations are needed to find these quantities. Since a given set of initial conditions always leads to the same pair of values of $\vec{v}_A{}'$ and $\vec{v}_B{}'$ some other requirement must be imposed on the system. Stress this point to prepare students to look for still another conservation law in the next experiment.

One special case is very well illustrated by the magnet disc experiments: In an (elastic) collision or interaction between two equal-mass bodies, one of which is initially at rest, the two velocities after the collision are perpendicular to each other. This is quite independent of the "off centeredness" of the collision.

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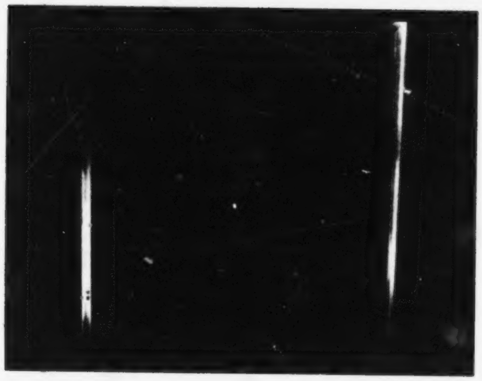
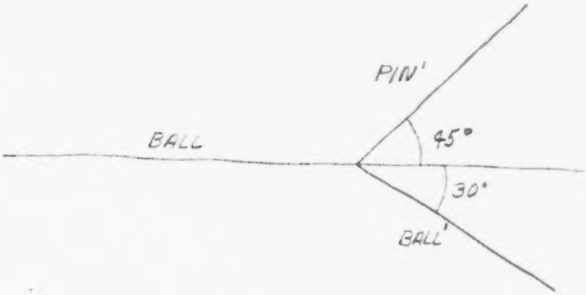
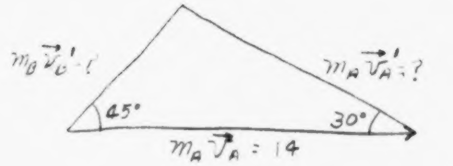
*From Star Band Co., P. O. Box 7068, Midtown Station, Portsmouth, Virginia 23707



Point this out to students, but don't expect them to be able to prove it, or understand the proof, which is given in the Teacher Notes to the next experiment.





Three known quantities (two angles and the length of a side) are enough to draw the vector triangle. By measurement we find

$$m_A\vec{v}_A' = 10.2$$

$$\therefore \vec{v}_A' = \frac{10.2}{7} = 1.5 \text{ m/sec}$$

$$m_B\vec{v}_B' = 7.3$$

$$\therefore v_B' = \frac{7.3}{2} = 3.7 \text{ m/sec}$$

The vector diagram is an octagon. Seven of its sides—representing the momenta of the yellow fireballs—are equal in length and at 45° to each other. The eighth side must therefore have the same length. Hence

$$(m\vec{v})_{red} = (m\vec{v})_{yellow}$$

$$\text{since } m_{red} = \tfrac{1}{2}m_{yellow}$$

$$\vec{v}_{red} = 2\vec{v}_{yellow}.$$

Addendum to E23

With a small modification, the Project Physics trajectory apparatus (Experiment 8, Unit 1) can be used in a collision in 2-D experiment.

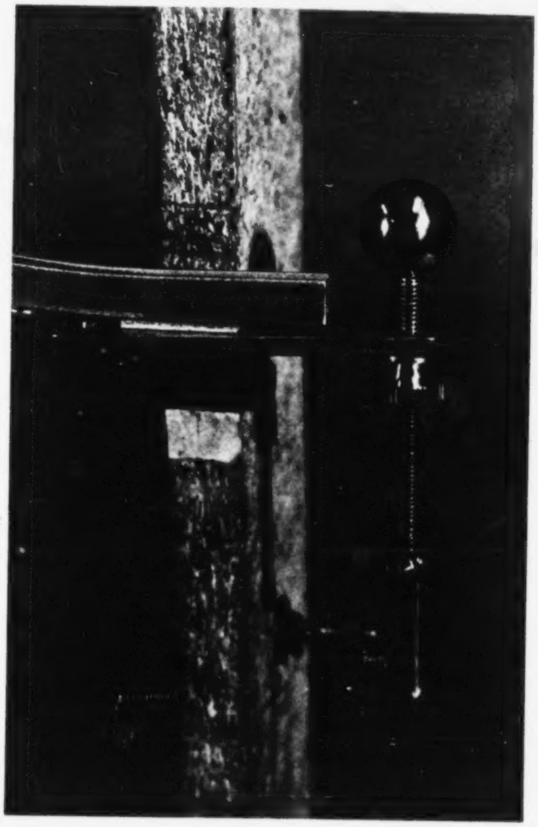
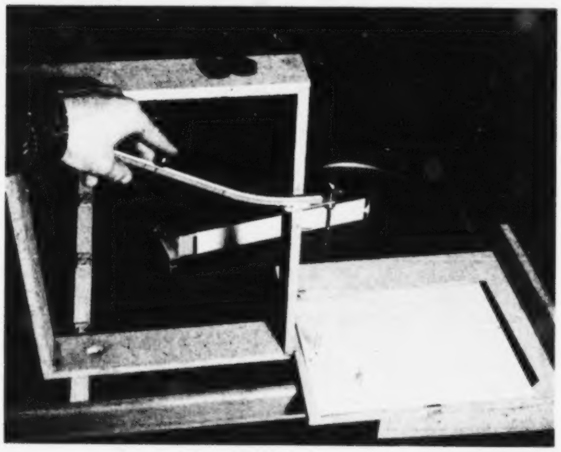


Figure 1 shows how the target ball is positioned at the end of the launching ramp. The attachment can be swung about a vertical axis to vary the impact parameter (amount of "off-centeredness" of the collision).

The plotting board is laid flat (at the corner of a table). On it are placed, first, a sheet of carbon paper (carbon side up), and over the carbon paper a sheet of onionskin paper. See Fig. 2.



With no target ball in position, a ball is released from a point on the ramp and allowed to fall on the plotting board where it records its point of impact. Repeat a few times to get consistent results. A target ball is now placed on the support and the impact ball released again from the same point on the ramp. Both balls record the positions at which they land on the impact board.

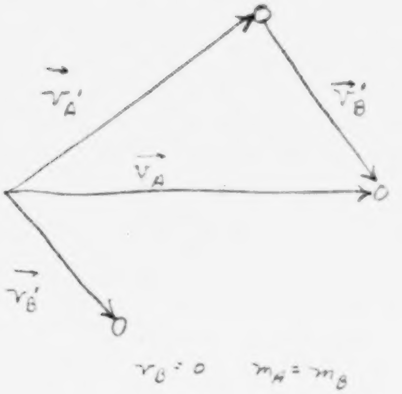
Analysis of Results

The balls all fall through the same vertical distance from collision to the board, and are therefore in flight for the same time. The horizontal distance that each ball travels between collision and impact is thus proportional to its velocity.

Distances are measured from the collision point, as indicated by the plumbline in Fig. 2.

Students can analyze for conservation of momentum in the horizontal plane by drawing vector diagrams as shown in Fig. 3. The launch position, mass ratio and impact parameter can all be varied to provide a wide range of situations.

The same data can be analyzed for conservation of kinetic energy (in the horizontal plane).



E24 Conservation of Mechanical Energy

Students could analyze their data from the previous (momentum) experiment, looking for other conserved quantities, as a homework assignment before beginning this experiment. For the elastic collisions they will find that mv^2 is conserved. (It will not be conserved in the explosion, of course. Other quantities like m/v and m^2v may be conserved in special cases, for instance, m^2v is conserved in an elastic collision between equal-mass objects.)

Before the students start photographing collisions, demonstrate a slow collision as follows: remove one screw from the spring bumper of each cart and arrange the carts as shown in Fig. 1(a). Add 1 or 2 kgs of extra mass to each cart. The combination of large mass and weak spring produces an extremely slow interaction; students can see clearly that there is an instant when both carts are moving quite slowly. Then replace the screws so the bumpers are in their original shape for taking photographs. Two kinds of spring are supplied in the dynamics cart kits; use the lighter one (0.010 inch) for this experiment.

You must limit the amount of compression of the bumpers by taping two corks or rubber stoppers to the front of each cart, as shown in Fig. 1(b). Otherwise too violent a collision

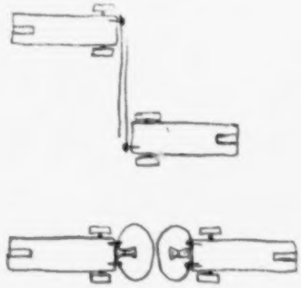


Fig. 1

may bend the bumpers permanently and make them unusable for subsequent experiments. Of course, a collision in which the bumper touches the stoppers will not be elastic and should not be analyzed.

If the carts are moving too slowly, successive strobe images will be too close together to measure.

It is essential to be able to identify simultaneous images of the two carts. The technique suggested to students is probably the simplest. Another is to close the camera shutter before either cart goes out of the field of view, and work backwards from the last image of each cart.

Transparency 22 is made from a photograph taken in this experiment. If your students don't succeed in getting a good photograph they could take data from the transparency and analyze it in the same way.

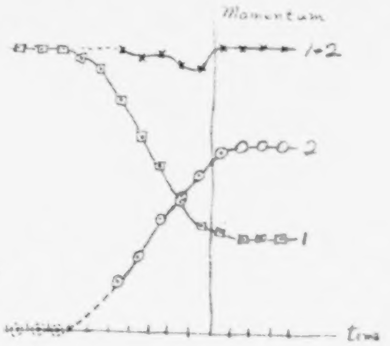


Fig. 2a

Fig. 2(a) Momentum of each cart, and total momentum, plotted against time.

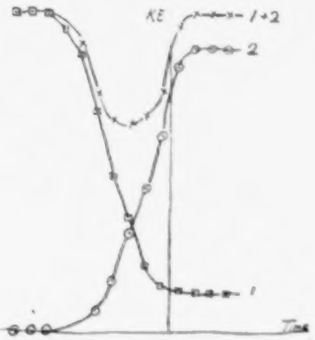


Fig. 2b

Fig. 2(b) Kinetic energy of each cart, and total kinetic energy, plotted against time.

Alternative technique for making a magnet "float" without using plastic beads: use ring magnets (e.g., from Edmund Scientific, Radio Shack, etc.). Glue a piece of card over the top of magnet. Put some dry ice, or "dry snow" from CO_2 fire extinguisher, inside the magnet.

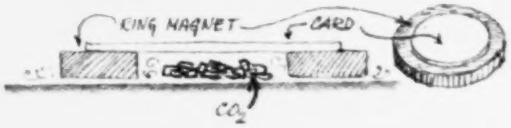


Fig. 3

Put the magnet down on a glass surface and it will float on a film of CO_2 gas (Fig. 3).

The steel ball is from the trajectory equipment used in Unit 1. Styrofoam spheres are available from: Star Band Co., P.O. Box 7068, Midtown Station, Portsmouth, Virginia 23707.

These photographs show up the "right angle" law very nicely. In an elastic collision between two equal mass objects the angle between the two velocities after the collision is 90°. This is quite independent of the impact parameter (amount of "off-centeredness" of the collision).

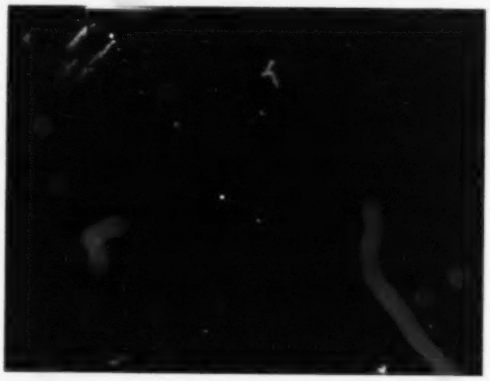


Fig. 4. Photo of a two-dimensional "collision" between disc magnets. Strobe rate about 30 per second.

Students should be aware of this 90° law, but need not necessarily follow the proof which involves both conservation of momentum, and of kinetic energy:

In $m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A{}' + m_B\vec{v}_B{}'$

$m_A = m_B$ and $v_B = 0$

$\therefore \quad \vec{v}_A = \vec{v}_A' + \vec{v}_B'$

i.e., the vectors $\vec{v}_A$, $\vec{v}_A'$, $\vec{v}_B'$ can be represented by three sides of a triangle.

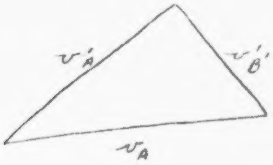


Fig. 5

The kinetic energy equation

$$\tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2 =$$

$$\tfrac{1}{2} m_A v_A'^2 + \tfrac{1}{2} m_B v_B'^2$$

simplifies to

$$v_A^2 = v_A'^2 + v_B'^2$$

This is the Pythagorean equation for the three sides of a right angle triangle. If $v_A^2 = v_A'^2 + v_B'^2$ then the angle between v_A' and v_B' is 90°.

In fact there usually is a steady loss of kinetic energy as the magnet moves across. Notice from the curve plotted below that the rate of energy loss is about constant. Although there is a significant drop in total kinetic energy during the interaction, the KE after the interaction is about what it would have been if no interaction had taken place.

This is not an easy experiment because the interaction is over so quickly—much more quickly than the collision between two carts with spring bumpers. In Fig. 6 (data from the photograph reproduced above) there is only one pair of values for velocities during the collision, but the kinetic energy minimum is nevertheless quick striking. The photograph was taken at the maximum setting of the Stansi strobe (about 30 per sec).

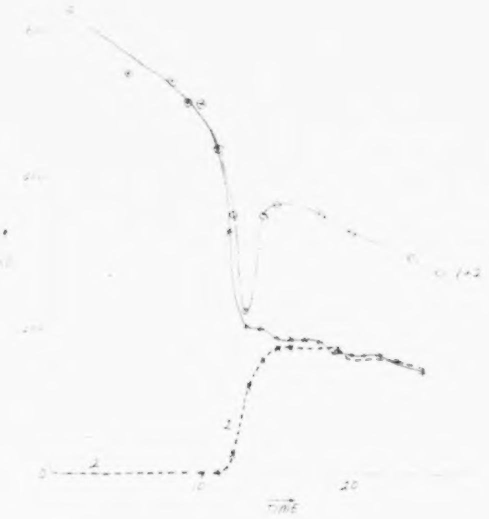


Fig. 6

The expression $\Delta(PE) = -\Delta(KE)$ describes "ideal" behavior rarely met in practice. Students will probably find that the airtrack is far from perfect. There may be some loss due to poorly fitting gliders. There may even be a gain in energy! The air pressure is slightly higher at the inlet end of the track: this tends to "blow" the glider away from that end when the track is horizontal, or push it uphill when the inlet is at the low end. It is more important that students analyze their results honestly, and can suggest reasons for any difference between $\Delta(PE)$ and $-\Delta(KE)$ than that they "prove" that mechanical energy is conserved.

$$\tfrac{1}{2} mv^2 = ma_g h$$

$$\therefore v = \sqrt{2a_g h}$$

v depends only on h, not on the slope of the track. In theory, therefore, v should be the same for all these trials. In practice we found that v_f is constant and is equal to $\sqrt{2a_g h}$ to within better than 10%.

Disc strobe technique with light source on the glider, or xenon strobe with a white or metallized drinking straw mounted on the glider. For this experiment it is essential to measure velocity in meters per second. Therefore the strobe rate and the scale factor (preferably 10:1) must be known.

In one trial (with the air inlet at the low end of the track) we found a significant decrease in total energy: the kinetic energy increase was less than the potential energy decrease, presumably due to the air pressure effect mentioned above. When the air inlet was at the top of the track we found the reverse:

$\Delta(KE) > -\Delta(PE)$.

When the glider is at rest at the bottom of the track all the energy is stored (potential energy) in the stretched rubber band. The stretching of the rubber band as the glider slows down and stops can be seen much more easily if the band is made quite slack. It is analogous to the slow collision between two dynamics carts with spring bumpers.

The simplest way would be to measure the height of successive rebounds.

$$E_1 = ma_g h_1$$

$$E_2 = ma_g h_2$$

$$\therefore \Delta E = ma_g \Delta h$$

These film loops could be used:

18 One-dimensional collisions
19 Further examples of one-dimensional collisions
21 Two-dimensional collisions
22 Further examples of two-dimensional collisions
31 A method of measuring energy in nails driven into wood
32 Gravitational potential energy
33 Kinetic energy
34 Conservation of energy 1—pole vault
35 Conservation of energy 2—aircraft take-off

Discussion

The different observations made by the various student groups should be summarized in a class discussion after the experiments have been done. Some of the points to bring out are:

1. Unlike momentum, kinetic energy is not conserved in all the situations investigated. It is not conserved in inelastic collisions or explosions, for example.

2. Although the kinetic energy after an interaction (e.g., slow collision) may return to its initial value there is a temporary disappearance of kinetic energy during the interaction, i.e., while the springs are touching or while the magnets are close to each other. This is the time to mention potential, or stored, energy and some of its forms, elastic and magnetic.

3. The airtrack glider pushed up the track comes to rest momentarily near the top, increases speed coming down again, comes to rest momentarily at the rubber band, moves back up again, and so repeats the cycle. The original kinetic energy is converted to gravitational potential energy, back to kinetic energy, to elastic potential, back to kinetic energy, and so on.

E25 The Speed of a Bullet

If the bullet bounces back a bit the cart will gain more momentum. Using $v = v' \frac{M+m}{m}$ would lead to too high a value of v.

a) Use time-of-flight technique with two beams of light and two photocells as described in notes on the use of oscilloscope, Teacher Guide 1, page 119.

b) Fire horizontally and measure the horizontal distance traveled as it falls to ground.

c) Fire vertically upwards: measure maximum height attained.

d) Time-of-flight measurement described in Unit 1 Student Handbook, Chapter 4, page 9.

Although the pendulum actually swings in an arc of a circle, the collision is over very quickly, which means that for the application of the conservation of momentum the two motions are in the same direction.

See the four methods suggested above.

E26 Hotness, Thermometers, Temperature

Students usually have not thought about what temperature means, are not aware that any problem of defining temperature exists, and may even be unwilling to admit that it does. Indeed many texts dismiss the problem with a statement like, "The temperature of a body is the scale reading on a suitable thermometer."

This experiment will have succeeded if the students have been brought, possibly for the first time, to think about the nature of the concept of temperature. The object of the experiment is NOT to have students calibrate other thermometers against the mercury-in-glass thermometer, but to realize that on first consideration the other devices are in themselves just as valid for use as thermometers (and for use when constructing temperature scales). It is only much later, when we have some theoretical basis for our ideas of heat and temperature, that we will have any reason other than convenience for choosing one device over another.

What follows is a more than usually lengthy outline of the sort of pre-lab and post-lab discussion that we believe would make the students aware of the problem. The title we have given to the experiment reflects the order in which the subject has to be developed: from the crude subjective sensation of hotness and coldness, through the invention of some objective device sensitive to changes in hotness, to the establishment of a temperature scale.

The major point to bring out is this: temperature, like all other ideas which have been of great value to science, is an invented concept. Acceleration is another example. It could have been defined as $\frac{\Delta v}{\Delta s}$ rather than $\frac{\Delta v}{\Delta t}$, and indeed Galileo considered this. But $\frac{\Delta v}{\Delta t}$ turns out to be a much more useful definition.

One point may summarize the whole problem. We frequently say and teach that Charles (or Gay-Lussac) found by experiment that the volume of a sample of gas is proportional to its absolute temperature. He probably used a mercury expansion thermometer. Suppose he had used a different sort of thermometer—one whose expansion was very non-linear with respect to the expansion of mercury—what kind of "law" would he have discovered?

Galileo used a "thermoscope" consisting of a glass flask with a long neck dipping into water. The water level in the tube rose or fell as the air in the bulb was cooled or heated. You might want to add this to the variety of thermometers used in the experiment.

Any convenient, easily measured property of any substance which changes with hotness (the original, subjective sensation from which we start) could be used to construct a thermometer, to define temperature quantitatively. Notice that we cannot at this stage of the argument say that the expansion of mercury (for instance) is a good system to choose "because mercury expands linearly with temperature." Only the reverse is possible: we could use the mercury expansion system to define temperature. But the choice is not obvious and a wide variety of systems are available.

An optical pyrometer measures the temperature of hot bodies by comparing the light they emit with a standard (hot wire).

We could not put a gas bulb between two colliding carts (or insert it under a patient's tongue) and hope to measure a temperature rise. We cannot use a mercury thermometer below the freezing-point or above the boiling-point of mercury.

The original fixed points on the Fahrenheit scale were:

0°, the temperature of a mixture of equal parts of ice and salt, and 100°, human body temperature.

Nowadays, the Fahrenheit scale is defined by:

32°F, the temperature at which ice melts, and 212°F, where water boils.

You might at this stage want to raise the question of temperatures below 0° or above 100°. How are they defined?

There is no reason at this stage, either theoretical or experimental, to suppose that the two thermometers should necessarily agree. There is a small but significant difference between a mercury expansion thermometer and an alcohol expansion thermometer. There is a very striking difference between the thermistor thermometer and almost any other thermometer.

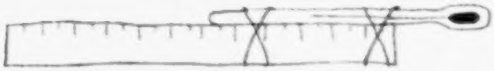
Use as wide a variety of devices as possible: uncalibrated mercury, uncalibrated alcohol, gas volume, gas pressure, thermistor, thermocouple,... Assign a pair of devices to each group of students.

Plotting a graph helps students to distinguish between systematic differences and random ones.

Have one ice bath and one boiling water bath and at least five other numbered baths covering the intermediate range. Note that except for the ice and boiling water, the baths need not be maintained at constant temperature; each group must, of course, take the readings of its two devices at the same time.

Thermometers

A. Uncalibrated mercury-in-glass (e.g., Cenco cat. no. 77320, $1.60 each). The volume of a confined sample of mercury is indicated by a thin thread of the sample that is free to rise in an empty tube. A centimeter scale should be fixed along the thermometer. A 1-foot wooden ruler (with the brass strip removed) is satisfactory when attached with rubber bands as in Fig. 1. The extra length provides a convenient handle. It is important that the scale be close enough to the mercury thread to facilitate reliable readings.



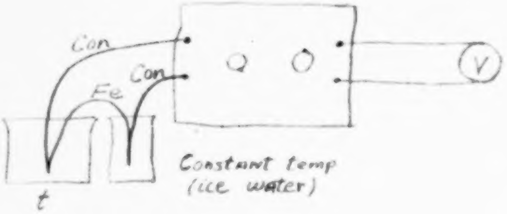
B. Uncalibrated alcohol-in-glass (e.g., Macalaster cat. no. 2666, $1.20 for four). Same principle as A. A 6-inch steel scale is long enough to serve here. Tapes do not hold up well in boiling water—rubber bands are much better, but care should be taken to bind them tightly enough to the thermometer and scale to prevent them from slipping out of alignment. A small stick or glass rod can be bound together with the thermometer and scale to help position the scale close to the liquid thread. With some maneuvering, the scale can be made to show through the glass directly behind the liquid thread.

C. Thermistor. The electrical conductivity of a semiconductor decreases rapidly with increase in temperature. The "100K" thermistor supplied by Project Physics has a resistance of about 400KΩ at 0°C, 100KΩ at 25°C and 4KΩ at 100°C. Use a volt-ohm-milliammeter to measure resistance directly, or connect a 6V dry cell and a milliammeter in series with the thermistor and use current as a measure of temperature. The maximum current will be about 1.5 mA at 100°C. (It would also be possible to use the Project Physics amplifier: connect the thermistor between the -6V terminal (white) and the (red) amplifier input. Set dc offset to zero. Connect a dc voltmeter to the output (negative terminal of meter to red output because of the polarity of the 6V supply) and set the gain control to give a reading of about 5V when the thermistor is in boiling water. This is approximately the maximum output of the amplifier, and increasing the gain will not increase the reading above this level.)

The thermistor and leads to it must be insulated to prevent conduction through the water. Slide a length of "spaghetti" or shrink tubing over it, or, if you use the thermistor embedded in lead, apply a coating of nail polish.

D. Thermocouple. The amplifier unit can also be used to detect the difference in voltage at the two junctions of a pair of wires made of different metals (Fig. 2). An iron-constantan couple develops about 5 mv for a temperature difference of 100°C. With the amplifier gain set to 100 the maximum output voltage will be about ½ volt.

One could simplify the circuit by connecting the iron wire directly to the amplifier terminal. This contact then becomes the reference junction and if it is less than room temperature, the voltage will be negative.



E. Gas Volume. Use a piece of capillary tubing closed at one end with a plug of silicone rubber cement. A mercury index traps a fixed amount of air—see part B of Experiment 29—The behavior of gases. Use as many different gases as possible.

F. Gas Pressure. "John's law" apparatus (i.e., a toilet reservoir float plus absolute pressure gauge—e.g., Welch Scientific, cat. no. 1602, $19.50) can be used here. Use as many different gases as possible.

E27 Calorimetry

This experiment introduces students to the idea that heat flow can be measured by observing the change in temperature of some standard substance, for example, liquid water. Using this method, students learn that measured amounts of heat produce different temperature changes in different substances. Also, they measure the latent heat of ice and the specific heat of a metal.

The foam plastic (styrofoam) cups are extremely useful for heat experiments and are very inexpensive. Keep a good supply on hand, and encourage students to use them to improvise experiments other than the ones specifically described here.

These cups are such good insulators that experiments can generally be done with the cup uncovered. For example, when water originally 15° C below room temperature is placed in an uncovered cup, it increases in temperature by less than 0.2° C/min. Since the marks on thermometer scales are usually a degree or more apart, the error introduced by heat leakage in the brief experiments suggested here is not much more than the uncertainty in the temperature measurement. The cups are also very light (between 2 and 3 grams) and therefore absorb very little heat. It is not necessary at this stage to require that students correct their calculations for this heat loss although the corrections could be made later as a separate experiment.

This preliminary experiment establishes the approximate rate at which heat leaks to or from the calorimeters, and also prepares students for a discussion of latent heat later in the period. This preliminary work can be started in the beginning of the period and carried along with the other experiments, or can be done the day before. It will give students some feeling for the insulating characteristics of their cups and also impress them with the fact that a water-and-ice mixture remains at exactly 0° C until all the ice is melted. Caution students that they should stir the water gently with the thermometer before taking each reading.

Background

The calorimeter was first used quantitatively to measure heat by Joseph Black (1728-1799). He made these three assumptions about the nature of heat.

Underlying the third assumption is the idea that temperature is a quantity which can be measured. The adoption of a temperature scale and basic improvements in thermometer design in the early eighteenth century by Fahrenheit and others made Black's work possible.

Around the beginning of the nineteenth century, the first assumption was shown to be inadequate to explain the relationship between work and heat. However, the caloric theory, as it was called, was extremely plausible; it served very well in developing early ideas about heat, and to this day is still implicit in many intuitive ideas used in calorimetry.

Students know from experience that when the two quantities of water are put together, the temperature of the resulting mixture will be between the two starting temperatures. Ask if they can predict the exact temperature of the mixture: this will be easy if the cups of hot and cold water have identical masses, but not so easy if their masses differ.

When unequal quantities of hot and cold water are mixed the relationship

$$m_h \Delta t_h = m_c \Delta t_c$$

holds.

The calorie is the CGS unit of heat. It is a convenient unit for the calorimeter experiments described here, but is too small for most practical applications. The MKS unit, the kilocalorie (kcal), is the heat that enters or leaves 1 kgm of water when the temperature changes by 1° C. This unit is the same as the Calorie used by dietitians.

The next reasonable question to ask is whether the factor c in the heat equation is the same for other materials as it is for water. Provide students with small samples of various metals and ask them to predict the equilibrium temperature when a hot metal sample is mixed with cold water in a calorimeter.

The thread should be tied to a wooden stick so that hands do not get too close to the steam.

No, $m_1 \Delta t_1$ is not equal to $m_2 \Delta t_2$.

aluminum	0.22 cal/gm C°
brass	0.08
copper	0.093
iron	0.11
lead	0.031
mercury	0.033

The ice-and-water mixture may still be at 0°C; even if the ice has all melted, it is probably cooler than the cup originally containing only ice water. Since heat is leaking into both cups, what is happening to the heat that enters the ice-and-water mixture? Because this added heat does not produce a change in temperature of the water while the ice is melting, this heat is called latent, or hidden, heat.

After the ice melts, the resulting ice water also absorbs heat as it warms up to its final temperature. The necessity of considering this additional amount of heat in the equation seems to be difficult for many students. Perhaps a comparison to compounding interest in a bank account, getting interest on interest, would be helpful.

If students use ordinary ice cubes, their values will generally be less than the accepted value of 80 calories per gram. This is probably due to the presence of air bubbles and impurities. Excellent results can be obtained using the plastic cups if you make ice cubes from distilled water. Start with the cup about half full of water at a few degrees above room temperature and record its mass. Add an ice cube which has been placed in a container of cold water for a few minutes so its temperature will be 0° C, rather than about -10° C as it comes from the freezer. Dry off the ice with a paper towel to avoid transferring excess water to the calorimeter.

Newton's law of cooling predicts that the rate of cooling is approximately proportional to the difference in temperature of a sample and its surroundings. This is an empirical result due to the combination of several physical processes. Students should be able to report qualitatively that the greater the temperature difference between the sample and its surroundings the quicker its fall (or rise) of temperature. But don't expect a formal statement of Newton's law.

Students who did not do part C (latent heat) will find that the ice water with ice in it warmed up less than the ice water. Use this observation to introduce latent heat, or let those students who did part C report their results to the rest of the class.

Because the conditions of each experiment (quantity of water, amount of stirring, etc.) are different, students can only get an estimate of the error due to loss of heat from the calorimeter.

In one trial, 100 g of water 25° C above room temperature lost about 2° C (ΔH = 200 calories) per minute.

Questions for Discussion

1. Students could be urged to look for the other factors which can introduce errors in calorimeter experiments, and to suggest possible remedies. Here are a few examples:

(a) The thermometer must take up some of the heat in the calorimeter. If we knew the specific heat of mercury and glass and the mass of each that is immersed in the water, we could make allowance for this factor. This would be very difficult to do because the relative amounts of glass and mercury are unknown. Perhaps it would be possible to measure the heat capacity of a thermometer directly by experiment, but in any case the error resulting from this is quite small.

(b) Heat is lost while samples are being transferred: the hot water and metal samples will lose some heat while being transferred; some ice will melt after it has been dried but before it is put in the calorimeter. These will be difficult to estimate, but they make relatively small contributions to the total error.

(c) The major source of uncertainty is probably the thermometer reading. Note that the larger the change in temperature during the experiment, the smaller will be the fractional error in Δt due to uncertainty in reading the thermometer.

Another source of uncertainty in the thermometer readings is the way in which the thermometers are calibrated. Some are calibrated to read correctly when the entire thermometer is immersed, while others should have only the bulb in the liquid. In the latter case, it is possible to correct for the length of the exposed thread of mercury. This correction will, however, be very small.

2. After students have measured the latent heat of melting ice, they should consider ways to measure the latent heat of steam. Section 11.3 of the text emphasizes the importance of Watt's improvement of the steam engine, which was based on his realization that the condensation of steam releases a large amount of heat. In fact the latent heat of vaporization of water is about seven times the latent heat of fusion of ice. The latent heat is measured by bubbling steam through cold water in a calorimeter cup; the procedure is the same as for ice. However, you may not wish to expose students to the possibility of burns from the live steam.

E28 Monte Carlo Experiment on Molecular Collisions

Both the formula for mean free path and that for viscosity are approximate. Rigorous derivation may be found in intermediate texts on kinetic theory.

You cannot give a precise definition of randomness nor a clear-cut exposition of this technique.

You might get a random set of numbers by opening a metropolitan telephone directory, taking the last two digits from the call numbers as they occur. Another way is to take a linear expression, say 11x + 7. Plug in a number for x, say 54. Compute the value of the expression, namely 11 × 54 + 7 = 601. Take the last two digits of this value for the next number of the random series and proceed as before. Thus 11 × 01 + 7 = 18, 11 × 18 + 7 = 205 etc. The series is 54, 1, 18, 5, 62, 89, 86, 53 ... etc. The table of random numbers given is from Statistical Tables by R. A. Fisher and Yates.

The first digit of the number taken from the table is the abscissa.

Use the last digit of the number on which the wheel stops and spin it twice, once for each coordinate.

In part A, both target molecules and bombarding molecule have the same finite diameter; but in Part B only the target molecules have a finite diameter. The bombarding molecule is a test particle with no diameter.

EXPERIMENT 28 (Part B)—Sample Results

Target marbles were set up, using random numbers, on a 6 by 6 grid (2-inch spacing). Bombarding marbles were released through the "pin-ball machine" array of nails illustrated in the student notes. Three experiments of fifty trials each were made with N (number of target marbles) = 8.

The observed ratios of hits: total trials were:

$$\frac{37}{50},\ \frac{31}{50},\ \frac{35}{50}.$$

Mean is $\frac{34 \pm 3}{50}$.

Applying $d = \frac{HD}{2NT} = \frac{H}{T}\frac{D}{2N} = \frac{(34 \pm 3)}{50} \times \frac{13''}{16}$

$= 0.55 \pm 0.05$ inches

Measured length of a line of 8 marbles is

$$4\ \frac{7}{16} \text{ inches.}$$

$\therefore\ d = \frac{4.44}{8} = 0.55$ inches!

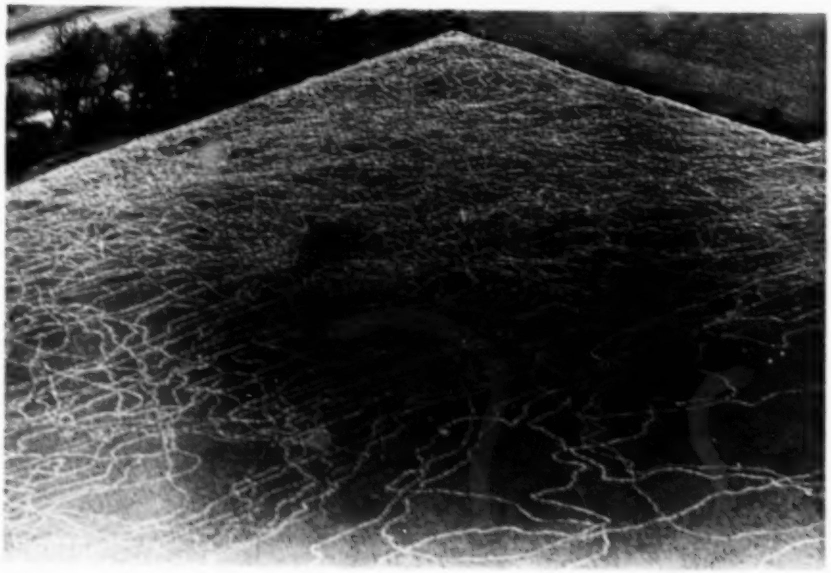
Shielding effect becomes important as N is increased.

For $N = 12$, $\frac{H}{T} = \frac{88}{150}$

giving $d = \frac{88}{150} \times \frac{13''}{24} = 0.32''$

and for $N = 20$, $\frac{H}{T}$ was $\frac{43}{50}$

giving $d = \frac{43}{50} \times \frac{13''}{40} = 0.28''$.



Wandering horse in a snow-covered field

E29 Behavior of Gases

For a complete account of Boyle's work, see "Harvard Case Histories in Experimental Science," edited by J. B. Conant. For an edited, annotated version see "Great Experiments in Physics," by Morris Shamos.

Detailed instructions for use of the conventional Boyle's law apparatus are found in most physics and chemistry laboratory manuals.

The simple plastic syringes are preferable because they are inexpensive and easy to use; students can obtain data from them with a minimum of preliminary discussion. No mercury is used; nor need one explain the relationship between height of mercury column and pressure. Here pressure is simply force (weight on piston) divided by the area of the piston. You will probably need to remind students that weight is measured in newtons and is equal to mass times $\frac{a}{g}$. The numbers written on "weights" (100g, 500g, etc.) are masses.

The data obtained with this equipment is not very precise, but since the primary purpose of the experiment is to show how the data are analyzed and interpreted, the syringes are probably adequate.

Grease the piston, with glycerine, vaseline, etc. to reduce friction. Make sure that no water gets into the cylinder.

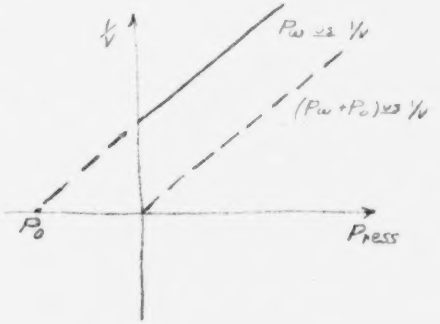
Another way to improve the data is to record two readings for each force applied to the piston, one when the plunger is lifted slightly and released, and the other when the plunger is depressed slightly and released. The mean of the two readings is the value used.

To get a p_w vs V plot that is convincingly not a straight line, one must work over a rather wide pressure range—weights of up to a few kilograms. But to get a good value for the intercept of the p_w vs $\frac{1}{V}$ plot (see below), data at relatively low p_w values are needed—below 1 kg added weight. So try to have (at least some) students work at both high and low pressures.

A common difficulty students encounter in working with gases is the tendency to confuse absolute pressure with gauge pressure. The procedure described is designed to show that Boyle's law as it is usually written,

$$PV = \text{constant}$$

is only true if P is the absolute pressure. Students vary the force exerted on an air sample by placing weights on the piston. Then they are to convert these forces to pressure by measuring the diameter of the piston and computing its area. This step would not be necessary if the purpose of the experiment were simply to demonstrate a linear relationship between $\frac{1}{V}$ and P. However, when students use values for P obtained from their data, not taking into account atmospheric pressure, they find that their graph of $\frac{1}{V}$ against P does not pass through the origin. Instead it should pass through the P-axis at about -10 nt/cm. Be sure students understand the significance of this intercept value.



Point out that if a constant term (P_o = 10 nt/cm) is added to each value of the calculated pressure, P_w, the graph would pass through the origin. This means that

$$(P_w + P_o) = k \frac{1}{V}$$

or

$$(P_w + P_o) V = \text{constant}$$

Explain that P_o is the additional, constant, pressure exerted on the piston by the atmosphere, and that the total pressure of the gas in the syringe is $(P_w + P_o)$. This is called the absolute pressure. The quantity P_w is usually called gauge pressure because it is the pressure which is most commonly measured by a gauge.

The similar behavior of different gases is an important point because it suggests that the same model (kinetic theory) could be used to explain the behavior of all gases.

Have different students work with different gases if at all possible.

In place of this experiment you can substitute a quick and effective demonstration: A small flask or test-tube is fitted with a one-hole stopper. A long capillary tube with a mercury pellet near one end is pushed through the stopper. The flask must be perfectly dry and the mercury pellet near the lower end of the tube at room temperature. The expansion of air is made vividly apparent as the flask is heated in a water bath.

Of course if you want to make quantitative measurements of change in volume with temperature you must relate the volume of the flask to the volume (per unit length) of the capillary tube.

The plastic syringes used in part A are supported by a ring stand and immersed in a beaker of cold water. Students record temperatures and volumes as they slowly heat the water. Friction between the piston and syringe is again the major source of uncertainty in measuring the volume. The change in temperature must be very gradual, and the water continuously stirred so that the air in the syringe will be at about the same temperature as the water. This apparatus will do little more than show that air expands as it is heated. Because of the large amount of friction in the syringe, no pretense should be made that students can determine the value of absolute zero by extrapolating their V-T curve to find the intercept on the T-axis.

Detailed directions are given in an accompanying equipment note for preparing the constant-pressure gas thermometers referred to in the student's notes.

Even if a graph of V against T is a straight line, this shows only that air expands with increasing temperature in somewhat the same way mercury in the glass thermometer does. If the optional experiment on Hotness, Thermometers, and Temperature (E26) is not done, there should be a brief discussion of temperature scales as outlined in the Teachers' Notes for that experiment.

When weights are added to the piston and the pressure increased, the temperature goes up because work has been done on the gas. However, since the sample in our case is small and is not in an insulated container, it quickly returns to room temperature.

Equipment Note: Assembling a Constant-Pressure Gas Thermometer

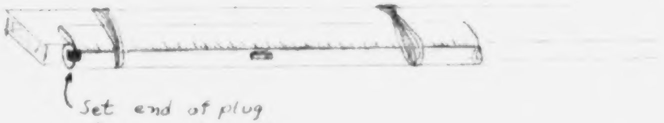
About 6" of capillary tube makes a thermometer of convenient size. The dimensions of the tube are not critical, but it is very important that the bore be dry. It can be dried by heating, or by rinsing with alcohol and waving frantically—or better still, by connecting it to a vacuum pump for a few moments.

Filling with air. The dry capillary tube is dipped into a container of mercury, and the end sealed with the fingertip as the tube is withdrawn, so that a pellet of mercury remains in the lower end of the tube.

The tube is held at an angle and the end tapped gently on a hard surface until the mercury pellet slides to about the center of the tube.

One end of the tube is sealed with a dab of silicone sealant; some of the sealant will go up the bore, but this is perfectly all right. The sealant is easily set by immersing it in boiling water for a few moments.

A scale now must be positioned along the completed tube. The scale will be directly over the bore if a stick is placed as a spacer next to the tube and bound together with rubber bands. (A long stick makes a convenient handle.) The zero of the scale should be aligned carefully with the end of the gas column—i.e., the end of the silicone seal.



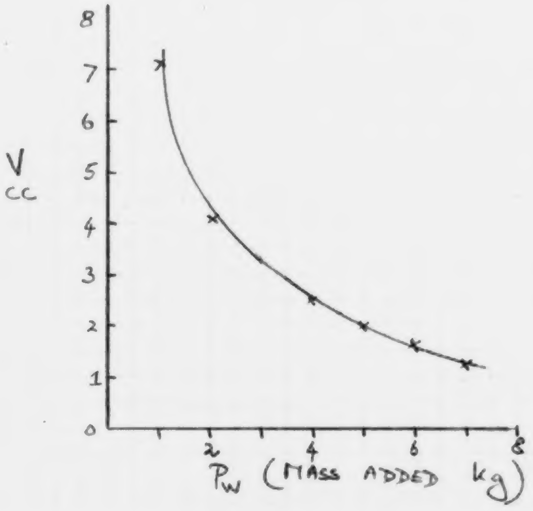
Set end of plug
at zero of scale

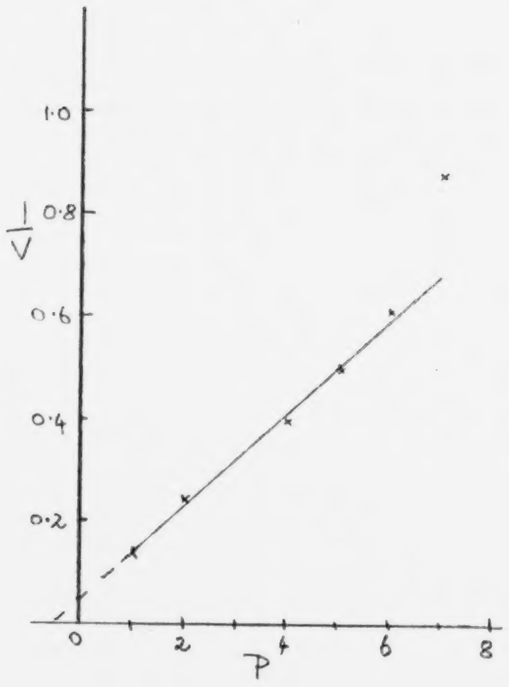
In use, the thermometer should be completely immersed in whatever one wishes to measure the temperature of, and the end tapped against the side of the container gently to allow the mercury to slide to its final resting place.

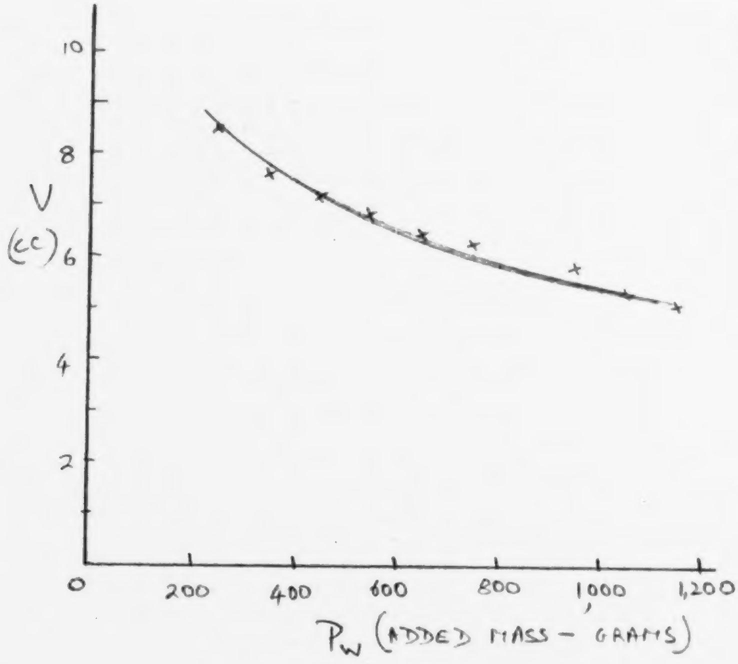
To fill the thermometer with some gas other than air: Connect a capillary tube to the gas supply by a short length of rubber tubing. Open the gas valve slightly to flush out the tube, and fill it with gas. Detach the rubber tube. Pick up a pellet of mercury as before. Keep your finger over the far end of the tube while you replace the rubber tube (gas valve shut). Lay the thermometer down flat. Work the mercury pellet to the center of the tube and, opening the gas valve slightly and very cautiously, release your finger for an instant.

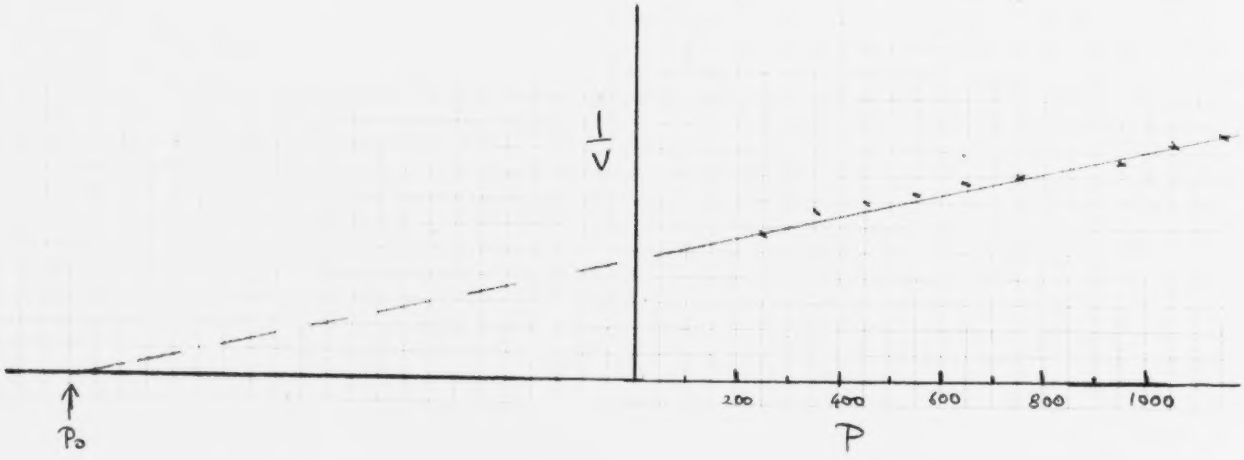
Remove from the gas supply, seal off as before (the end that was connected to gas supply) and attach scale.

Sample Results




$\frac{1}{V}$
1·0
0·8
0·6
0·4
0·2
0
2
4
6
8
P


V (cc)
10
8
6
4
2
0
200
400
600
800
1,000
1,200
P_W (ADDED MASS – GRAMS)


$\frac{1}{V}$
200
400
600
800
1000
P
P_0

E30 Introduction to Waves

Note that there is enough here for two lab periods, and it would be difficult to complete all of the material in one lab period.

Students should experiment with longitudinal pulses long enough to appreciate the difference between these and the transverse pulses with which the rest of the experiment deals.

Q1 The amplitude changes because of friction: the energy of the pulse is a function of its amplitude and as the energy is dissipated by friction, the amplitude of the pulse decreases.

Q2 That depends on whether the end is free or fixed. Presumably the other fellow is holding it down, so that the reflected pulse will be upside down.

Q3 The speed of a wave along a slinky, rope, or what have you, is proportional to $\sqrt{[\text{Tension }(T)/\text{mass per unit length }(\mu)]}$. Therefore increasing the tension should increase the speed along the slinky.

Q4 The conclusions ought to be consistent with $v \propto \sqrt{T/\mu}$, but of course one would not expect quantitative results here.

Q5 & 6 The wavelength will, of course, change consistent with $\lambda = v/f$. The polarity of the reflected pulse can easily be remembered by considering the two limiting cases: at a fixed end (incident pulse on lighter spring) there is a change; at a free end (incident pulse on heavier spring) there is no change.

Some ambitious students might want to determine the amplitude of the reflected pulse as well as its polarity. The mathematical expression for it is easily derived. If the incident pulse is produced by wiggling one's hand with a displacement equal to $f(t)$, the shape of the pulse will be given by $f(t - x/v_1)$; the reflected pulse will be $r \cdot f(t + x/v_1)$, where r is the reflection coefficient; and the transmitted pulse will be given by $c \cdot f(t - x/v_2)$. At $x = 0$ (the position of the junction) the string must be continuous, so [incident pulse at $x = 0$] + [reflected pulse at $x = 0$] = [transmitted pulse at $x = 0$], and the same will be true of the slopes: [slope of incident pulse at $x = 0$] + [slope of reflected pulse at $x = 0$] = [slope of transmitted pulse at $x = 0$]. These equations are

$$f(t) + rf(t) = cf(t)$$

$$-\frac{1}{v_1}f(t) + \frac{r}{v_1}f(t) = -\frac{c}{v_2} \cdot f(t)$$

Solving for r is now easy:

$$r = \frac{v_1 - v_2}{v_1 + v_2}$$

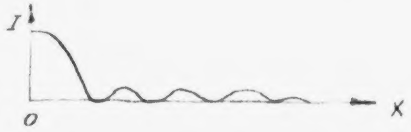
Q8 If the reflected wavefronts do not form a straight line, the mirror is not parabolic or the source was not at the focus. (See text p. 13f.)

Q9 The speed of the waves decreases in a shallow area, so that they are bent toward the normal to the boundary when they enter a shallow area; on leaving it they are bent away from the normal again.

Q10 The nodal lines move closer together as the frequency is increased and the wavelength decreased. (The position of the nodes is a function only of λ and not of f; one can get the same effects by either increasing f or decreasing the speed while keeping f constant.)

Q11 & 12 The angular width of the diffraction pattern (assuming perfectly plane incident waves) is just a function of λ/d (d is the size of the opening). For $\lambda = d$ the first minima are at ±90° from the direction of propagation, while for $\lambda << d$ the diffraction pattern is concentrated in the forward direction.

It might be interesting for some of your students to note that the intensity of the diffraction pattern has a graph of roughly the following shape:



(It is symmetrical about $x = 0$.)

There is, of course, another way to measure λ, and that is to measure the distance between the minima or maxima of an interference or diffraction pattern—but that comes later, in Experiment 31. (Unless, of course, you want to do it now.)

Q13 The node-to-node distance is $\lambda/2$.

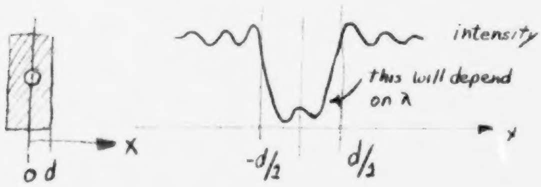
Q14 & 15 The quantitative agreement that students are likely to find is not too spectacular. Difficulties with timing the transit times of the ripples, and the rotation speeds of the strobes will make it so. Hand-driven strobes are probably the easiest to use and time (by just counting).

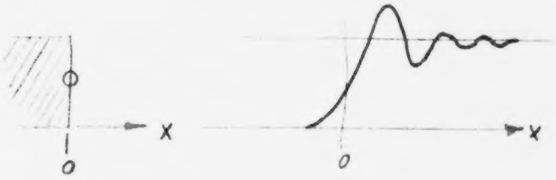
E31 Sound

Again, there is a lot of material here; one could easily spend several lab periods on these experiments. If your lab time is limited, however, we suggest that you cut down on the time spent on parts A and B (you might leave B for a classroom discussion) so that students can do a good job on part C which contains important experiments on interference. If you have difficulty with leaving the equipment out for several days, you might eliminate track III (the ripple tank), or include it in Experiment 30, when the ripple tanks are still set up.

Q1 The sound is absorbed. Heavy cloth does very well (velvet or terrycloth even better).

Q3&4 The best patterns are probably obtained when the board is perpendicular to the line straight out from the speaker. The narrow board (two-edge diffraction) should be symmetrically placed, while the wide board (single-edge diffraction) should have its edge on that center line: the diffraction patterns are just those familiar from optics (a wave is a wave is a wave, after all). They should roughly have the following forms:





Q5 The speed of sound is about 330 m/sec in dry air. The measurements of λ should be consistent with this. If students get about 150 m/sec, they have forgotten that the distance between nodes is λ/2 and not λ.

Q8&9 See comments for Q3 and 4 above. With the ultrasound apparatus students can get quantitative results. You might have them plot up the single-edge diffraction pattern and compare with the predicted result (see any book on optics, such as Jenkins & White).

Q10 The velocity of these waves is 333 m/sec in dry air. Again, the students' results ought to be consistent with this value. Note that the frequency of the oscillator is probably ± 10 per cent.

(Track III)

Note that these experiments parallel those in part C; the experiments equivalent to those of tracks I and II have been done with a ripple tank in Experiment 30—much more qualitatively, of course.

Figure 12: missing arc points A, B and C. A and C are on an antinodal line, and B is on a nodal line. A and C correspond to A and C in Fig. 13.

Figure 13: note that the distances S_1A, S_2A and S_2C are all L and S_1C is $\lambda + L$.

Q11 It should not be surprising if the results do not agree very well; the interference method is probably far more precise.

Q12&13: The positions of the maxima are separated by a distance

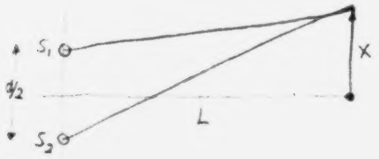
$$x = L \cdot \frac{\lambda}{d} = L \cdot \frac{v}{d \cdot f},$$

where v is the speed of sound. Thus changing d and changing f have completely equivalent effects.

Q15 $v \cong 330$ m/sec in dry air; it does not change by very much when it is humid (no more than 5 per cent).

Error: Q5 should read Q16.

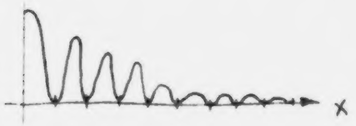
Q16&17 See discussion for Q12 and 13 above. Some students might want to make a plot of the intensity as a function of distance along the meter stick. If we let the symbols have the meanings indicated in the figure:



then it is easy to show that if the sources are in phase, the intensity is proportional to

$$\cos^2\left(\pi \frac{d}{\lambda} \cdot \frac{X}{L}\right).$$

Of course, the maxima will drop off in intensity after awhile, since the speakers or transducers do not emit in all directions uniformly, and the resulting pattern will look more like this:



Q18 Again, $v \cong 333$ m/sec.

Power Supply

The DAMON "Mark VII" Power Supply serves as a power source for all of the Harvard Project Physics experiments. In addition, this unit also contains the transistor switch used for demonstrations in Units 3 and 5.

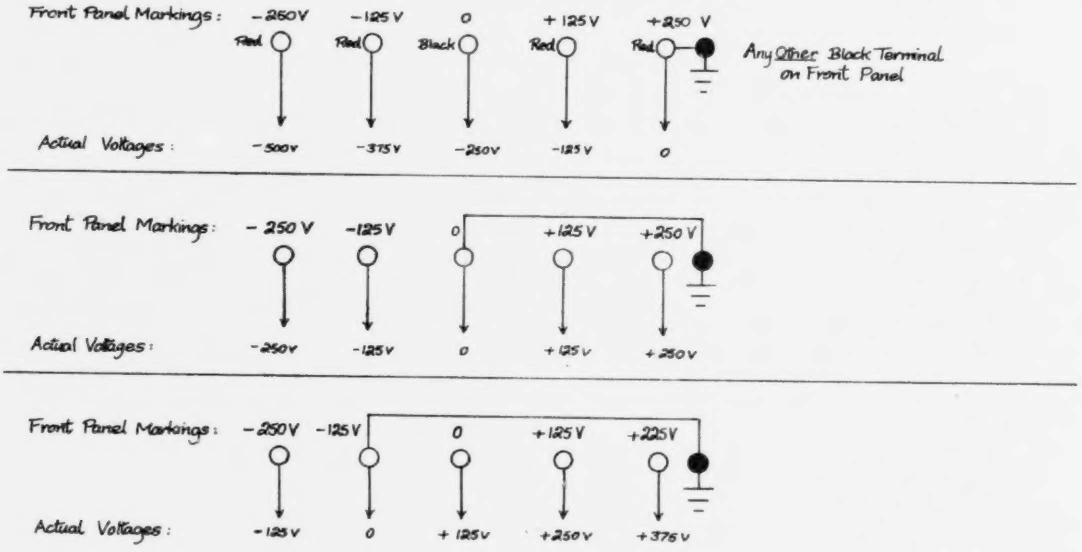
The characteristics of the various power sources available from this unit are:

1. Regulated +8 volts and -8 volts at 250 milliamperes.
2. Unregulated 6 volts at 5 amperes.
3. Unregulated 6 volts at 5 amperes, pulsed by a transistor switch that is triggered by an external signal.
4. Adjustable unregulated 0 to 6 volts at 5 amperes.
5. -250, -150, 0, +150, +250 volts unregulated at 100 milliamperes.

The regulated +8 volt and -8 volt power sources are used to run the amplifier and oscillator units. Depending on the loading conditions of the various amplifiers and oscillators, up to a total of six units may be connected to the +8 volt and -8 volt outputs. These supplies may also be used for other applications requiring a regulated voltage source of +8 volt, -8 volt, or a floating supply of 16 volts.

The unregulated 6-volt output can furnish approximately 5 amperes of current. A 1-ohm resistor is internally connected in series with this output to limit the maximum current.

The unregulated high voltage power supply (+250, +125, 0, -125, -250 volts dc) can furnish 100 milliamperes of current. These output voltages can be injurious. The power supply should be off when making connections and care should be exercised not to touch exposed wires or terminals with the supply on. The high voltage supply is floating with respect to the other power sources in the DAMON "Mark VII" Power Supply. This feature permits the selection of a wide range of discrete step voltages. For instance, if the +250-volt terminal is connected to one of the black front panel ground (⏚) terminals, a voltage of -500 volts dc @ 100 mA is available at the -250-volt output. The other voltages for this particular connection would be as shown in Fig. 1. The unregulated 0 to 6-volt dc output and the 6v transistor switch pulsed output shave the same output terminals. Selection between the two modes of operation is accomplished by the slide switch in the lower right hand corner of the power supply. With the switch down, a 0 to 6-volt dc voltage is controlled by the front panel knob and is available at the terminals to the left (labeled 0→6 V 5 Amp). With the slide switch in the up position, the transistor switch function is enabled and the 0 to 6-volt adjustment

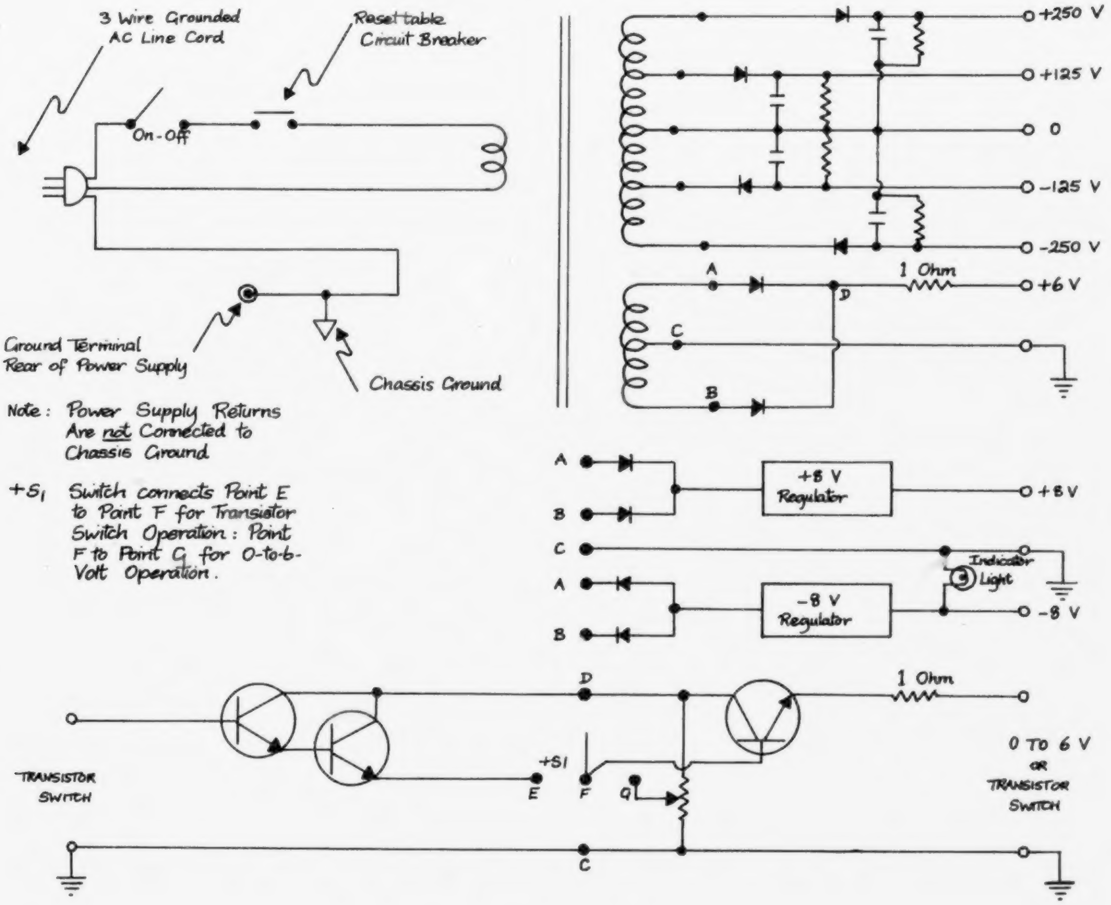

Front Panel Markings:
-250V
-125V
0
+125V
+250 V
Red
Red
Black
Red
Red
Any Other Black Terminal on Front Panel
Actual Voltages:
-500v
-375v
-250v
-125v
0
Front Panel Markings:
-250 V
-125V
0
+125V
+250 V
Actual Voltages:
-250v
-125v
0
+125v
+250v
Front Panel Markings:
-250V
-125V
0
+125V
+225V
Actual Voltages:
-125v
0
+125v
+250v
+375v

function is disabled. For example, a variable frequency source (the square wave output of the DAMON Oscillator) can be connected to the transistor switch input to provide a 6-volt square wave at the 0→6 V output terminals. This output is capable of driving a short circuit load at high current levels. The transistor switch is required for several demonstrations described in the Teacher's Guide for Units 3 and 5. These include:

1. Producing transverse standing waves in a wire under tension (D45).
2. Producing longitudinal standing waves in a spring (D45).
3. Producing standing waves in a circular ring (analogy to bounded de Broglie waves) (Unit 5).

Figure 2 displays the experimental setup for producing transverse standing waves in a wire.

DAMON MARK VII POWER SUPPLY FUNCTIONAL DIAGRAM


3 Wire Grounded AC Line Cord
Resettable Circuit Breaker
On-Off
Ground Terminal Rear of Power Supply
Chassis Ground
+250 V
+125 V
0
-125 V
-250 V
A
D
B
C
1 Ohm
+6 V
A
B
C
A
B
+8 V Regulator
+8 V
Indicator Light
-8 V Regulator
-8 V
D
1 Ohm
+S1
E
F
G
C
TRANSISTOR SWITCH
0 TO 6 V OR TRANSISTOR SWITCH

Note: Power Supply Returns Are not Connected to Chassis Ground

+S1 Switch connects Point E to Point F for Transistor Switch Operation: Point F to Point G for 0-to-6-Volt Operation.

Amplifier

Supply Voltage Requirements:

+8 V (red) at 60 mA
-8 V (green) at 60 mA } with a 50-ohm load
ground (black)

Controls:

Voltage Gain: Adjustable, 0 to 100.
Current Gain: Adjustable, up to 40,000.
Output Offset: Adjustable from +2 V to -7 V, with a calibration point at 0 volts.
AC/DC Switch

Characteristics:

Input Impedance: 20,000 ohms in parallel with 100 pf
Bandwidth: DC position - dc to 600 KHz
AC position - 7 Hz to 600 KHz

Output Impedance: 30 ohms

Output Power: 320 mW dc or 160 mW ac

Output Voltage: Up to +6 V to -6 V into high impedance load.

Output Current: Up to 150 mA or -150 mA into a short circuit.

Notes:

1. Output Offset Control: The signal appearing at the output can be offset over a -7 to +2 volt range by means of the OUTPUT OFFSET control. This enables one to "buck out" a constant dc input. One can then use a sensitive meter to show small changes in the input voltage signal above or below the preset level. (This is explained in detail in the notes for Demonstration 33: An Inelastic Collision, in which a thermistor probe is used with the amplifier to detect the small temperature rise in a strip of lead which absorbs the kinetic energy of a pair of colliding dynamics carts.)

2. Note the maximum output values given above. These mean that for fairly large input signals the maximum possible gain will be less than 100. Thus if the input is 0.1 volt the output will go on increasing as the gain control is turned up to 60X (6 volts output), but further increase in gain setting will not affect the output.

3. With *no* input signal applied to the amplifier, the dc appearing at the output should be a function only of the output offset adjustment and not of the gain control. If the variation of output dc with gain setting exceeds about 0.5 V, the gain balance may require adjusting. This adjustment is made by inserting a screw driver into the hole at the back of the unit and proceeding as follows:

   a. With no input signal, turn the gain control to zero (completely counterclockwise).

   b. Adjust the output offset for zero volts at the output terminals.

   c. Turn the gain control to maximum.

   d. Adjust the gain balance to restore zero volts at the output terminals. The gain is now balanced.

Typical uses of the amplifier:

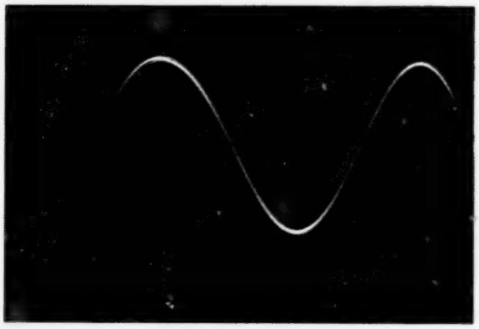
With an input from an iron-constantan thermocouple and a maximum gain setting, a temperature difference of 45°C between hot and cold junctions (or between hot junction and input terminal of amplifier) gives an output of 1/4 volt.

The amplifier can be used to amplify the signal picked up by an ultrasound microphone. The output (ac) can then be fed through a simple diode (to produce half-wave rectified current) and read on a meter. (See Teacher's Notes and Fig. 3 below.) For amplifying ac signals with a frequency below 20 cycles/sec., the AC/DC switch should be on dc.

Oscillator

Supply Voltage Requirements:

+8 V (red) at 50 mA } 50-ohm load
- 8 V (green) at 40 mA } 50-ohm load
ground (black)



Characteristics:

Waveform - Sine and square waves available simultaneously from separate outputs.
Frequency - 5 Hz to 50 KHz (4 decade ranges)
Amplitude
  Sine: Adjustable, 0 to ±10 V peak-to-peak
  Square: Fixed, 0 to +8 V peak-to-peak
Output Impedance -
  Sine: 150 ohms or less
  Square: 1 1,000 ohms

Notes:

1. The oscillator is capable of driving the 50-ohm loudspeaker directly without the need for a matching transformer or an additional amplifier. When driving a low impedance load such as the loudspeaker from the sine-wave output, the output level control should be set low enough to avoid distortion of the waveform.

2. The square wave output has an impedance of 1 1,000 ohms and is not intended for driving low impedance loads such as a loudspeaker. It is intended primarily for controlling the transistor switch contained in the DAMON "Mark VII" Power Supply unit.

Turntable Oscillator

Assembly and Operation Procedure: Turntable Oscillators

1. Remove existing turntable from base (simply by lifting off) and replace with modified turntable.

2. Mount both spool brackets and drive assembly to oscillating platform, as shown in Illustration Sheet #1, with hardware provided.

3. Place platform on turntable and drop nylon bushing into position over adjustment spindle.

4. Place rubber bands over spool brackets.

5. Insert aluminum dowel into the roll of recording tape and position between spool brackets.

6. Adjust the height of both rubber bands until the roll of tape is slightly suspended; this will eliminate unnecessary surface drag during operation. As the size of the roll decreases, height adjustments should be made accordingly.

7. Feed the end of the tape under the wheel of the drive assembly and start the motor. By means of the adjustment wing screw, bring the roller in contact with the tape. Apply only enough pressure to pull tape steadily. Too much pressure will only cause the unit to bind. (It is recommended that when not in use, tension should be relieved.)

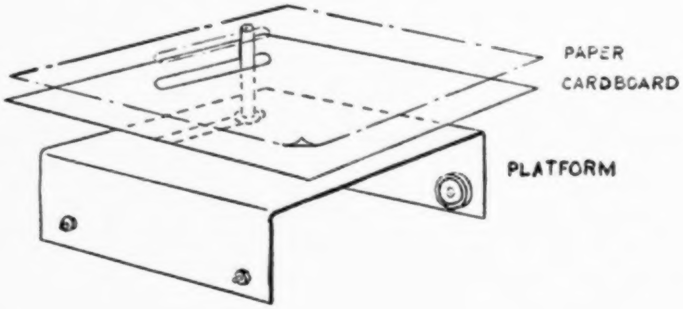
8. To change amplitude of oscillating frequency adjust spindle location by turning counterclockwise and sliding to desired position. Pull spindle lightly upwards and retighten.

9. To mount accessories, see Illustration Sheet #2.

Utilizing Platform with Ellipse Arm

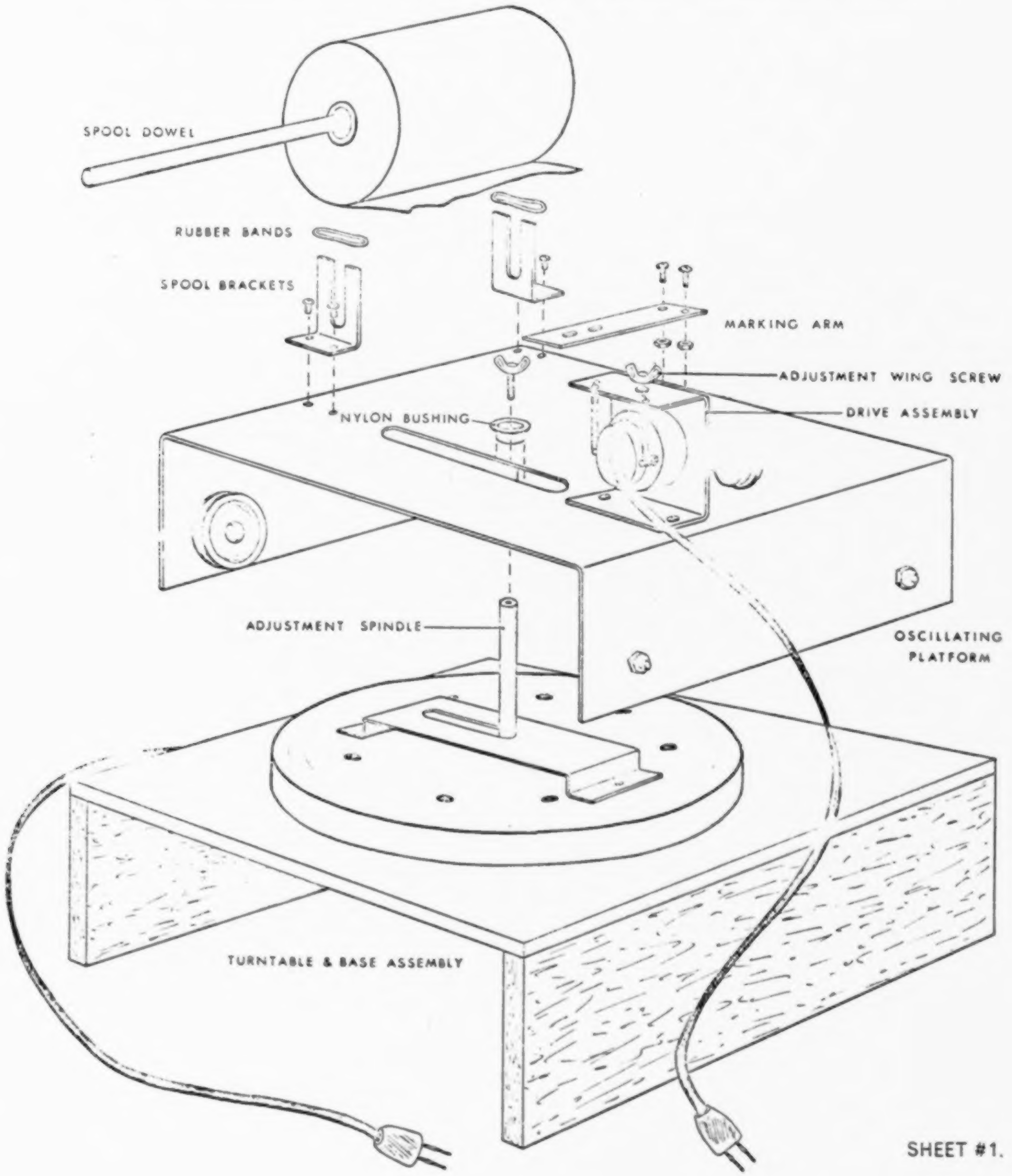
1. Using platform only, place a piece of cardboard with a clearance slot as shown below over adjustment spindle. Cardboard should be large enough to extend beyond the edge of the platform. (This will act as a pad for a recording sheet. Both pad and paper should be taped to platform.)

2. Attach ellipse arm, as shown in Illustration Sheet #2, and slide to desired position. Insert pen into holder, you will find that the pen fits loosely; to correct this, wind masking tape around the pen until it becomes stable in the pen holder. A final positioning of the pen height may be obtained by adjusting the threaded pen holder.



HARVARD PROJECT PHYSICS

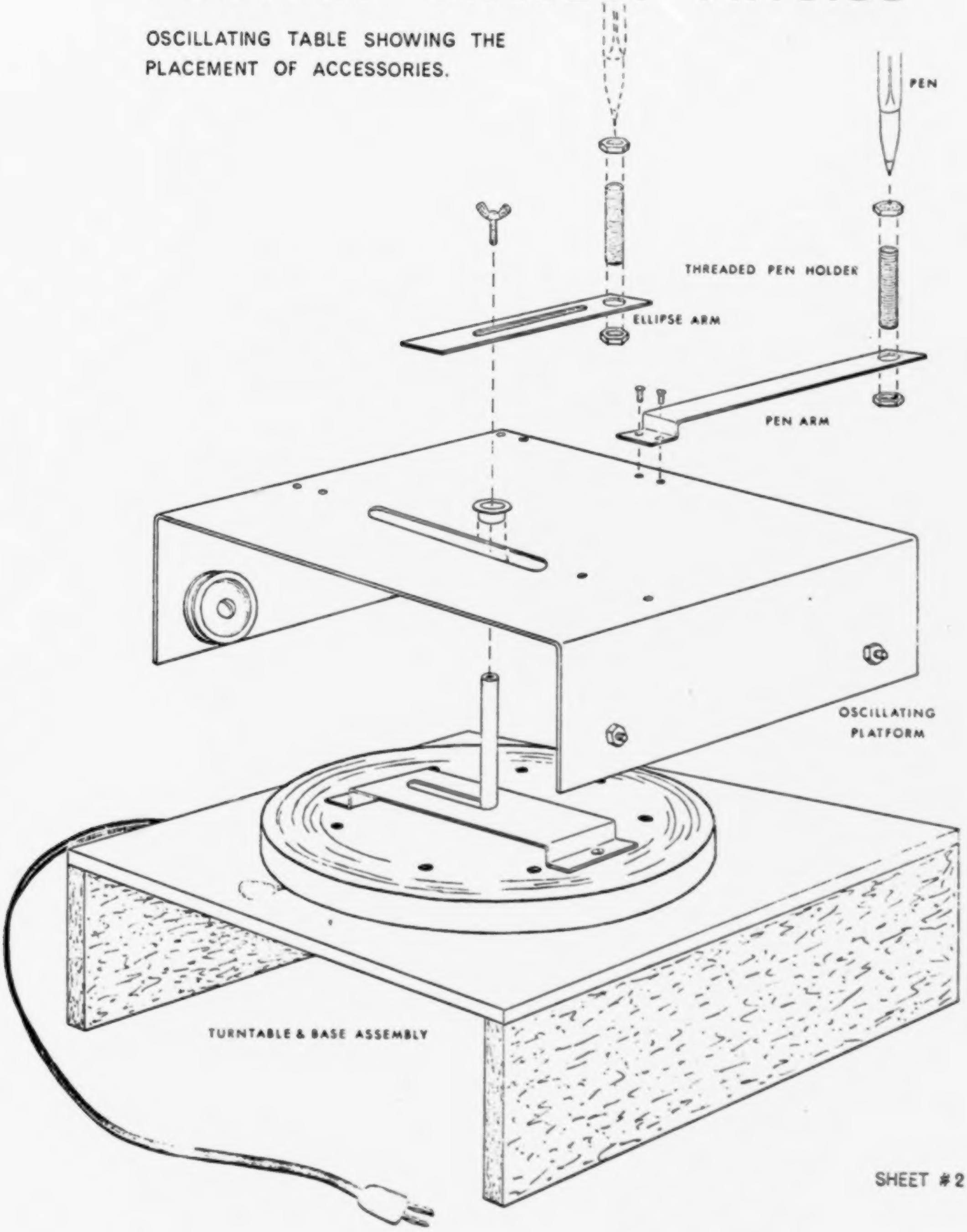
OSCILLATING TABLE WITH CHART RECORDER ATTACHMENTS


SPOOL DOWEL
RUBBER BANDS
SPOOL BRACKETS
MARKING ARM
ADJUSTMENT WING SCREW
NYLON BUSHING
DRIVE ASSEMBLY
ADJUSTMENT SPINDLE
OSCILLATING PLATFORM
TURNTABLE & BASE ASSEMBLY

SHEET #1.

HARVARD PROJECT PHYSICS

OSCILLATING TABLE SHOWING THE PLACEMENT OF ACCESSORIES.


PEN
THREADED PEN HOLDER
ELLIPSE ARM
PEN ARM
OSCILLATING PLATFORM
TURNTABLE & BASE ASSEMBLY

SHEET #2

A vertical rod is attached to a rotating phonograph turntable. This rod extends up through a long slot cut into a rectangular platform. The platform is constrained to move in a direction parallel to the slot and in a horizontal plane. As the turntable rotates, the platform moves with the SHM. This combination of turntable and platform is referred to as a turntable oscillator (Fig. 1).

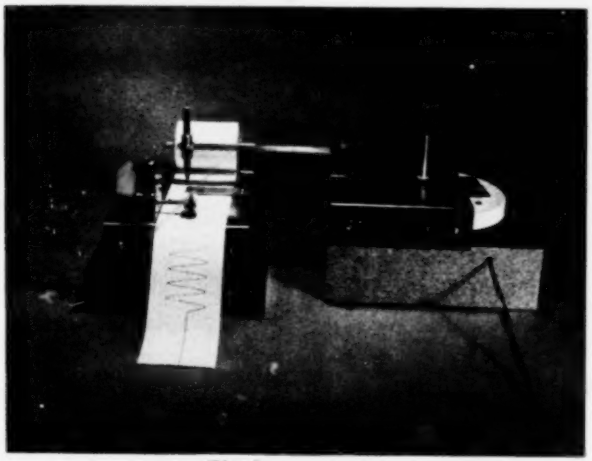


Fig.1

Sine Curves

The turntable oscillator is shown in operation with a pen attached to the platform in Fig. 1. A sine curve, drawn by the pen, can be seen on the strip chart recorder on the left. Fig. 2 is a reproduction of the trace. The SHM is displayed as a function of time.

Fig. 2

In order to emphasize oscillations and periodic functions in a simple way, students should be encouraged to produce a few hand-drawn traces on moving strip charts. The strip chart recorder has a plate with two slots mounted above the moving paper (see Fig. 3). The student, using a sharp pencil or felt-tip pen, should make periodic movements back and forth.

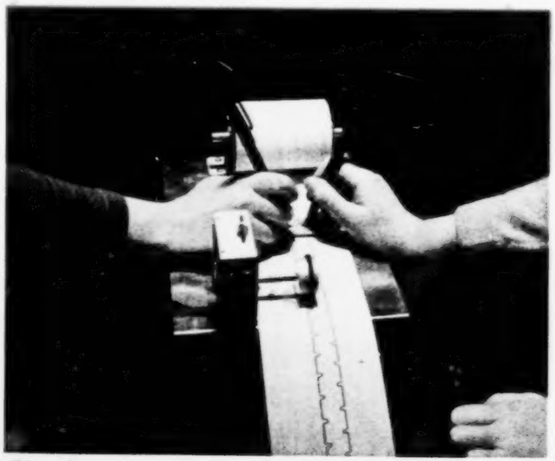


Fig. 3

Fig. 4 is a sketch of six such hand-drawn traces. Students were here asked to produce traces of (a) a sine curve, (b) a square wave, (c) a sawtooth, (d) an exponential relaxation curve, (e) rectified half waves and (f) rectified full waves. It can be seen that quite respectable-looking traces can be produced in this way, and it seems likely that efforts to produce the traces might be accompanied by improved understandings of the kinematic and dynamic requirements for such motion in physical systems. Ask students to give actual examples of each motion they are graphing.

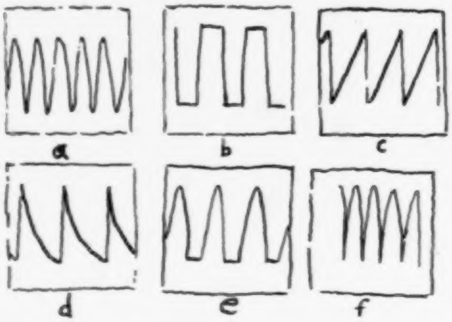


Fig. 4

Harmonic Synthesis

The superposition of two waves can be demonstrated qualitatively and quantitatively with two turntable oscillators arranged so that the reciprocating motions are parallel to each other (see Fig. 5).

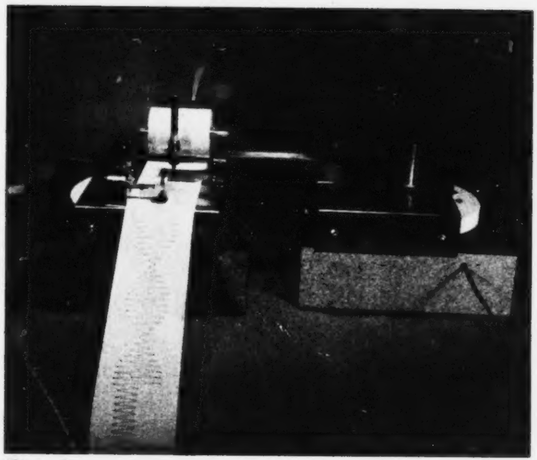


Fig. 5

A pen mounted on one oscillator leaves a trace on a strip chart recorder mounted on the second oscillator. The paper of the recorder moves perpendicularly to the direction of oscillation. If the frequency of one oscillator is a multiple of the other, the resulting trace illustrates the elements of harmonic synthesis, that is, the production of complex periodic functions by the addition of harmonics to the fundamental frequency. The traces produced in this way are shown in Fig. 6 through Fig. 9.

The amplitude of the oscillation is increased by moving the vertical peg toward the rim of the turntable. Adjust amplitude so that either trace alone gives only half paper width.

The frequency of oscillation depends upon the rotational speed of the turntables. The turntable speed selector provides the "coarse tuning" at 16, 33, 45 and 78 revolutions per minute. Fine tuning of one turntable can be accomplished by using a variac or powerstat voltage control. Always reduce the speed by lowering the voltage. Voltages in excess of 120 v may damage the phonograph motor. (CAUTION: Do not use the transistorized speed controls now available for drills, and other ac-dc motors, as they may be damaged when used with phono motor.) The speed can also be adjusted (slowed) by mechanically loading the motor, i.e., by adding weights to the platform. Weights should be placed in pairs, symmetrically on the two sides of the platform. The phase relationship can be altered by adjusting the positions of the two recorders before switching on, and then turning them both on simultaneously.

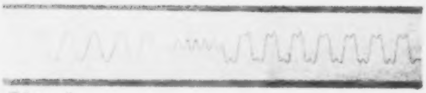


Fig. 6

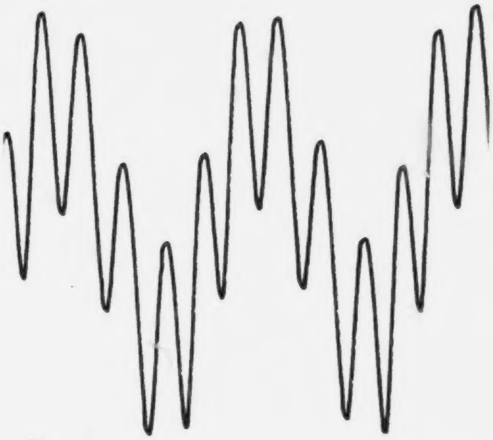


Fig. 7

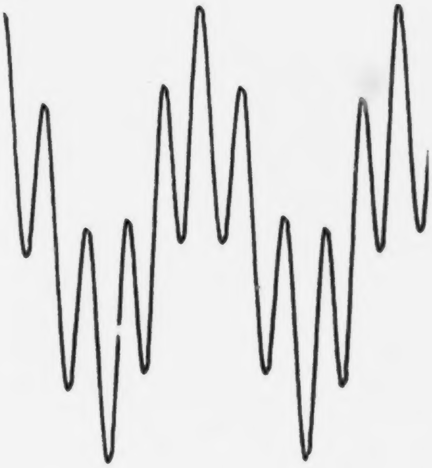


Fig. 8

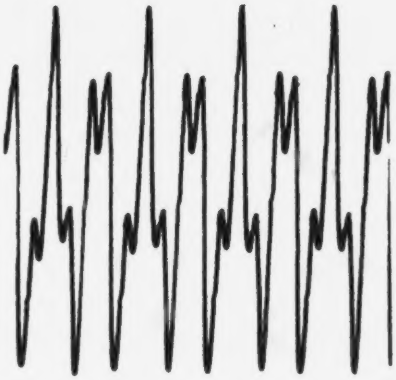


Fig. 9

ACTIVITIES

Some student activities associated with the coupled oscillators follow:

1. Produce superposed traces of two sine curves of different frequency ratios, amplitudes and phases.

2. Attempt to analyze a harmonic synthesis in order to identify the components.

3. Compare the sum of two sine curves with the original curves: the ordinates of the original curves should be first added arithmetically, point for point, and the resulting "theoretical" curve can then be compared with the one obtained by actual mechanical addition of two sinusoidal motions.

4. Apply the skills learned from the above to the analysis of oscilloscope traces of simple sound waveforms from musical instruments, tuning forks, combined output of two audio oscillators, etc. Fig. 10 shows the output of two oscillator-amplifiers fed simultaneously into the vertical input of an oscilloscope.

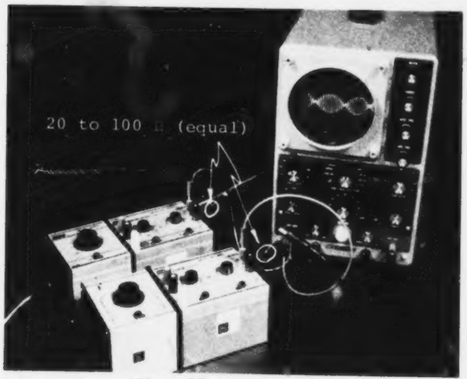


Fig. 10

Beats

The outputs of two coupled oscillators produce noticeable beats if the frequencies f_1 and f_2 are nearly the same. Beats are commonly demonstrated by simultaneously sounding two tuning forks of slightly different frequencies. Beats can easily be produced on a piano or organ by playing two adjacent notes (a black key and a white key) simultaneously.

The superposition of two waves to produce beats can be demonstrated quantitatively by coupling the outputs of the two turntable oscillators as decribed in the previous section. Set each oscillator for equal amplitude, and the "coarse tuning" controls for the same frequency. The amplitude of each oscillator should not be greater than one-half the paper width.

Fig. 11 shows the trace with, first, only oscillator #1 running at f_1 and then only oscillator #2 running at f_2. The beats are produced on the strip chart recorder when both oscillators are operated simultaneously.

When two functions have the form $y_1 = \sin a$ and $y_2 = \sin b$ and these functions are added, the result is:

$$y = y_1 + y_2 = \sin a + \sin b$$

$$[2\cos(\frac{a-b}{2})]\sin(\frac{a+b}{2}). \qquad (1)$$

If $a = 2\pi f_1 t$ and $b = 2\pi f_2 t$ and $f_1 > f_2$, Eq. (1) becomes:

$$y = [2\cos 2\pi(\frac{f_1 - f_2}{2})t]$$

$$\sin 2\pi(\frac{f_1 + f_2}{2})\ t. \qquad (2)$$

You now see that y is a periodic function with frequency

$$f_{av} = \frac{f_1 + f_2}{2} \text{ (the average frequency)}. \qquad (3)$$

The amplitude of y varies in time with the lower frequency

$$f_{amp} = \frac{f_1 - f_2}{2} \text{ (the amplitude frequency)}. \qquad (4)$$

With two oscillators operating at slightly different frequencies, one gains on the other. Assume that they start at the same time, but 180° out of phase (Fig. 12): their outputs add to zero (a null). As #1 oscillator overtakes #2, they come into phase and their outputs add as in (b). As #1 continues to

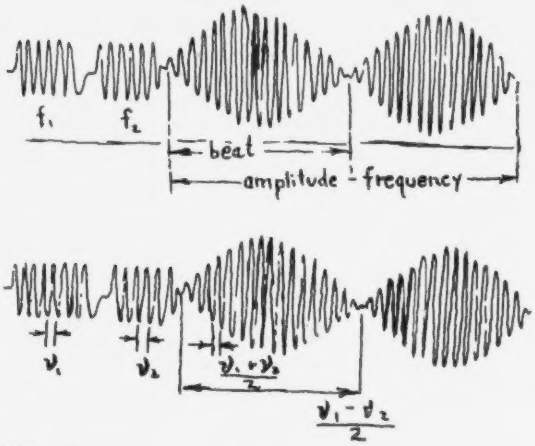


Fig. 11

gain on #2, they again go out of phase (the null at c). The two are again in phase at (d) but now in the opposite phase, and again out of phase at (e), and so on. Since the positions at (a) and (e) are identical, one complete amplitude cycle has elapsed. The beat frequency cycle is between consecutive nulls or maxima, and occur twice during each amplitude cycle.

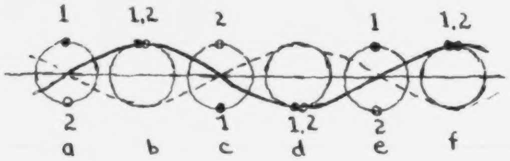


Fig. 12

Spatial Frequency and Wave Number

From the trace on a strip chart recorder one can measure the "spatial frequency", i.e., the number of oscillations per centimeter on the chart. The symbol "ν" (Greek letter "nu") is used for spatial frequency; the units are cm^{-1}. (The spatial frequency is quite analogous to the familiar time frequency f, i.e., the number of oscillations per second, which is measured in sec^{-1}.) Just as time frequency is the reciprocal of period ($f = 1/T$), so spatial frequency is the reciprocal of wavelength, $\nu = 1/\lambda$. Incidentally, the term wave number, used by spectroscopists, is also the reciprocal of wavelength.

If the tape moves through the recorder at a uniform rate you see that $\nu \propto f$. If v is constant, then

$$f = \frac{v}{\lambda} \propto \nu. \tag{5}$$

You now see that y is a periodic function with the spatial frequency, ν_{av}. Equations (3) and (4) can now be written in terms of spatial frequency:

$$\nu_{av} = \frac{\nu_1 + \nu_2}{2} \tag{6}$$

(the average frequency)

and

$$\nu_{amp} = \frac{\nu_1 - \nu_2}{2} \tag{7}$$

(the amplitude frequency).

From Equations (6) and (7) division gives

$$\frac{\nu_{av}}{\nu_{amp}} = \frac{\nu_1 + \nu_2}{\nu_1 - \nu_2} . \tag{8}$$

These three relationships could be verified with records similar to the one shown in Fig. 11.

Lissajous Figures

Lissajous figures can be produced by coupling two oscillators with the output of one perpendicular to the output of the other (see Fig. 13). Demonstrate Lissajous figures first with two turntable oscillators, and then with an audio oscillator and an oscilloscope. Set the oscilloscope sweep control to "line" position and adjust the sweep width to about one-half the screen diameter. Connect the audio oscillator to the vertical input, and adjust the amplitude of the signal until the signal is about one-half screen diameter. Stationary Lissajous figures will appear on the oscilloscope screen when the oscillator frequency is exactly a multiple, or submultiple, of the line frequency. If the phase changes slightly, the shape of the figure is altered (see Fig. 14).

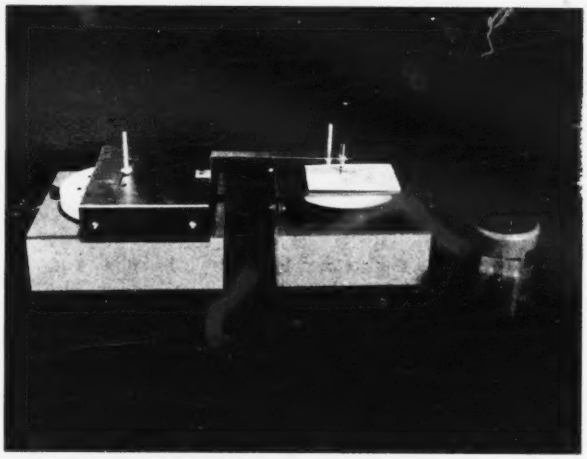


Fig. 13

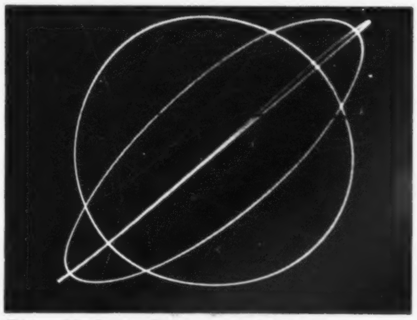


Fig. 14 Same frequency; phase difference changing from zero (straight line) to 90° (circle).

If the frequencies of the oscillators are proportional to whole numbers, the trace closes upon itself and may be repeated again and again. Figure 15 is a reproduction of actual traces produced by a pair of turntable oscillators.

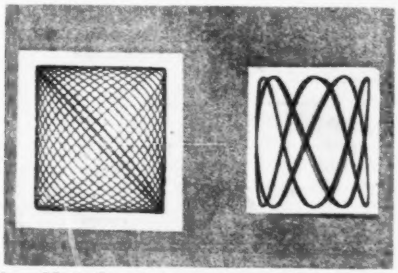
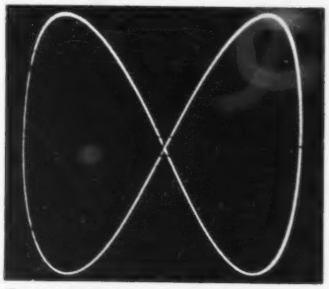
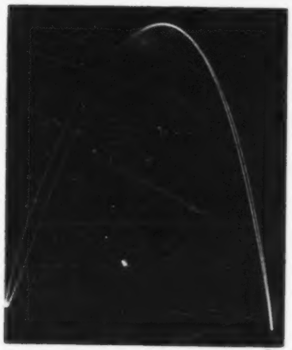


Fig. 15 Left picture shows almost equal frequencies, drifting in phase. Right picture shows frequency 3 to 1; drifting in phase.

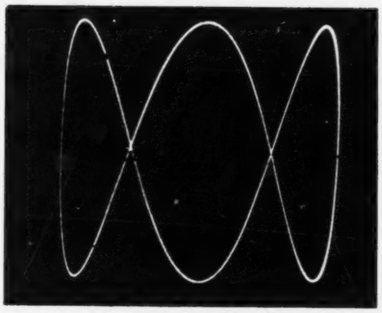
Some of the traces show that a given frequency ratio can produce a variety of figures if the relative amplitude and starting phase are changed.



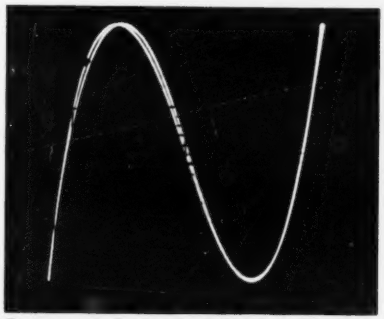
Frequency 2 to 1 with zero phase difference



Frequency 2 to 1 with 90° phase difference.



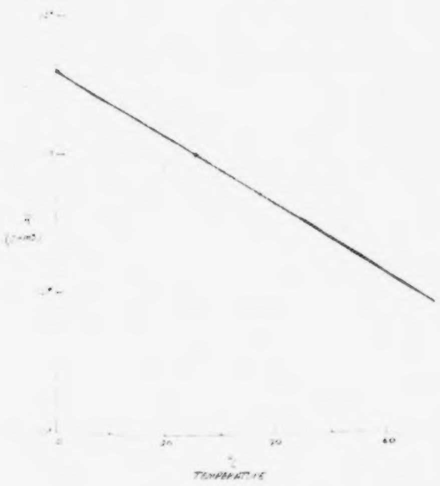
Frequency 3 to 1 with 90° phase difference.

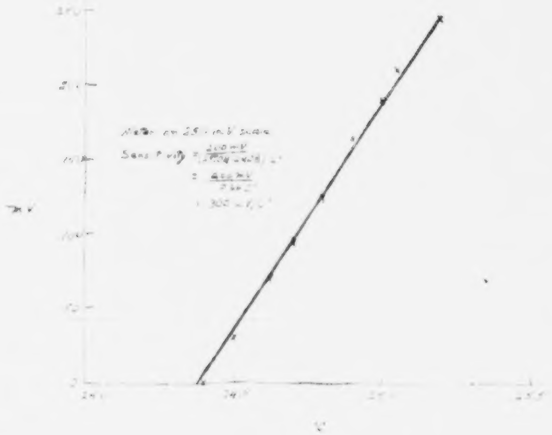


Frequency 3 to 1 with zero phase difference.

The Thermistor

The thermistor (Keystone Carbon Company, Saint Marys, Pennsylvania, type AL32E1T) has a resistance of about 100 K at 25°C, and its temperature coefficient is approximately -5%/°C (i.e., resistance at 26°C is 0.95 × 100 K, at 27°C it is 0.95^2 × 100 K and at (25+t)°C it is 0.95^t × 100 K). A plot of R vs temp on semilog paper is a straight line:

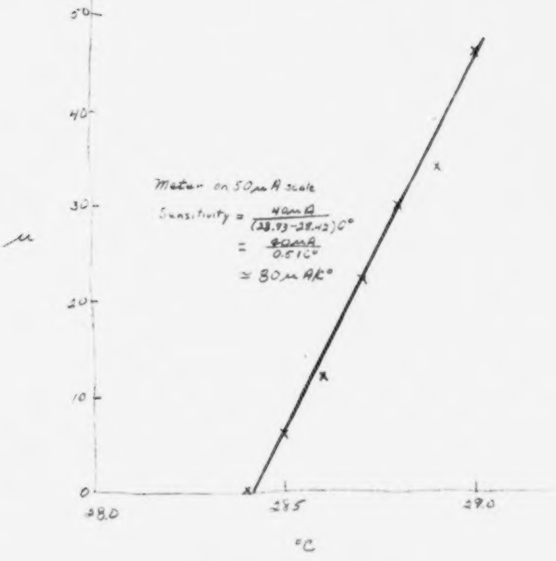




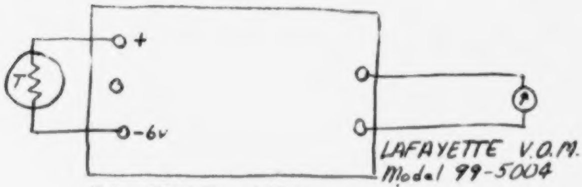
Sample Calibration Curves for Thermistor-Amplifier Combination

The graphs are intended as examples only; the slope of the line (sensitivity) will vary with the gain setting and the intercept will depend on the actual thermistor used, the gain and dc offset settings.

Note that over small temperature ranges the response is linear.



Meter on 50μA scale
Sensitivity = $\frac{40\mu A}{(28.93-28.42)C°}$
= $\frac{40\mu A}{0.51C°}$
≃ 80μA/C°



LAFAYETTE V.O.M.
Model 99-5004

DC OFFSET : +6 Volt
GAIN : ~6

To measure small changes at other temperatures adjust the gain and dc offset as necessary to get a near-zero reading on a sensitive scale.

To measure temperatures over a larger range it is simplest to measure the thermistor's resistance directly in ohms with a volt-ohm-milliammeter (see also Experiment 26: Hotness, Thermometer and Temperature). The resistance vs temperature (°C) plot is not linear over a wider range (see above).

Note on the State of Physics as a Science at the Beginning of the Nineteenth Century

It was not until the beginning of the nineteenth century that practical sources of electrical current were devised so the study of electricity was not well developed. Heat and electricity were both considered to be weightless fluids, in accordance with the mechanical point of view. An adequate theory of heat was not developed until the middle of the nineteenth century. The nature of light was not known at the beginning of the nineteenth century; the particle hypothesis of Newton was favored against wave hypothesis by Huygens—but both were mechanical theories.

It should be noted that those who contributed to the development of mechanics, like Newton and the following generations, were great mathematicians. This put a definite stamp of precision and logic on the development of mechanics. Mathematics seemed to be a science of unlimited possibilities. One of the French mathematicians once said that, given sufficient time, it will become possible to express human thoughts in the form of a mathematical formula. Mechanics was considered as an applied aspect of mathematics (remember the full title of Newton's Principia—Philosophiae Naturalis Principia Mathematica, "Mathematical Principals of Natural Philosophy") and as such also as a science of unlimited possibilities. This was true both for "pure" mechanics (e.g. laws of motion) and technological or engineering applications of mechanics. As we will see later in this unit, Heron of Alexandria was able to make certain mechanical devices utilizing the power of water steam in about 100 A.D. In the seventeenth and eighteenth centuries, many sophisticated mechanical devices and automata were built (such as Besancon's duck which could swallow food, or the mechanical boy who could play a musical instrument). The Swiss watchmakers produced ingenious devices, some of them able to give a whole theatrical performance staged by mechanical dolls; one needed only to wind the spring and the mechanism started to operate. The possibilities of these mechanical devices (from the clocks of the thirteenth century on) greatly stimulated human ingenuity in the West. (See L. Mumford, Technics and Civilization.)

This development of mechanics and mathematics on one side, and the constructing of mechanisms of high complexity on another side, induced scientists to think of God as the greatest scientist himself. God had built the world-machine, the universe in which we live, and had wound it up for all time. As Kepler put it, the task of a scientist consisted therefore in following the thoughts of God which He had at the moment of creation. But of course not all scientists accepted this view. This lack of concensus is illustrated by the famous anecdote involving the French scientist Laplace, who had suggested a mechanical account of the origin of the earth and our solar system. The French Emperor, Napoleon I, to whom Laplace explained his theory, asked him, "Where is God in your theory?" to which Laplace is said to have replied, "I have no need of that hypothesis." (Je n'ai pas besoin de cette hypothese-la.")

Note on Conservation Laws in Physics

The discovery of conservation laws has been one of the most important achievements of science. These laws, which are perhaps the most powerful and certainly the most prized tools of analysis in physical science, say in essence that, whatever happens within a system of interacting bodies, there are certain measurable quantities that can be counted on to remain constant so long as the system remains isolated.

The list of conservation laws has grown in recent years, particularly as the result of work in the area of fundamental (or "elementary") particles. Some of the newer ones are imperfectly and incompletely understood. There are others that are on tenuous ground and are still being argued.

Here is a list of conservation laws. It would be foolhardy to say the list is complete or entirely accurate. Only too recently we have had to surrender long held, cherished conceptions that appeared almost self-evident. But this list includes those conservation laws that comprise the working tool-kit of physicists today. Those which are starred are discussed in the basic six units of this course; the others are scheduled to be treated in supplementary (optional) units.

1. Linear momentum*
2. Energy (including mass)*
3. Angular momentum (including spin)
4. Charge*
5. Electron-family number
6. Muon-family number
7. Baryon-family number
8. Strangeness number
9. Isotopic spin

Those listed in numbers 5 through 9 result from work in nuclear physics, high energy physics, elementary or fundamental particle physics (some use these terms interchangeably). Even if these laws are unfamiliar, you will find Kenneth Ford's "Conservation Laws" (in the Reader) interesting and enlightening. It appears with the Unit 6 selections, but it is worth reading at this stage. Ford discusses the first seven laws in that selection, and, even for those uninitiated to the mysteries of elementary particles, he gives a clear presentation of many ideas basic to contemporary physics. The selection is a chapter from Ford's fine little book The World of Elementary Particles (Blaisdell Publishing Co., 1963). The book is a well written introduction that should appeal to anyone wanting to learn about one of the leading edges of current physics.

Note on the Gain and Loss of Matter by the Earth

The earth has a mass of about 6×10^{27} g, and has existed for a time of about 5×10^9 years, or 1.5×10^{17} sec. We want to find how the present estimated rates of gain and loss of matter by the earth measure up against these numbers.

The most important process now operating seems to be the influx of dust particles. These are detected by their impacts on satellites that are outside most of the atmosphere, and by the light and ionization that they create when they pass through the atmosphere and are slowed down by it. The number of such particles one detects increases very rapidly as the size diminishes. In fact, the bulk of the mass influx is due to very tiny particles (on the order of 10^{-4} cm diameter). Such small particles cannot be individually detected from the ground when they enter the atmosphere. They are far too small to appear as meteors, which result from the vaporization of particles at least several millimeters in diameter. The total estimated influx of all these particles, large and small, is about 10^5 g/sec over the whole surface of the earth. This influx is not balanced by any loss of matter in dust particles or larger sizes of particles, save for a few spacecraft and their debris we have sent out in recent years.

On the molecular level the earth does lose mass by evaporation from the top of the atmosphere. The rate of this evaporation depends on the number of molecules which are near enough the top of the atmosphere to escape without collisions with other molecules, and which have velocities high enough for them to escape the earth's gravitational pull. Because the velocities of the molecules are determined by the temperature of the upper atmosphere, the rate of evaporation depends critically on this temperature. At present this rate is probably something less than 5×10^3 g/sec over the whole earth, and is thus negligible in comparison with the influx of dust. (No H_2O would be likely to be lost by atmospheric "evaporation," because it would previously have been dissociated into hydrogen and oxygen.) The earth also collects some of the hot gas evaporating from the sun, but this amount is small.

Is Mass Really Conserved?

The chemist Hans Landolt conducted extensive research during the two decades after 1890, making very accurate measurements of the masses of systems in which chemical reactions were taking place. In 1909 he stated his conclusion:

> The final result of the investigation is that no change in total weight can be found in any chemical reaction.... The experimental test of the law of conservation of mass may be considered complete. If there exist any deviations from it, they would have to be less than a thousandth of a milligram [10^{-6} g].

Landolt's result is typical of the experimental data of the physical sciences: one can never claim to have proved that something is exactly zero (or exactly any other number). One can only say that experiment shows the number to be zero within a specified margin of error.

Landolt's work should not be represented as being of critical importance in the acceptance of the conservation of mass. The physicist's faith in the conservation of mass goes beyond the accuracy of any chemical or physical apparatus—it is philosophically pleasing and makes his laws simple.

In some cases 10^{-6} g can be quite a potent quantity (for example, that amount of LSD has terrifying effects on humans). A cube with a 1/10 mm edge length has a volume of 10^{-6} cm^3. With a density near that of water, such a cube has a mass of about 10^{-6} g. One can probably see several specks of dust on any desk top with about that mass.

If 18 g of methane CH_4 are burned with 64 g of oxygen (one gram molecular weight of each), 211 kilocalories of heat are released. This amounts to a change in rest mass of about 10^{-8} g or about one part in 10^{10}, as the following shows.

$$\Delta m = \Delta E/c^2$$

$$= \frac{(211 \text{ kcal})\ (4.19 \times 10^{10} \text{ ergs/kcal})}{(3.00 \times 10^{10} \text{ cm/sec})^2}$$

$$= 9.8 \times 10^{-9} \text{ g.}$$

This is a relatively energetic reaction, so that this fractional mass change is very much larger than for the reactions Landolt studied.

Using a typical modern precision balance, chemists commonly measure one-g masses to an accuracy of about 10^{-6} g. Indirect measurements, e.g., on the mass spectrometer, give masses of individual atoms with an accuracy of about 10^{-6} g. Indirect measurements, e.g., on the mass spectrometer, give masses of individual atoms with an accuracy of about one part in 10^5 of an atomic mass unit—which is on the order of 10^{-24} g.

Elastic and Inelastic Collisions

The terms "elastic" and "inelastic" appear on p. 28; there is a marginal note on p. 38 that gives some justification for the usage. It is difficult to discuss these terms fully without using the concept of energy. For an elastic collision between bodies A and B, B's velocity relative to A after collision is just the negative of what it was before collision; the distance between them afterward will increase at the same rate by which it decreased before collision. For a completely or perfectly inelastic collision, the relative velocity is zero after collision. For a collision somewhere between completely elastic and completely inelastic, the bodies will separate more slowly than they approached.

More usually an elastic collision is defined as one for which the kinetic energy after the collision is the same as the kinetic energy before. Below is a proof that this is equivalent to the above definition.

In either treatment, an elastic collision is "reversible," i.e., it would be impossible to tell whether a motion picture of the collision was running forwards or backwards.

Notes on the Equivalence of the Definitions of "Elastic Collision"

Before	After
$A \xrightarrow{V_A} B \xrightarrow{V_B}$	$A \xrightarrow{V_A'} B \xrightarrow{V_B'}$
$(V_B - V_A)$ is the velocity of B relative to A before	$(V_A' - V_B')$ is the velocity of B relative to A after

If kinetic energy before and after are equal, then

$$\tfrac{1}{2}m_A V_A^2 + \tfrac{1}{2}m_B V_B^2 = \tfrac{1}{2}m_A V_A'^2 + \tfrac{1}{2}m_B V_B'^2 . \quad (1)$$

Since the linear momentum is always conserved,

$$m_A V_A + m_B V_B = m_A V_A' + m_B V_B' . \quad (2)$$

We can rearrange (1)

$$m_B(V_B'^2 - V_B^2) = m_A(V_A'^2 - V_A^2) \quad (3)$$

and (2)

$$m_B(V_B' - V_B) = -m_A(V_A' - V_A) . \quad (4)$$

If we divide (4) into (3), we get

$$V_B' + V_B = V_A' + V_A$$

or

$$V_B' - V_A' = -(V_B - V_A) , \quad (5)$$

which says that the velocity of B relative to A after collision is equal to the negative of what it was before.

Energy Reference Levels

It might be advisable to make the point that the zero level for measuring potential energy (or even for measuring kinetic energy!) is arbitrary. Because the conservation principle really deals only with changes (which is more easily seen if the general formulation used for momentum is used again here— $\sum_i(\tfrac{1}{2}m_i v_i^2 + F_i h_i) = 0$, the zero levels can be chosen as whatever is most convenient at the moment. Lest students conclude that playing with the zero levels is only "mathematical," some examples can be given to show that there is really no meaning to absolute zero levels for potential or kinetic energy. It is common, for example, to take the lowest point of the pendulum's swing as the zero potential level. But if the string is cut, the bob can fall to the floor and decrease its potential energy still more. The potential energy at the floor level could be taken as "negative," or the floor level could be chosen as a new zero level. But if a hole were to be dug next to the bob, it could decrease its gravitational potential still further by falling into the hole—all the way to the center of the earth! (What would happen if it went beyond the center?) But neither will the center of the earth do as an absolute zero of potential energy, since there is still the possibility of falling all the way into the sun...and so on, ad infinitum. The topic of "inertial" frames of reference is too subtle to take up here so it is not easy to discuss the "zero level" for speed. The "zero level" for kinetic energy is not so easy to handle. The change in v^2 in one frame of reference will not be the same in some other frame of reference moving with uniform translation relative to the first—but neither will the observed distance through which the force acts. Very often (e.g. in nuclear physics) velocities are referred to the center-of-mass of the interacting bodies to simplify calculation.

After all this, the main point to be made is as follows:

Intuitively, $\frac{1}{2}mv^2$ is not an obvious choice for "quantity of motion," but neither is it particularly objectionable; also, weight times height is not objectionable as a measure of a "potential" something.

However, by making these choices we can compute a quantity, the sum of -mv and Fh, which is conserved—at least in frictionless rising and falling near the surface of the earth. It will turn out to be valid also for the frictionless "rising and falling" in the solar system and in many other situations (when some other distance measure than distance from the earth is used). We have given the sum the name "energy." When related to ideas of "heat" and "work," "energy" proves to be one of the most powerful ideas of physics, making many connections with other sciences as well as within physics. The extension of appropriate terms to preserve conservation principles is beautifully treated in Feynman's "Conservation Laws" in the Reader.

Note on Food Energy

Our fuel-foods combine with oxygen to produce carbon dioxide and water. A given food releases the same amount of energy from chemical form, when "burned" to carbon dioxide and water, whatever the chain of chemical stages between—if not, we could disprove the conservation of energy by building up some food by one process and breaking it down by another process that released more energy! So we can measure energy-values of foods by burning samples in a laboratory. Then we can calculate the fuel supply in our diet and in the diets of people all over the world. We can calculate the fuel used in various activities from the CO_2 breathed out; in an hour's walking, in a night's sleeping, in a football game, in a day's shift in the machine shop, in a 3-hour mathematics examination. Then we can estimate the cost for a specimen day. The measurements are made by placing a mask on the victim's face and collecting the breath he breathes out in a short measured time. The volume collected is measured (with a gas meter) and a sample of it is analyzed to find how much CO_2 has been added to it in place of oxygen. This chemical change, multiplied up to the full volume of breath for the whole job, tells us the total amount of food fuel "burned" by the victim during the job. Basic living, keeping the machinery of heart, lungs, digestion just idling, demands a certain amount of energy each day. This minimum energy-turnover is called our basal metabolism. In cold weather, body heating demands some more. Walking and other mild activities add further demands, and violent exercise requires much more. For heavy physical work we must take in a great deal more energy in food than we need for the job itself, because our bodies are only about 25 per cent efficient, wasting the other 75 per cent as heat.

(From Eric M. Rogers, Physics for the Inquiring Mind, Princeton University Press, 1960.)

Note on the Temperature of Outer Space

When we use the word "space" in this context we don't mean a region with nothing in it, for then the idea of temperature has no meaning. What we mean is some representative region of the universe, outside the earth's atmosphere, not too close to the sun or any bright star, and perhaps not within a galaxy. Such a region is not necessarily empty since it has (within one galaxy, at least) on the average about one hydrogen atom per cm, and many photons of radiation flowing through it. If we consider a region of space large enough, i.e., including enough atoms and photons, so that the statistical concept of temperature makes sense, then the temperature of that region is determined by how the energy within it is distributed among the possible forms—just as it is for ordinary gases. Often the radiation component is more important than the particle component. It is, for example, a lot "colder" in the region of space shadowed by the earth than it is in the sunlight—even though the particle contents are the same. An earth-satellite with no internal heat source and the same kind of surface on both dark and light sides would come to a temperature a little above 0°C—unless it moves into the earth's shadow.

Recent measurements of the temperature of outer space, made by measuring the radiation reaching the earth (with great care taken to eliminate radiation from the sun and other extraneous, as well as man-made sources), indicate that outer space is at 3.0°K - 0.5°K. Even if the temperature of space were very high, however, it would not necessarily be "hot." A low density of molecules ("space"), even if moving very fast ("high temperature"), wouldn't be able to transfer much heat to a body.

Note on Classifications of Energy

For some purposes there are advantages to classifying energies. But the kind of classification that one might best make depends upon the situation he is considering and is, in any case, arbitrary.

Suppose we are looking at a physical system: a bowl of soup, a soap bubble, or maybe our solar system. Someone asks us, "What kinds of energy does the system possess?" We might answer that it has kinetic and potential energies, but that, while true, is likely too general—too all inclusive.

We might go further. We might say that the system has kinetic energy as a consequence of the motion of the system as a whole. We may have dropped the bowl of soup; the soap bubble may be floating away; or our solar system may be moving relative to the center of our galaxy. The magnitude of this energy depends upon the reference frame in which we choose to measure the system's motion.

In addition, the system as a whole may possess potential energy as a consequence of its position relative to external sources of force. The soup and soap bubble have gravitational potential energy as a consequence of experiencing the earth's gravitational force. Similarly the solar system may experience the gravitational forces due to nearby stars.

For some purposes the kinetic and potential energies of these kinds are not very important. For example, we do not consider them at all when we ask about the soup's temperature or about the way the planets move relative to the sun. When that is the case, we generally say that all the rest of the energy is internal energy V. For most thermodynamics problems, this is the sort of classification we make. (In fact, the laws of thermodynamics are independent of the detailed nature of the system.)

How we go about describing the internal energy depends upon our purposes. If we are dealing with helium gas at moderate temperature and pressure, it is reasonable to say that the internal energy is just the translational kinetic energy of the atoms, where we measure their speeds relative to the container. That is the approach taken in Secs. 12.4 and 12.6 of Unit 4. This works reasonable well for He, because the helium atoms exert only very weak forces on each other when not colliding, and because there is no molecular structure to the monatomic He molecule.

If we are dealing with a gas composed of more complex molecules, then we would need to consider other forms of internal energy; for example, the kinetic and potential energies associated with the vibrational motion of the atoms within the molecule and the kinetic energy associated with the molecule's notation. We might even need to include the energy associated with the interactions that the molecules have with each other, often referred to simply as chemical energy.

Of course, even these do not exhaust the sources of internal energy. There is the energy associated with the binding of an atom's electrons to its nucleus and that associated with the forces holding the particles together in the nucleus. There are instances for which we might need to include the system's radiation energy (light).

It should be clear that there are many different ways of making energy classifications. There are times when the classifications; elastic, electrical, and chemical energies are helpful, even though we know that elastic and chemical energies are predominantly electromagnetic in character. (Think of a charged compressed string in a jar of sulfuric acid.) It is probably worthwhile to give some thought to the sources of internal energy, but it is not the sort of thing one wants to bludgeon students with. Detailed classifications can sometimes lead to confusion, and are always somewhat arbitrary.

Note on Suction

The apparently simple question "How do you drink through a straw?" can be asked at this point, but not answered very completely until more attention has been given to kinetic molecular theory. An explanation appropriate to Sec. 12.4 would be as follows. One does not "inhale" a coke. The back of the mouth is closed off with the back of the tongue, thus forming a closed space of mouth and straw. The palate is raised and the jaw lowered, thus increasing the volume of the closed space. The increase in volume means a drop in pressure in the mouth and on the surface of the liquid in the straw. The greater pressure of the atmosphere is able to force liquid up into the mouth, decreasing the amount of space until the air pressure in the mouth returns to slightly below atmospheric pressure. The inward distortion of the cheeks, often proposed as the mechanism of sucking, results from air pressure inside being less than the atmospheric pressure. The cheeks may be stiffened to prevent their distortion inwards.

Eventually, in Chapter 11 it will be possible to explain in more detail the drop in pressure. When the volume of the space in the mouth is increased, the random thermal motion of the air molecules will result in their moving apart to fill the greater volume. The molecules being further apart means that the bombardment of the surface of the liquid in the straw will diminish. The undimished bombardment of the liquid from below will drive it up the tube.

Some Notes on Watt

(from James Watt, Craftsman and Engineer, H. W. Dickinson, Cambridge University Press, 1936.)

"The problem now was to make the apparatus into an engine capable of repeating its motion indefinitely. Watt started on the construction of a model with a cylinder 2 in. diam. While thus engaged Robison's story is that he burst into Watt's parlour and found him with a 'little tin cistern' on his lap. Robison began to talk engines, as he had done previously, but Watt cut him short by saying: 'You need not fash yourself about that, man; I have now made an engine that shall not waste a particle of steam.' Robison put to Watt a leading question as to the nature of his contrivance but 'he answered me rather drily and vouchsafed no explanation.' If an artist ever wishes to paint a genre picture of Watt, instead of perpetuating the unfounded story of his playing as a boy with the steam issuing from the spout of a kettle, he might limn the young workman in his leathern apron with the separate condenser on his lap and Robison trying to quiz him.

"If Watt lacked experience in the construction of engines 'in great,' i.e. of full size, he had the advantage of being free from preconceived ideas of what engines should be like. In fact he had in view two engines, one reciprocating and the other rotary, entirely different in design from anything that had gone before. This very fertility of mind, and his resource in expedient, may almost be said to have delayed his progress. Of a number of alternatives he does not seem to have had the flair of knowing which was the most practicable, hence he expended his energies on many avenues that led to dead ends. In truth this is the attitude of mind of the scientist rather than that of the craftsman. Still, unless he had explored these avenues he could not be certain that they led nowhere."

Discussion of Conservation Laws

Conservation laws are important because they enable us to make predictions. This ability has a far wider significance than its application to colliding carts in the laboratory. For example, much of what is known about subatomic particles comes from analyzing scattering (collision) experiments. Students hear a great deal about bubble chambers and particle accelerators without understanding the way in which information from these devices is used.

A detailed account of Chadwick's discovery of the neutron and his calculation of its mass is found in Chapter 23 of the text, and could well be used here as an example in which both conservation laws are put to use to make new discoveries.

We suggest the following procedure as an effective way to show the use of conservation laws to make predictions. It will also introduce a method of using graphs which is very similar to that used by engineers and physicists. Although the computations and plotting may be assigned as homework it is important that you go through the analysis of the graph carefully with the class so that they will appreciate the full significance of the curves they have drawn. Data from one of the collisions you photographed could be used; however, if you feel that students would get bogged down in arithmetic, you could give them easy round numbers such as those used in the example below.

First, ask students to assume only that momentum is conserved in a collision. For example, let $m_A = 1$ kg, $m_B = 2$ kg, $v_A = 0$, and $v_B = 10$ cm/sec. Then,

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A' + m_B\vec{v}_B'$$

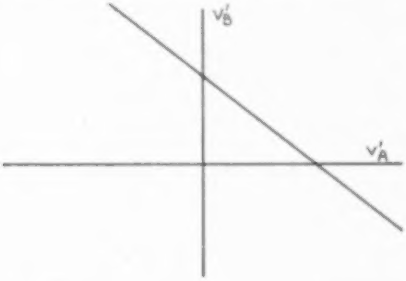
$$0 + 20 = v_A' + 2v_B'$$

$$v_A' = 20 - 2v_B'$$

Have the students assign values for v_B' and compute v_A' :

v_B'	v_A'
0	+ 20
5	+ 10
10	0
15	- 10
- 5	+ 30

Plot these points as in Fig. 1.



Point out that there is an infinite number of values for v_B' and v_A' which would satisfy the above equation, and ask if all of them might actually be observed if the experiment were done enough times. Make sure that the significance of + and - values is clear; for example, ask students to predict from their graphs what the velocity of the smaller cart would be if the large cart rebounded with a speed of 10 cm/sec ($v_A' = 40$ cm/sec).

Next, ask students to assume only that kinetic energy is conserved in this collision.

$$\tfrac{1}{2}m_A v_A{}^2 + \tfrac{1}{2}m_B v_B{}^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2$$

$$0 + 200 = v_A'^2 + 2v_B'^2$$

$$v_A'^2 = 200 - 2v_B'^2$$

Constructing a table of values for v_B' and v_A' is a little more time-consuming because of the square roots; however, a number of shortcuts are available. Slide rules with A and B scales can be used. If they are not available, students might plot values of x^2 as a function of x, draw a smooth curve, and use this as a simple computer.

Again, any number of values for v_B' and v_A' can be calculated, but most of them are physically meaningless. The table of values could be constructed as follows:

v_B'	$v_B'^2$	$2v_B'^2$	$v_A'^2$	v_A'
0	0	0	200	± 14.1
± 2	4	8	192	± 13.8
± 4	16	32	168	± 12.9
± 6	36	72	128	± 11.3
± 8	64	128	72	± 11.3
±10	100	200	0	± 8.5
±12	144	288	- 88	(imaginary)

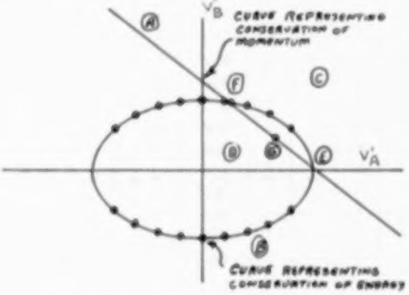
Plot these values of v_B' and v_A' on the same axes as the preceding graph (Fig. 2). An equation in the form

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$$

results in the curve called an ellipse. The equation resulting from the conservation of energy in the collision has this form. Students may be familiar (from Unit 2) with the mathematics involved here, but may not appreciate that these curves can be used to infer a number of interesting facts about real physical events.

Use the key letters on the graph, Fig. 2, to develop the following ideas:

A. Any point lying on the straight line represents values of v_B' and v_A' for which momentum is conserved.



B. Any point lying on the ellipse represents values for which kinetic energy is the same after the collision as before.

C. For any point lying outside the ellipse, the corresponding kinetic energy of the carts would have to be greater than before the collision. In other words, kinetic energy was added to the system. This could not happen in this kind of collision; it would require that an explosion be set off during the collision.

D. For any point inside the ellipse, the total kinetic energy of the carts after the collision is less than before. In other words, kinetic energy was lost from the system, for example, by friction between the bumpers.

E. This point on the ellipse represents the case in which the cart with velocity v_B goes past or through the other cart having velocity $v_A = 0$, without exerting any force on it. This case may seem trivial or physically meaningless, but a comment about neutrinos may be in order here.

F. This point on the ellipse represents the only values for v'_B and v'_A (other than case E) for which both momentum and kinetic energy are conserved, as shown by the intersection of the two curves. If students used starting data from one of their photographs, they should see where the values for v'_B and v'_A obtained from their photograph plot on the graph.

G. This point on the line shows v'_B and v'_A set for a collision in which the carts stick together after the collision. In other words, $v'_B = v'_A$. Since momentum is always conserved, the point must fall on the straight line, and so it falls inside the ellipse. Kinetic energy was lost, so this represents an inelastic collision. Points along the line segment FG represent all possible values of v'_B and v'_A, for which the cart with v_B = 10 cm/sec pushes the other ahead; the line segment GE represents all values for which the first cart overtakes or passes through the second one.

If the values of v'_B and v'_A obtained from the photograph do not plot at point F, ask the students for possible reasons. (It should fall in the region FG.)

The Crisis of World Hunger

At the time of the first Pugwash Conference 10 years ago, about one half of the world's population was continuously on the verge of starvation, and another 25 per cent was seriously undernourished. That situation has not changed appreciably in the last 10 years except for the fact that the total number of people in the world has increased by about 500 million persons, some 400 million of whom have been added to the ranks of the hungry.

Actually during the last 10 years the total food production in the developing countries has risen at about the same rate as that in the developed countries—some 30 per cent in the decade. The discrepancy in food availability has arisen from the considerable differential in population growth rates: currently about 2.8 per cent per year in the poorer countries and 1.1 per cent per year in the richer ones. The net result has been a substantial increase in the per capita availability of food in the richer countries and an actual decrease in the per capita availability of food in the poorer ones.

Why have the poorer countries not been able to increase their agricultural production more rapidly? We know that crop yields can be greatly increased by applying modern technology. The average rice yield per hectare in Japan is 3.5 times that in India and 4 times that in Nigeria. Such differences have led many persons to believe that simply by applying technology we could easily expect annual increase in yield of 2 per cent and that with effort 4 per cent might be acheived in a number of areas. Indeed, the development plans of virtually every developing country have set high targets for increases in agricultural productivity—some as high as 8 per cent per year. Yet, in the last 10 years the increase in crop yield per hectare in the poorer countries have averaged less than 1 per cent per year. By far, the greater part of the total increase of food production has come about as the result of increase of farm acreage. By contrast, the greater part of the increased production in richer countries has resulted from increased productivity.

Some time ago the President's Science Advisory Committee in the United States became interested in this critical problem of world food production and appointed a World Food Panel to investigate the situation. The Panel's findings have now been published.

The Panel points out that one of the deficiencies in increasing agricultural productivity has been lack of adequate education and training, which have played such key roles in the rapid rise of productivity in the richer countries. Japan, for example, produces 7,000 college graduate agriculturalists per annum compared with only 1,100 per year in all of Latin America. In Japan there is one farm advisor for each 600 farms. Compare this with perhaps one advisor for 100,000 farms in Indonesia and one advisor per 10,000 farms in Colombia!

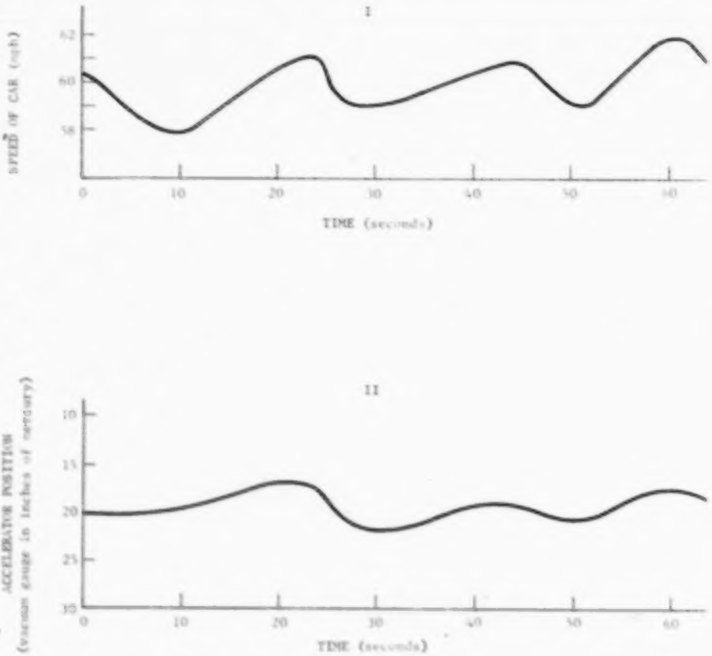
As essential aspect of applying technology is learning what technology to apply and how to apply it. We know that application of fertilizers can increase crop yields and we know how to build fertilizer plants. But we generally do not know how the fertilizers will interact with the plant being grown and with the soil. We do not know how much water will be required. We do not know what genetic variety of the desired plan is the best to use within the local ecological framework. Indeed, our knowledge of how to grow things efficiently in the tropics is lamentably sparse. Learning the answers to such questions requires research and development which in turn requires trained research workers and research facilities.

Equally important, the World Food Panel concludes that the problem of increasing food production in the poorer countries greatly transcends that of simply changing agricultural practices. Virtually all facets of the economies and cultures of the poorer countries are involved, ranging from food pricing policies to urbanization, to industrial growth to public health. The Panel emphasizes "if the world's supply of food is to grow substantially, there must be general economic development in the developing nations and not just a change in farming itself."

Feedback

Between the practical world of applied science and engineering and the idealized world of pure science and theoretical engineering an enormous difference often seems to exist. This is because it is often difficult of describe commonplace physical situations theoretically by exact mathematical relationships, while simple quantitative theoretical ideas, such as frictionless surfaces, constant speed, point sources, pure sine waves and a host of other theoretical concepts, may be difficult to produce experimentally Usually, we have to be satisfied to produce these basic physical—or is it mathematical—ideas to within acceptable limits of error. To do this, we often make use of a system of control called feedback. Before attempting a definition of feedback let us consider a situation that any motorist meets and solves daily.

Suppose the driver of an automobile on a lightly traveled superhighway is asked to hold a speed of 60 mph for five minutes. Assuming that he cooperates to the best of his ability, the chances are that it will take quite a bit of manipulation of the gas pedal for him to keep his car's speed constant to within a few miles per hour of 60 because of the different road conditions he will encounter in five miles. For example, his car will tend to slow up when climbing hills unless he gives it more gas and speed up on going downhill unless he lets up on the accelerator. Various road surfaces also require different accelerator settings. Thus in loose terms, we realize by experience or observation that a motorist who attempts to maintain a constant speed is continually moving the accelerator by small amounts. In engineering terms, we describe this situation by saying that the motorist forms a feedback system for his car: he attempts to keep the car moving with constant speed under a changing load (the hills and road surface) by observing the deviation from the desired output signal (the 60 mph on his speedometer) and then correcting the input signal (the flow of gas controlled by the accelerator) in such a way as to bring the output signal back to the desired value (60 mph). The process just described is the essence of what we mean when we use the term feedback in a technical sense.



Simulated graphs of automobile speed and vacuum gauge readings as a function of time. The vacuum gauge, an indicator of gasoline flow to the carburetor as governed by the accelerator, is a fast response instrument compared with the speedometer with this difference in response contributing to phase differences in the two curves. Though simulated, the values are realistic.

There are some other things about a feedback system that are characterized by this situation. For one, it is quite evident that the more closely the motorist tries to hold his speed to 60 mph, the more tiring it is to him. Or we can say that a tightly coupled feedback system, one allowing only small deviations from a given value, generally requires a greater expenditure of energy than a system in which larger deviations are allowed. In general, we also note that the fluctuations in the speedometer and the variations in the accelerator position do not occur together because it takes the driver and car a certain amount of time to react to the error signal—a reading different from 60 mph. Thus, we see that the simple mathematical statement v = 60 mph, used to describe the speed of the car in a theoretical problem, may in practice require good equipment, concentration and skill to fulfill.

In engineering, one often wishes to dispense with the human element, replacing it with some physical device that doesn't tire yet performs its function in the feedback loop. To show in some detail how feedback operates in a physical system, we shall choose a simpler and more direct example than that of the automobile and driver. Let us consider how we might meter and control the flow of a liquid in a completely automatic way. In this metering system, the objective will be to maintain a constant flow of liquid under a pressure variation stemming from a change in the height of a liquid in a tank at the high pressure side of the tube.

In order to make a liquid metering device, theory shows that all that is needed is a tube of constant length and uniform bore. For this tube and a specified liquid, the volume of liquid passing through the tube in a given time is directly proportional to the pressure difference across the ends of the tube. In a common arrangement, one end of the tube is open to the atmosphere and the other end is fed from a column of liquid of some height H. As the pressure on the side where the liquid flows out of the tube is atmospheric pressure, while the high pressure side is at atmospheric pressure plus a term proportional to the height of liquid, the pressure difference across the tube is directly proportional to the height H of liquid. From this analysis, we see that by maintaining a constant height of liquid feeding the tube, a constant flow of liquid through the tube is assured. The problem therefore resolves itself into finding a means for keeping H constant as liquid flows through the tube.

The classical way of solving this problem is through use of a weir. This arrangement is shown in Fig. 1. The weir is the open tank labeled W. The metering system is the horizontal tube of length D. The weir maintains a constant liquid level at height H above the tube by means of an overflow pipe, the curved tube inserted in the tank. Liquid flows through the supply pipe, entering the system at the point labeled input, and flows out into the weir. From the weir, the liquid leaves the system at the output point after having first flowed through the metering tube, the tube of length D. After the supply pipe fills the weir to height H, this level is maintained by allowing the excess liquid to run out of the overflow pipe as waste. Once this condition is reached, the output through the metering tube is constant for all variations of input through the supply tube, provided there is always some liquid flowing out the overflow pipe.

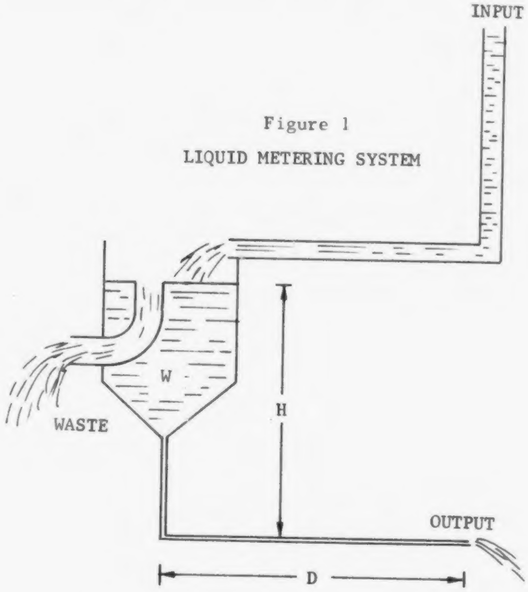


Figure 1
LIQUID METERING SYSTEM

This system for metering a liquid is very precise and as an apparatus for research in the viscosity of liquids, for example, yields definitive results. For less exacting uses such as in the metering of fuel to the carburetor of an internal combustion engine, it is more convenient to dispense with the overflow pipe and devise a system where the height of liquid in the tank is controlled automatically by a valve in the supply line. Two ways of accomplishing this are suggested, both of which use feedback—one purely mechanical and the other electromechanical feedback. The aim here is to illustrate the concept of feedback—not to design the most practical metering system possible.

In Figs. 2 and 3, the height of liquid is determined by a float controlled valve. The simplest of the two arrangements is shown in Fig. 2 where a float is suspended at one end of a beam balance and a valve V is suspended at the other end. The float and valve—here shown as a simple plate damming the liquid in the supply pipe—are adjusted at a certain level H for a fixed pressure on the input side of the supply line. The subsequent action of the float and valve is such that when an increase in pressure and hence increase in flow of the liquid occurs in the supply line, the resulting flow through the valve leads to an increase in height H in the tank. This causes the float to rise. Being connected to opposite ends of a beam balance, the rise of the float depresses the valve, decreasing the flow of liquids and leading to a fall in the liquid level H. When this level falls, the float falls and the valve opens. This compensating action tends to keep level H constant, because the float always moves the valve in such a way to keep the liquid surface at a fixed height. The float and valve constitute a mechanical feedback system.

Figure 2

LIQUID METERING SYSTEM WITH FEEDBACK

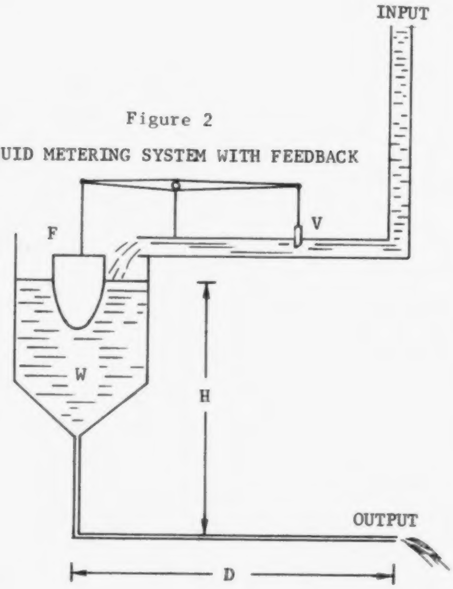


Figure 3 is an electromechanical adaptation of Fig. 2.

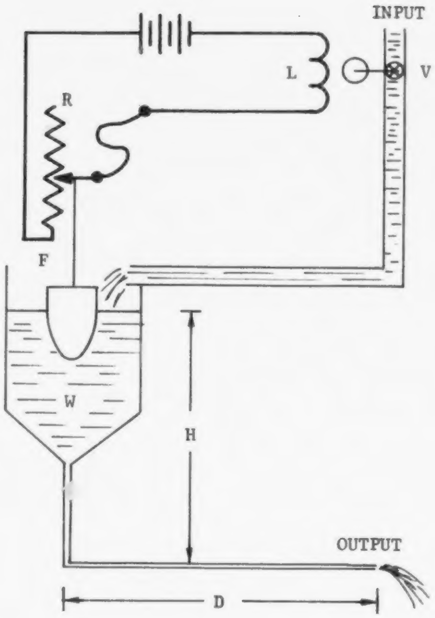


Figure 3

LIQUID METERING SYSTEM WITH ELECTRO-MECHANICAL FEEDBACK

In this feedback system, float F and valve V are linked electrically, not mechanically. The change in the height of the float changes the resistance and hence the current in the electrical circuit shown. A change in current also occurs in the coil L, changing the strength of the magnetic field in the coil. By controlling the magnetic field of a coil that is part of a motor or relay, one controls the position of a valve connected to the motor or relay and hence controls the flow of liquid in the supply line in the same sense as in Fig. 2. The advantage here, of course, is that F and V need not be close to one another, making this arrangement the more flexible feedback system.

Historically, feedback goes back to the fly-ball governor on James Watt's steam engine. In order to keep his engine running at constant speed under differing mechanical loads, it was necessary to control the amount of steam admitted to the cylinder containing the piston. This control was achieved by linking a valve in the steam line (a valve acting like the gas pedal of a car) to a fly-ball governor rotated by the shaft of the engine. This governor operates on the same principle as a stone on a string. That is, when a stone on the end of a string is swung in a horizontal circle at constant speed, the angle between the string and the vertical increases as the speed of the stone increases. On the fly-ball governor, this change in angle with speed can be used to control the action of the valve in the steam line.

Since Watt's time, and especially now, feedback has become an important and sophisticated part of the structure of modern devices, ranging from the oscillator in tiny radio sets to the automatic pilots of our largest jet airliners.

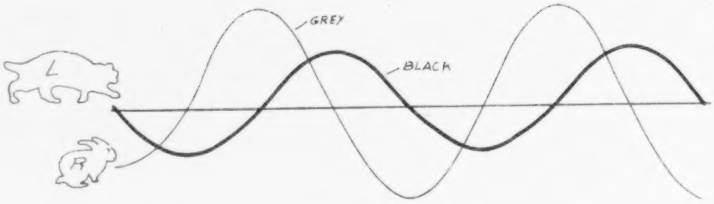
Feedback in Animal Population Cycles

Quoted from A. Tustin's article entitled "Feedback." Scientific American, September (1962), Vol. 187, page 53.

Fascinating examples of the consequences of interdependence arise in the fluctuations of animal populations in a given territory. These interactions are sometimes extremely complicated. Charles Darwin invoked such a scheme to explain why there are more bumblebees near towns. His explanation was that near towns there are more cats; this means fewer field mice, and field mice are the chief ravagers of bees' nests. Hence near towns bees enjoy more safety.

The interdependence of animal species sometimes produces periodic oscillation. Just to show how this can happen, and leaving out the actual complications that are always present in an actual situation, consider a territory inhabited by rabbits and lynxes, the rabbits being the chief food of the lynxes. When rabbits are abundant, the lynx population will increase. But as the lynxes become abundant, the rabbit population falls, because more rabbits are caught. Then as the rabbits diminish, the lynxes go hungry and decline. The result is a self-maintaining oscillation, sustained by negative feedback with a time-delay (see diagram below).

Curves of variation such that when R is large L is rising, but when L becomes large R is falling must have the periodic oscillatory character indicated. This is not, of course, the complete picture of such phenomena as the well-known "fur cycle" of Canada, but it illustrates an importnat element in the mechanisms that cause it.



Rabbit and lynx population cycles are an example of a feedback system in nature. Here a fall in the relatively small population of lynxes (black curve) is followed by a rise in the larger population of rabbits (gray curve). This is followed by a rise in the lynx population, a fall in the rabbit population and so on.

A Method for Calculating the Pressure of the Atmosphere

Assume any cross-sectional area A for the column of height h and liquid density ρ. The volume of the column of height h will be Ah, the mass will be $\rho V = \rho Ah$ and the weight will be $ma_g = \rho Aha_g$. The pressure on the bottom needed to support the column will be the weight divided by the area

$$P = \frac{\rho Aha_g}{A} = \rho ha_g$$

Using mks units, $\rho = 1.4 \times 10^4$ kg/m^3, $h = 0.76$ m and $a_g = 9.8$ m/sec^2, so $P \approx 10^5 \frac{\text{kg m}}{\text{sec}^2} / \text{m}^2 = 10^5$ N/m^2. You may prefer to find numerical values for each step instead of waiting until the last step.

The detailed history of Boyle's law is not important in itself. It takes on importance in this chapter because it can be used to test models of gases. It should be made very clear that Pv = const. is an empirical rule, summarizing the data of many experiments.

You might want to give students a rough idea of the range of pressures it is possible to obtain in earthly laboratories—from 10^{-11} atmosphere or even less in the best vacuum pumps to 10^7 atmospheres or more in special high-pressure apparatus. The initial pressure produced by a hydrogen bomb is on the order of several billion atmospheres.

The Newtonian mechanics of collisions can be used to derive the relation between P and v for a collection of perfectly elastic, vanishingly small particles. It turns out that the product of pressure and volume is proportional to the total kinetic energy of the particles. This idealized model does not appear to match real gases, however, because Newtonian mechanics shows also that the kinetic energy, and therefore the product Pv, changes when the volume is changed.

It is essential that students can distinguish between the empirical relation Pv = const. (Boyle's law) and the hypothetical relation derived from a simplified kinetic model Pv = 2/3N(KE). The latter appears not to agree with the former, because some very simple analysis of moving a piston to compress a sample of gas shows that KE does not remain constant.

We can claim that the two agree only if we can show why KE should remain constant. The problem is resolved when the 'constant KE" of the hypothetical relation is claimed to be equivalent to the "constant temperature" of the empirical relation. This solution might be spotted by students, many of whom have long been exposed to the identification of molecular motion and temperature, especially if the conditions are emphasized in your presentation: "Pv = const. if the temperature stays constant." "Pv = 2/3N(KE) if the KE of the particles stays constant."

The statements about absolute zero are correct but they fail to make explicit two qualifications which you may wish to present briefly to students. The first is sometimes called the third law of thermodynamics: it is impossible for any process to reduce the temperature of a system to absolute zero in a finite number of steps. The approach to absolute zero becomes progressively more difficult as the temperature nears zero, so that each successive step becomes smaller. Classically then, absolute zero may be approached as closely as desired, but can never be reached exactly. Another qualification is the result of quantum mechanics that a system has a finite least energy, the "zero-point energy;" thus even at a theoretical temperature of absolute zero, the particles of the system would not have zero kinetic energy.

Photographing Standing Waves

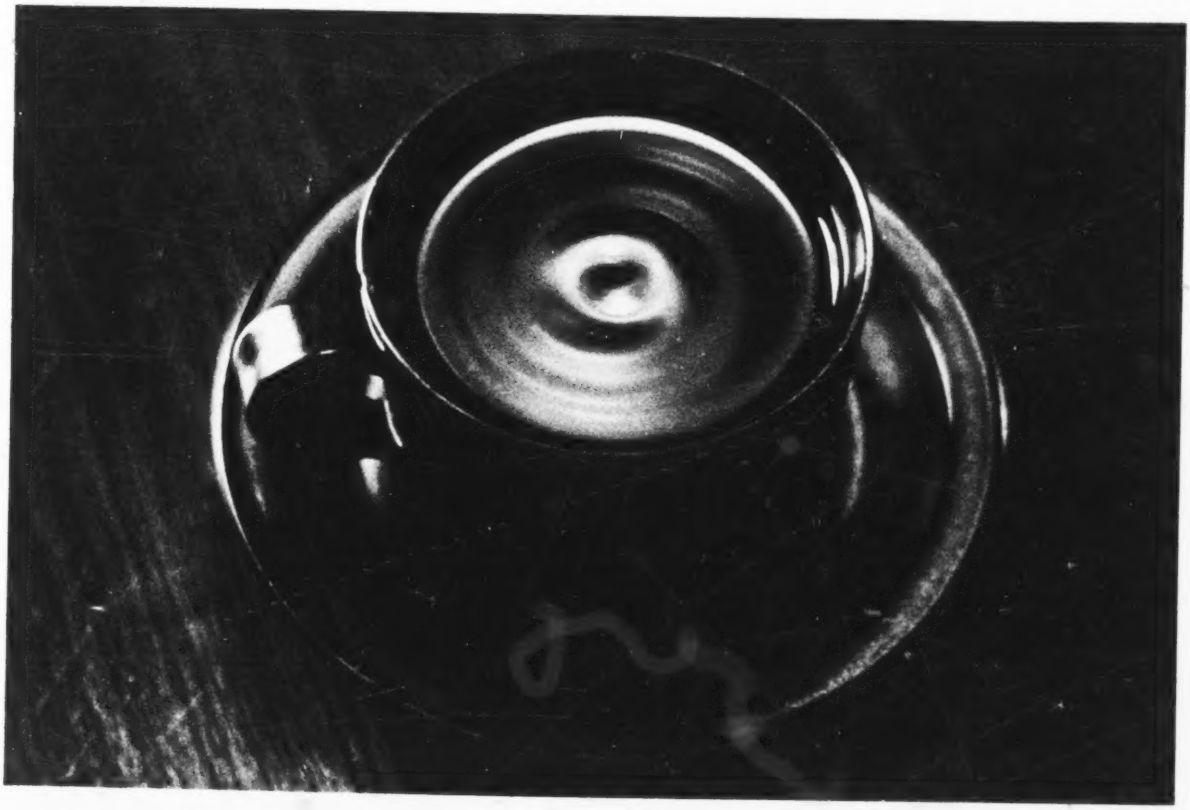
1. Put a dark liquid (like coffee or cocoa) in a dark container (like a dark coffee cup). The dark liquid reflects light well. And the dark container keeps visual distractions to a minimum.

2. Set the container on a slightly unstable platform (like a wobbly table). The standing waves will be set up by striking the platform immediately before releasing the shutter.

3. Arrange the light source (almost any light source will do), the container, and the camera (mounted on a tripod) so that light is reflected from the liquid's surface into the lens. This will be facilitated by focusing the lens not on the surface but on the light source reflected in the surface. Once the light is seen clearly in the liquid, the alignment is correct.

4. Read the light reflected from the surface with a light meter to determine the proper exposure. The meter should be placed close to the surface and along the lens axis.

5. Make a series of exposures at shutter speeds between 1/80 and 1/20 second, striking the platform before each exposure. The best picture can then be chosen from the series. (Remember to readjust the lens aperture when changing the shutter speed. Closing the lens one full stop cuts the light exactly in half.)



Bibliography for Unit 3

KEY:

(T) Recommended for teacher background
(S) Student supplementary material
(T,S) Teacher should read first
(S,T) Student can read, teacher would find useful
* Highly recommended
** Essential

Good texts for general use in classical particle physics, heat and conservation laws are: Kemble; Rogers.

(T,S) Abro, A. D' — The Rise of the New Physics: Its Mathematical and Physical Theories Vol I. Dover paperback $2.00 (formerly: Decline of Mechanism, Van Nostrand, 1938)
A sophisticated but not too technical survey of classical mechanics, thermodynamics and kinetic theory.

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(T) Ashton, T. S. — The Industrial Revolution 1760-1930 Oxford Univ. Press, Galaxy Paperback $1.00
Partly a defense of the Industrial Revolution against left-wing attacks, but also presents much fascinating information about the historical interactions between economic factors and technology. Would be recommended for students but for high frequency of unfamiliar (to Americans) words.

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One of the Science Study Series which discusses wave motion in the ocean, the biggest "ripple tank."

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See page 32—Boltzmann's suicide due to rejection of atomism by scientists, page 37—atomic theory's recent successes show fallacy of positivist criticism of hypotheses.

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The physics of sound. Physiological aspects of human hearing are included as well as a subjective treatment of music.

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Background for mass conservation, theological view of "matter and motion" (page 185—Charles II's dig at Royal Society for weighing air).

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(T)	Brush, S. G.	"The development of the kinetic theory of gases," Annals of Science 13 188, 273 (1957), 14, 185, 243 (1958); Amer. J. Phys. 29, 593 (1961), 30, 269 (1962) Historical background for Chapter 12.
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(T)	Brush, S. G.	Kinetic Theory: Vol. I. The Nature of Gases and of Heat Pergamon Press, Oxford (1965) Reprints of original works of Boyle, Newton, Daniel Bernoulli, Helmholtz, Joule, Mayer, Clausius, Maxwell, with historical introduction (background for energy conservation and Chapter 12). Volume II includes more technical papers, but also some of the Boltzmann-Zermelo, etc. discussions on irreversibility mentioned at end of Chapter 12.
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#2 The overthrow of the phlogiston theory: the chemical revolution of 1775-89
#3 The early development of the concepts of temperature and heat: the rise and decline of the caloric theory
#4 The atomic-molecular theory (mainly chemistry)

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(T) Crew, Henry The Rise of Modern Physics Williams & Wilkins Co (1935)
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Historical background for the Newtonian cosmology

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Background for Chapter 10

(T) Dugas, R. La Théorie Physique au Sens de Boltzmann et Ses Prolongements Modernes Editions du Griffon Neuchatel-Suisse (1959)
Boltzmann's defense of the Newtonian cosmology; its relevance for twentieth-century physics (background for Chapter 12)

(T) Duhem, P. The Aim and Structure of Physical Theory (translated from French) Princeton Univ. Press (1954)
See especially page 80ff for criticism of English physics based on mechanical models. Duhem was one of the positivist (??) critics of nineteenth-century classical physics

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Recommended for school libraries. Note chapters on measurement of time, scientific instruments, exploitation of electrical energy, "The Age of Steam," turbines, internal combustion engines, nuclear reactions, electronic age, etc.

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Survey articles on energy in modern physics and technology; good for Units 3 through 6.

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(T,S)	Mach, Ernst	Popular Scientific Lectures (translated T. J. McCormack) Open Court Pub. Co., LaSalle, Ill., 5th edition Paperback $1.40 (1943) See especially "On the Principle of the Conservation of Energy" P. 137; and "On the Economical Nature of Physical Enquiry"
(T,S)	Magie, W. F. (ed.)	*A Source Book in Physics Harvard Univ. Press (1965) Descartes, Leibniz and D'Alembert on quantity of motion; Young on energy; Galileo on rise of water in a pump; Torricelli on the barometer; Pascal on barometer experiments and fluid pressure; Boyle on PV relations of air (but note that Magie cuts off just before the paragraph in which Boyle gives Towneley the credit for suggesting the hypothesis!). Also, Newton's scale of degrees of heat; Amonton's air-pressure thermometer; Fahrenheit temperature scale; Black on specific and latent heats; Rumford and Davy on frictional heat; Gay-Lussac, Joule, Thomson on expansion of gases; Mayer and Helmholtz on conservation of energy; Joule on mechanical equivalent of heat; Carnot and Clausius on second law; Kelvin temperature scale; Bernoulli, Joule, Maxwell and Boltzmann on kinetic theory and probabilities
(S)	Magnus, Rudolf	Goethe as a Scientist (translated from German, originally published 1906) Collier Books, NY 95¢ Mainly about his work in biology, but includes chapters on his theory of color
(T,S)	Matson, F. W.	The Broken Image: Man, Science and Society Braziller, NY (1964) A personal viewpoint on the implications of Newtonian and evolutionary cosmologies. "The mechanization of man," etc.
(T,S)	McClintock, Michael	Cryogenics Reinhold, NY (1964) $10.75 This book describes various phenomena that occur in matter at low temperatures and also gives applications of cryogenics
(T)	McKie, Douglas	Antoine Lavoisier: The Father of Modern Chemistry Lippincott, Philadelphia (1935) History of conservation of mass
(T)	Merz, J. T.	A History of European Thought in the Nineteenth Century 4 vols. Dover $2.75 per volume The first two volumes concentrate on science

(T)	Millikan, R. A.	Mechanics, Molecular Physics, Heat and Sound Ginn & Co. (1937) Reprinted by MIT Press Paperback $3.45 (1965) According to the publisher, "This classic text has probably been used at one time by virtually every leading physicist of today... the chief value of this work lies in its presentation of physics in an historical context..."
(S,T)	Mott-Smith, M.	Heat and Its Workings Dover Paperback $1.00 (1962) Good account of temperature, gas properties, latent heat, change of state, triple point, convection, conduction, etc., but not as relevant to our purposes as his other book, The Story of Energy
(T,S)	Mott-Smith, M.	*The Story of Energy Appleton-Century (1934); Dover Paperback Reprint (revised edition 1964) Excellent survey of history and present knowledge of steam engines, thermodynamics—good background for Chapter 11
(T)	Moulton Schifferes	The Autobiography of Science Pages 218-220 on latent heat, suggested by a teacher
(T)	Needham, J.	Science and Civilization in China Vol. 3 Mathematics and the Sciences of the Heavens and the Earth Cambridge Univ. Press (1959) See page 154ff for comparison of social and other factors influencing development of science in Europe and China
(T,S)	Nef, John U.	*Cultural Foundations of Industrial Civilization Cambridge Univ. Press (1958); Harper Torchbook (1960) Very good historical background for Units 2 and 3
(T)	Newton, I.	Opticks Dover Paperback $2.25 See especially the queries at the end of the book for Newton's opinions about the nature of matter. The difference between the Boyle and Newton models for gas pressure (which Chapter 12 treats as if equivalent) is indicated by the remarks on pages 395-6, where Newton argues the need for repulsive forces and says that the contraction and expansion of air "seems unintelligible, by feigning the Particles of Air to be springy and ramous..."
(T)	Partington, J. R.	A History of Chemistry St. Martin's Press, NY Vol. IV (1964) Comprehensive reference work
(T,S)	Partington, J. R.	A Short History of Chemistry Harper Torchbook $1.95
(T)	Partington, J. R. McKie, D.	"Historical studies on the Phlogiston Theory" Annals of Science 2, 361 (1937); 3, 1, 337 (1938) This is related to history of conservation of mass
(T)	Partington, J. R.	"Joan Baptista van Helmont" Annals of Science 1, 359 (1936) includes history of conservation of mass
(T)	Pendse, C. G.	"A Note on the Definition and Determination of Mass in Newtonian Mechanics" Philosophical Magazine 24, 1012 (1937); 27, 51 (1939) "On Mass and Force in Newtonian Mechanics" Philosophical Magazine 29, 477 (1940) Discussions of the limitations of Mach's procedure for defining mass: an observer can't determine mass ratios if there are more than seven particles in the system
(S,T)	Planck, Max	*A Scientific Autobiography and Other Papers Williams and Norgate, London (1950) (American paperback reprint may be available) Shows very clearly the connection between some ideas of Units 3 and 5
(T)	Power, Henry	Experimental Philosophy 1664. To be reprinted in the Sources of Science Series (Johnson Reprint Corp.)
(T)	Randall, J. H.	The Making of the Modern Mind: A Survey of the Intellectual Background of the Present Age Houghton Mifflin Co., Boston (revised edition 1940) Good historical background on the Aristotelian and Newtonian cosmologies, relations between science and philosophy. See also chapters on reaction against science and mechanism; evolutionary cosmology

(T)	Rankine, O.	"On the General Theory of Thermodynamics" in Reynolds' Papers on Mathematical and Physical Subjects Cambridge Univ. Press Vol. II, P. 138 (1901) Mechanical illustrations of second law of thermodynamics (heuristic demonstrations)
(T,S)	Rogers, Eric	*Physics for the Inquiring Mind Princeton Univ. Press (1960) See Part 3: Molecules and Energy; also Chapter 8 on Momentum
(T)	Rosen, George	"The Conservation of Energy and the Study of Metabolism" pages 243-263 in The Historical Development of Physiological Thought (edited C. M. Brooks and P. F. Cranefield) Hafner, NY (1959) Connections between physics and biology (Chapter 11 of Unit 3)
(S)	Ruchlis, Hy	The Wonder of Heat Energy: A Picture Story of the Vital Part Heat Plays in Our World Harper, NY (1961)
(T,S)	Sandfort, J. F.	*Heat Engines: Thermodynamics in Theory and Practice Science Study Series, Doubleday/Anchor 95¢ (1962) Semi-historical, semi-descriptive: very good for teachers and boys interested in engineering applications of physics
(T)	Schanck, R. L.	The Permanent Revolution in Science Philosophical Library, NY (1954) Transition from Newtonian to evolutionary cosmology
(T)	Scherer, F. M.	"Invention and Innovation in the Watt-Boulton Steam-Engine Venture" Technology and Culture 6, 165 (1965) See text and notes on pages 167-8 for discussion of Watt's models of separate condenser
(T,S)	Schofield, R. E.	The Lunar Society of Birmingham: A Social History of Provincial Science and Industry in Eighteenth-Century England Clarendon Press, Oxford (1963) Background for Chapter 11
(T,S)	Sears and Zemansky	University Physics Section on momentum, suggested by a teacher
(T)	Singer, C. Holmyard, E. J. Hall, A. R. Williams, T. I.	A History of Technology Oxford Univ. Press, 5 volumes; the last one published in 1958 on "late nineteenth century" Valuable reference book for details and illustrations, but not suitable for schools unless you can get it at a bargain from one of the book clubs. Suffers from lack of coordination between articles written by different authors—no sense of overall perspective
(T,S)	Shamos, H. H. (ed.)	Great Experiments in Physics Holt, Rinehart and Winston, NY $6.35; text edition $4.75 (1959) The experiments of classical physics and the physicists who performed them
(T,S)	Shapiro, A. H.	Shape and Flow: The Fluid Dynamics of Drag (Science Study Series), Garden City, Doubleday Paperback $1.25 (1961) Some very interesting experiments and applications of fluid dynamics.
(S,T)	Sporn, Philip	Energy: Its Production, Conversion and Use in the Service of Man Pergamon Press (1963) Brief survey of history, present use and future prospects
(T)	Stallo, J. B.	The Concepts and Theories of Modern Physics Appleton, NY (1882) Reprinted recently Penetrating critique of Newtonian physics, including atomic and kinetic theory—provides good ammunition for debate on existence of atoms
(T)	Tabor, D.	"Large-scale Properties of Matter" Contemporary Physics 6, 112 (1964) Suggestions for teaching about gas, liquid and solid states

(T)	Taton, R. (ed.)	Science in the Nineteenth Century (History of Science, Vol. 3) Basic Books, NY (1965) See earlier volumes. General background
(T)	Thurston, R. H.	A History of the Growth of the Steam Engine Cornell Univ. Press (1939)
(T,S)	Toulmin, S. Goodfield, J.	*The Architecture of Matter (Ancestry of Science #2) Hutchinson, London (1962) Background for the entire unit; other volumes in this series are also recommended
(T)	von Laue, M.	History of Physics Academic Press, NY (1950)
(T)	Whitehead, A. N.	The Concept of Nature Cambridge Univ. Press (1920) Critique of "matter and motion" idea
(T)	Whitehead, A. N.	*Science and the Modern World Mentor Paperback 35¢ Tough reading, but well worth it. Transition from Newtonian to evolutionary cosmology
(S,T)	Wilson, Mitchell and Editors of Life	Energy (Life Science Library) NY, Time, Inc. Book Division $3.95 (1963) (order from Silver Burdett Company) An excellent source of pictures and illustrations in color to show to students of physics
(T)	Wolf, A.	A History of Science, Technology, and Philosophy in the Eighteenth Century 2 vols., Harper Torchbook $2.50 each. See also book with similar title for sixteenth and seventeenth centuries. Good reference book

Momentum and Energy Articles

Scientific American

1966	June	River Meanders (Leopold and Langbien) A river develops those lazy curves because that is the way it does the least work.

Physics Today

1964	March	Symmetry and Conservation Laws (Wigner)

American Journal of Physics

1964	March	A Versatile Ballistic Pendulum (R. W. Barton) (Kinetic Energy)
1964	April	Newton's Laws of Motion and the Seventeenth Century Laws of Impact (Arons and Bork)
1964	Nov	A Laboratory Demonstration of the Equivalence of Potential Energy and Work (O. H. Zinke)
1966	June	Additivity of Mass in Newtonian Mechanics

Physics Teacher

1963	Oct	Momentum and Energy (Carpenter)
1964	Jan	Momentum (R. L. Teat)
1964	April	Momentum and Energy (F. Robillard)
1964	May	"The Falling Monkey" Problem (H. R. Cohen)
1964	Sept	Proof of Archimedes' Principle (V. M. Rudiac) potential energy
1964	Oct	Falling Bodies (Bray)
1965	Jan	Squeaking Chalk (Gale)
1965	Sept	More Chalk Talk (Gerhard)
1965	Oct	Account for the Lost Kinetic Energy
1965	Oct	Skidding Automobile
1966	Jan	Lost Kinetic Energy (Little Stinkers)
1966	Jan	Conservation of Momentum and Energy: Collision of Particles (T. Toepker)
1966	Jan	Skidding Automobile (Little Stinkers)
1966	Jan	Friction (H. H. Gottlieb)
1966	Feb	A Whipped Cream Pendulum (Leonard)
1966	Feb	Some Laboratory Work with Pendulums (Horton)
1966	Feb	Functional Dependence of the Elongation of a Coil Spring on the Load Applied (Sears)
1967	March	Momentum Transfer Demonstration (H. Kruglak and H. M. Waage)

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Engines

Scientific American

1964	Jan	Origins of the Steam Engine (Ferguson) Thomas Newcomen built a practical engine 50 years before the time of James Watt.

American Journal of Physics

1966	Oct	Carnot Cycle Diagrams (Kalantar)

Physics Teacher

1966	March	Reciprocating Engine (Kolb)

GAS

Physics Teacher

1966	April	Heat Required to Expand a Gas
1967	May	Simple Introduction to the Kinetic Theory of Gases (G. R. Henry)

Suggested Answers to Unit 3 Tests

Test A

ITEM	ANSWER	SECTION OF UNIT	PROPORTION ANSWERING ITEM CORRECTLY
1	C	9.2	0.66
2	C	9.3	0.72
3	E	12.4	0.78
4	D	12.3	0.70
5	E	9.3, 10.1, 10.3	0.69
6	B	10.7	0.60
7	E	11.6	0.32
8	C	11.3	0.46
9	C	10.3	0.55
10	D	12.4	0.50
11	A	12.4	0.82
12	C	11.6	0.74
13	B	11.3	0.74
14	D	10.3	0.57
15	E	10.3	0.49

Answers Test A

Group I

1. Section of Unit: 12.3

Since the speed of the wave cannot change, the only change is a decrease in wavelength.

2. Section of Unit: 9.2

a) $p = mv$

$= (0.14 \text{ kg})(30 \frac{\text{m}}{\text{sec}})$

$= 4.2 \text{ kg} - \frac{\text{m}}{\text{sec}}$

b) $p = (2.0 \times 10^{-3} \text{ kg})(300 \frac{\text{m}}{\text{sec}})$

$= .6 \text{ kg} - \frac{\text{m}}{\text{sec}}$

3. Section of Unit: 11.2

A gas consists of a large number of very small particles (molecules) which move in rapid disordered motion.

Forces between these particles act only at very short distances.

Collisions among these particles are perfectly elastic.

4. Section of Unit: 10.6

$\text{Power} = \frac{\text{work}}{\text{time}}$

$= \frac{Fd}{t} = \frac{(100 \text{ N})(10\text{M})}{50 \text{ sec}}$

$= 20 \text{ watts}$

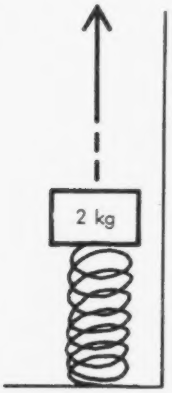
5. Section of Unit: 12.4

If the distances $PS_1 - PS_2 = (n + \frac{1}{2})\lambda$, P will lie on a nodal line.

If the distance $PS_1 - PS_2 = n\lambda$, P will lie on an antinodal line.

Group II

6. Section of Unit: 10.3



Since we assume that mechanical energy is being conserved, the potential energy of the 2-kg mass at the peak of its trajectory must equal the potential energy originally stored in the spring.

$$10 \text{ joule} = ma_g h$$

$$h = \frac{10 \text{ kg-}\frac{m^2}{sec^2}}{(2 \text{ kg})(10 \frac{m}{sec^2})} = .5 \text{ m}$$

7. Section of Unit: 9.4

$$\vec{F} = m\vec{a}$$
$$= m\,\frac{\Delta\vec{v}}{\Delta t}$$

If the mass of the body is constant,

$m\Delta\vec{v} = \Delta(m\vec{v}) = \Delta\vec{p}$.

Therefore

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}.$$

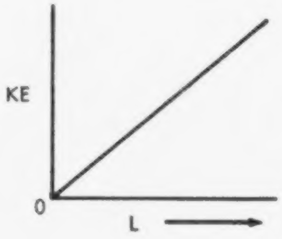
Suggested Answers to Unit 3 Tests

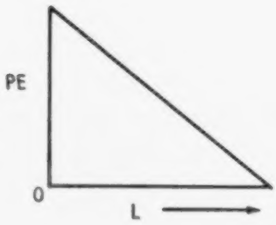
Test B

ITEM	ANSWER	SECTION OF UNIT
1	B	9.2
2	D	11.1
3	A	10.10
4	C	10.3, 9.6
5	A	10.3, 9.6
6	C	9.6
7	A	9.6
8	B	12.4
9	A	11.3
10	D	11.6
11	B	12.8
12	A	general
13	A	11.6
14	C	12.7
15	C	12.5

Group I

1. Section of Unit: 10.3





2. Section of Unit: 11.6

An infinitely long list of physical phenomena are explained by the second law of thermodynamics. The student should pick one, and then describe its connection with the second law.

For example: A cold spoon is dropped into hot water. Soon both the spoon and water will be at the same middle temperature.

This situation is described by the second law in that

a) the process is not reversible—the spoon and water will not return to their previous temperatures, and

b) the order of the process is from order—two distinct distributions of molecular kinetic energies, to disorder—one distribution of molecular kinetic energies, i.e., the entropy of the system has increased.

3. Section of Unit: 9.2-3

Although Alouette's speed is almost constant, its velocity—a vector—is not. Since Alouette's path is circular, the direction of its velocity is always changing. In fact, the direction of this change is always towards the center of the circular orbit.

Therefore, since momentum ($\vec{p}$) = $m\vec{v}$, Alouette's momentum cannot be constant.

4. Section of Unit: 9.1

a) The mass measured before and after the reactions were always the same.

b) These results lead to the conclusion that mass is conserved in a closed system.

5. Section of Unit: 11.2

The assumption of disordered or random motion of gas molecules allows only a consideration of "net" effects, the average effect of the motions of large numbers of molecules. This statistical treatment leads to a description of the properties of the gas as a whole rather than a description of the motions of the individual constituent particles.

Answers Test B

Group II

6. Section of Unit: 9.1

This question may be dealt with in many different ways. A student may answer yes or no and explain his answer in terms of

a) the points of view which motivated the discovery of the law of conservation of momentum.

b) a discussion of deterministic versus non-deterministic points of view.

c) a discussion of whether the universe may be viewed as a closed system in the same way as the isolated collision of two billiard balls.

d) questioning the applicability of our physical laws throughout the universe.

7. Sections of Unit: 9, 10

A student may indicate belief or disbelief in this story. In either case, however, he should justify his decision

a) with a qualitative discussion of the nature of the impact, the meaning of the word abrupt, the phrase "straight down," and the applicability of the laws of conservation of momentum and conservation of energy.

b) with a quantitative approach, assigning realistic numerical values to the respective masses and speeds of the bullet and the lion. He should indicate that the collision is unelastic, and discuss the applicability of the laws of conservation of momentum and conservation of energy.

Suggested Answers to Unit 3 Tests

Test C

ITEM	ANSWER	SECTION OF UNIT	PROPORTION ANSWERING ITEM CORRECTLY	ITEM	ANSWER	SECTION OF UNIT	PROPORTION ANSWERING ITEM CORRECTLY
1	A	9.2	0.81	21	B	10.7	0.61
2	C	10.3	0.51	22	E	11.6	0.39
3	E	9.5	0.33	23	C	11.3	0.38
4	C	9.6	0.86	24	C	10.3	0.95
5	B	12.2	0.68	25	D	10.10	——
6	B	9.6	0.67	26	E	10.1	0.69
7	D	11.1	0.58	27	A	10.10	0.73
8	B	11.3	0.74	28	D	12.4	0.60
9	A	11.6	0.58	29	A	12.4	0.78
10	D	9.2	0.41	30	C	10.3, 9.6	0.67
11	E	10.3	0.41	31	A	10.3, 9.6	0.79
12	B	10.1	0.50	32	C	9.6	0.40
13	B	12.7	0.57	33	A	9.6	0.46
14	A	12.5	0.55	34	A	12.5	——
15	D	12.4	0.71	35	C	11.6	0.60
16	A	9.3	0.53	36	A	12.8	0.74
17	B	9.6	0.68	37	E	general	0.72
18	A	10.3	0.40	38	A	11.3	0.66
19	B	12.2	0.73	39	D	10.6	0.42
20	B	9.2	0.60	40	C	9.1	0.75

Suggested Answers to Unit 3 Tests

Test D

Group I

1. Section of Unit: 12.3

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

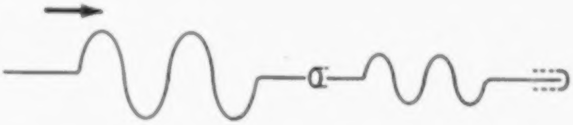
$$= \frac{340 \ \frac{\text{m}}{\text{sec}}}{440 \ \frac{1}{\text{sec}}}$$

$$= .77 \text{ m}$$

2. Section of Unit: 9.1

The volume of a substance may change, without there being an accompanying change in the amount of substance. For example: if water is put in a container, the water level marked and the water then frozen, we find that the volume of the ice is larger than the volume of the water we started with.

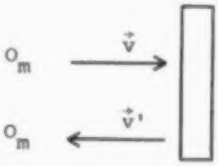
3. Section of Unit: 12.7



The pulses will be traveling at lower speed, with shorter λ and with smaller amplitude.

4. Section of Unit: 9.3

The ball's momentum is not the same before and after the collision.



M = mass of wall (and earth)

Momentum is conserved, therefore

$$\underbrace{m\vec{v}}_{\substack{\text{momentum of ball}\\ \text{before collision}}} = \underbrace{m\vec{v}\,'}_{\substack{\text{momentum of ball}\\ \text{after collision}}} + \underbrace{M\vec{V}}_{\substack{\text{momentum of wall (and earth)}\\ \text{due to collision}}}$$

The ball's kinetic energy is not the same before and after the collision. Since the ball is soft, the collision was not perfectly elastic. The energy was used to deform the ball during the collision—this energy, stored in the ball like energy stored in a compressed spring, was not completely returned when the ball was restored to its original shape. This lost energy reappeared as heat. In addition, a very small amount of energy is transferred to the wall, in the form of mechanical kinetic energy.

$$(\frac{MV^2}{2}, \quad V \approx 0)$$

5. Section of Unit: 11.2

A gas expanding while pushing a piston does work in pushing the piston, decreasing its internal energy. The gas expanding in a vacuum does no work; its internal energy remains the same.

Group II

6. Section of Unit: 9.3

The system is isolated, therefore the total momentum of the system $\vec{P}_{total}$ must be constant throughout.

a) Initially the whole system is at rest. Therefore

$$\vec{P}_{total} = 0.$$

b) While the box is in the air

$$\vec{P}_{box} + \vec{P}_{boat + men} = \vec{P}_{total} = 0.$$

Therefore the boat moves in the opposite direction to the box.

c) After the box is caught, $\vec{P}_{box} = 0$,

and since $\vec{P}_{total} = 0$,

therefore $\vec{P}_{boat + men} = 0$, and the boat is at rest.

7. Section of Unit: 10.8

The engineering analysis of the steam engine led to several new important discoveries in physics. A student in responding to this question may select any one of the following list of discoveries, which is by no means inclusive, and explain its link to the steam engine and its importance to the study of physics.

a) the nature of heat—J.P. Joule's contribution to physics

b) the discovery of the law of conservation of energy

c) energy conversion—Carnot's heat engine, the second law of thermodynamics

d) energy conversion—efficiency, power, etc.

8. Section of Unit: 10.10

A satisfactory answer to this question should include

a) a brief description of nature and of science as viewed by nature-philosophy.

b) an explanation of the link with the law of conservation of energy. For example:

  i) nature-philosophy searches for the underlying reality of nature—a sweeping generalization to fully describe nature.

  ii) nature-philosophy states that the various forces of nature—gravity, electricity, magnetism, etc.—are not really separate from one another, but are simply different manifestations of one basic force.

c) a statement that by encouraging scientists to look for connections between different forms of energy, nature-philosophy stimulated the experiments and theories that led to the law of conservation of energy.

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